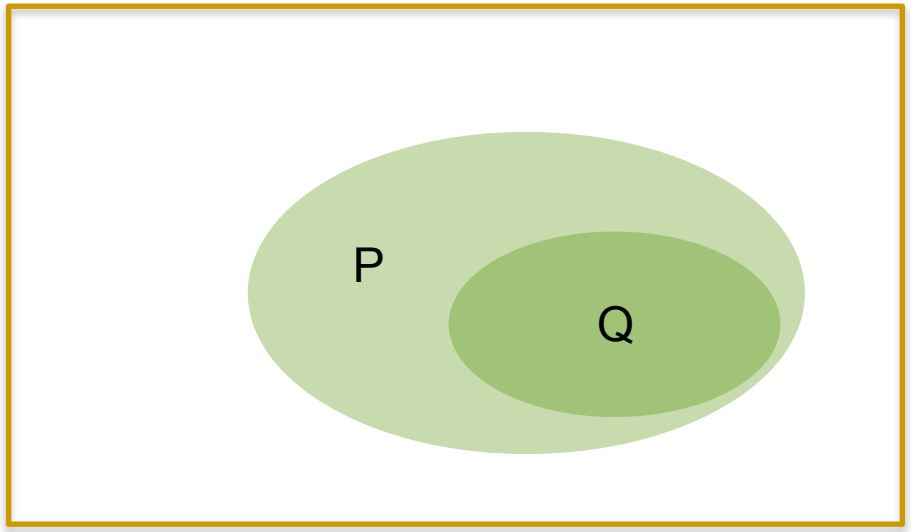
Interlude - weak and strong statements

- What does weak and strong mean in this context?
- When is a statement P weaker than a statement Q?
  - if Q implies P
  - set interpretation: Q is a subset of P



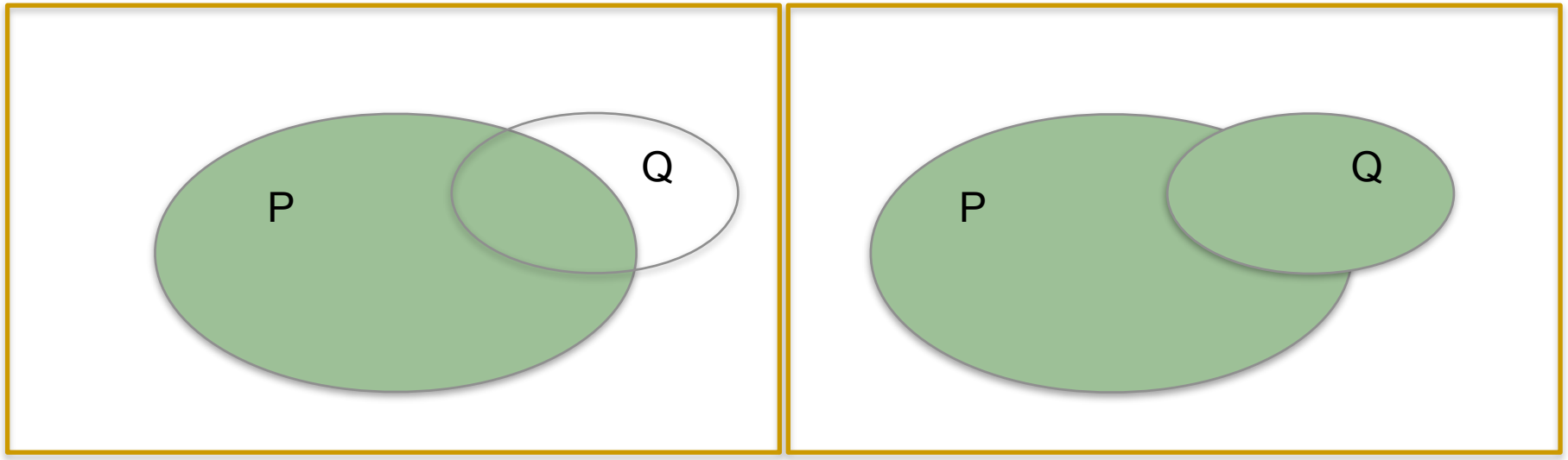
- ‘False’ corresponds to the empty set, so strongest possible statement
- ‘True’ to the whole universe, so weakest

Interlude - weak and strong statements

- But how do operators affect this?

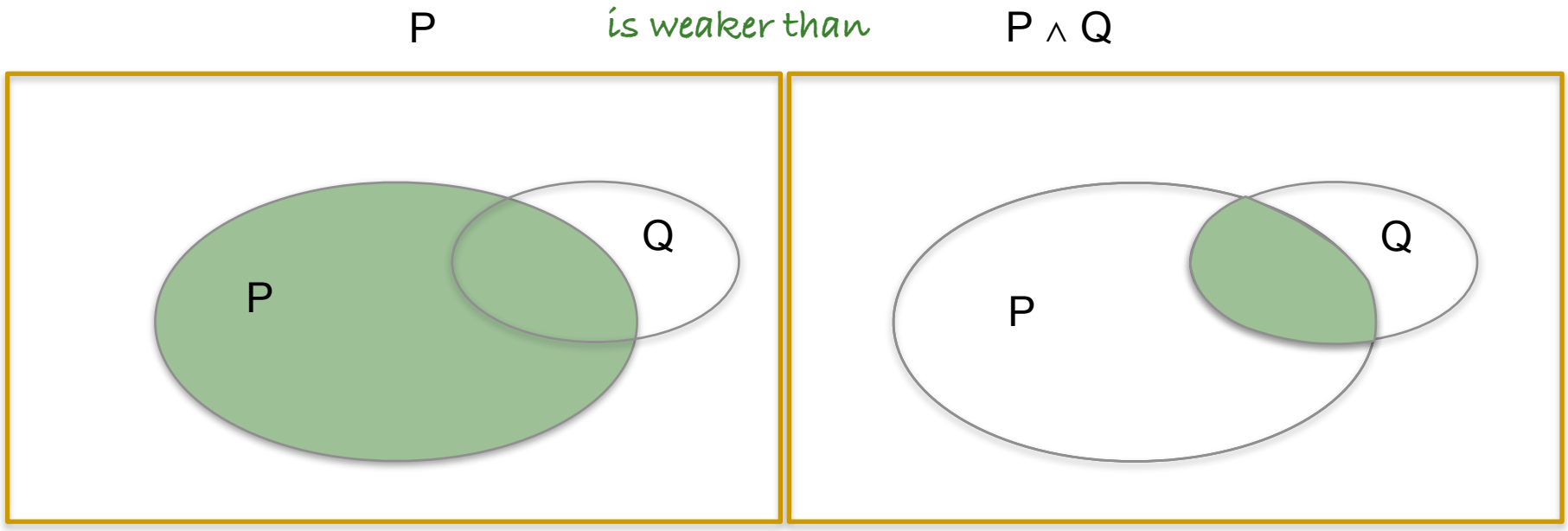
- E.g., is P weaker or stronger than P ∨ Q, P ∧ Q for an arbitrary Q?

P is stronger than P ∨ Q

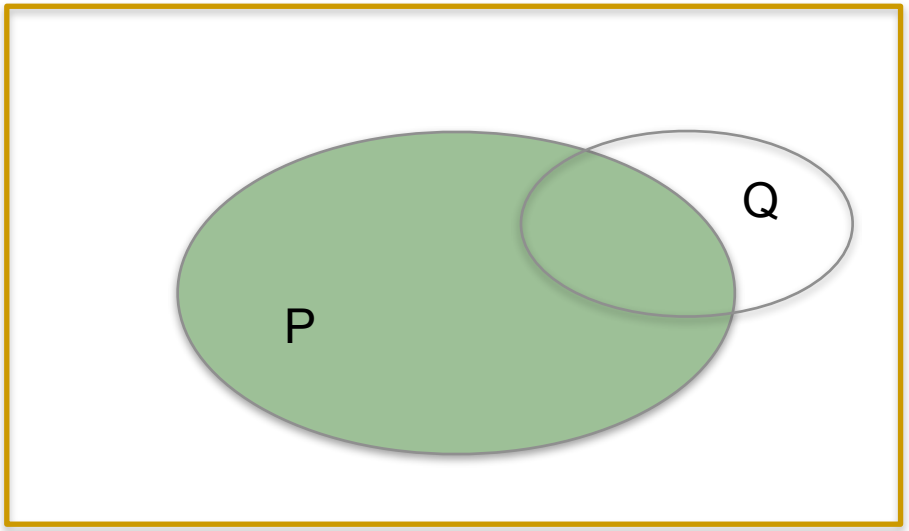


Interlude - weak and strong statements

- That's easy, but how do operators affect this?
- E.g., is P weaker or stronger than P ∨ Q, P ∧ Q?

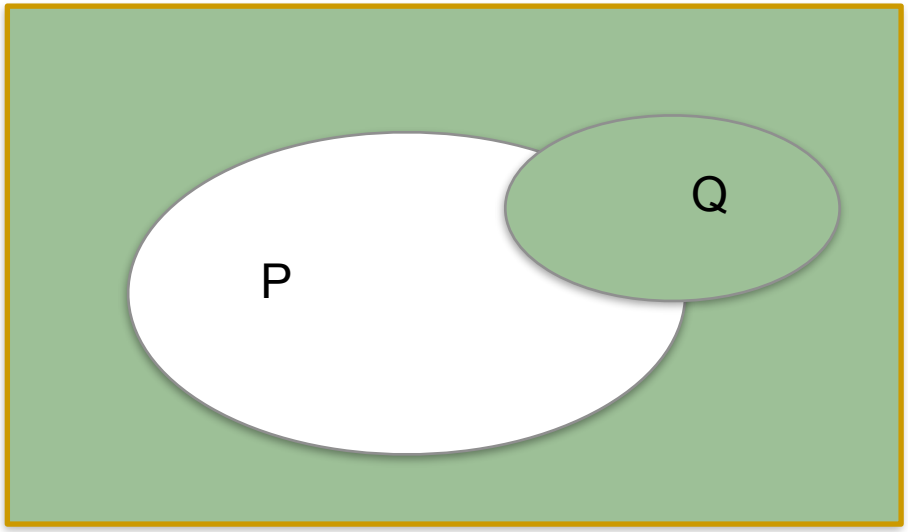


Interlude - weak and strong statements



P

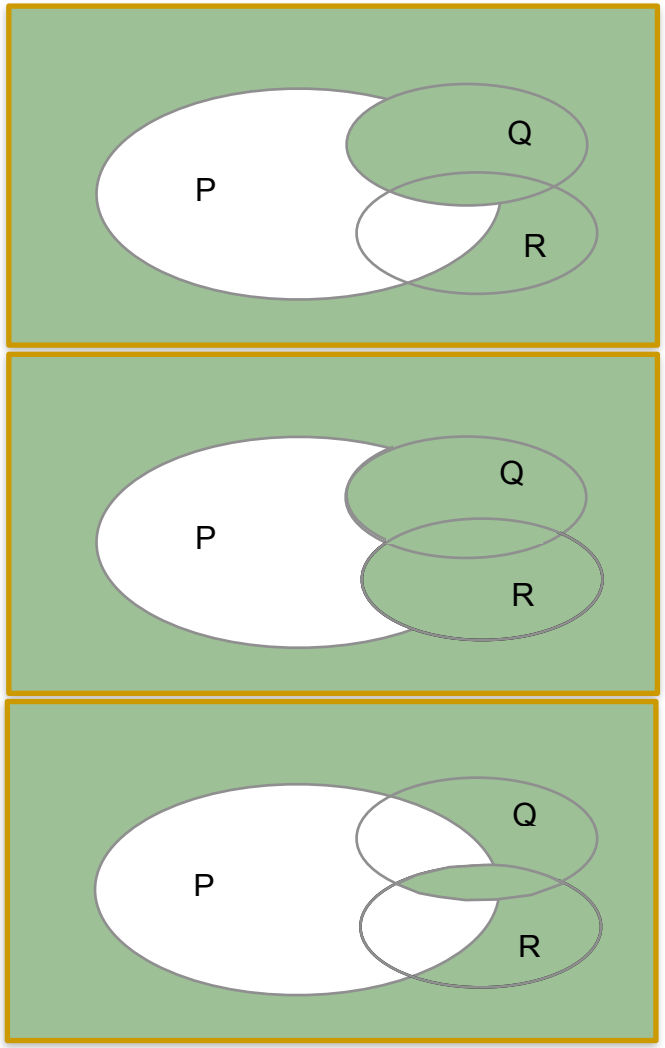
neither stronger nor weaker than



$P \Rightarrow Q$

Q is stronger than $P \Rightarrow Q$

Interlude - weak and strong statements



$P \Rightarrow Q$

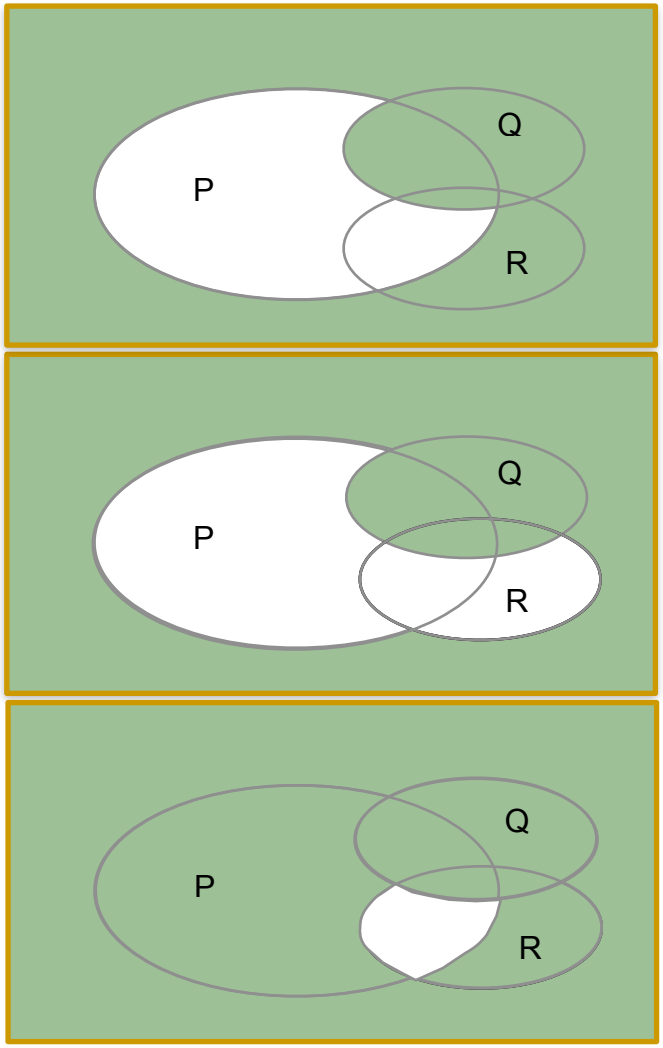
is stronger than

$P \Rightarrow Q \vee R$

and both are weaker than

$P \Rightarrow Q \wedge R$

Interlude - weak and strong statements



$P \Rightarrow Q$

is weaker than

$P \vee R \Rightarrow Q$

and both are stronger than

$P \wedge R \Rightarrow Q$

Interlude - weak and strong statements

- Hoare triples are a kind of implication:
  - {* P *} S {* Q *}:
    - **if** P holds before the execution of S, **then** Q holds after S is executed and terminates (partial correctness)
    - **if** P holds before the execution of S, **then** S terminates and Q holds after S is executed (total correctness)
- Therefore, the same rules hold wrt strengthening/weakening the conditions
  - if {* P *} S {* Q *} and
    - R ⇒ P, then {* R *} S {* Q *} (strengthen the pre-condition)
    - Q ⇒ R, then {* P *} S {* R *} (weaken the post-condition)