Universiteit Utrecht

[Faculty of **Science**
Information and Computing Sciences]

Talen en Compilers

2023 - 2024

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2023-12-14

9. Simple stack machine

Midterm Summary lecture

- Tuesday
- Short summary on contents
- Send requests for demo-ing old questions!

Recap: Semantic functions

In the previous lectures, we have seen how to evaluate (interpret) expressions.

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- We have added variables and talked about environments.
- We have added local definitions and talked about nesting and blocks.
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In the previous lectures, we have seen how to evaluate (interpret) expressions.

- We have added variables and talked about environments.
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- We have added (mutually) recursive definitions and talked about scoping.

Now we are going to generate code in a low-level language instead of interpreting the expression directly.

This lecture

Simple stack machine

- Architecture of the simple stack machine
- Instructions
- Translating programs
- Functions / methods

9.1 Architecture of the simple stack machine

Simple stack machine

A virtual machine that executes programs consisting of assembly language instructions.

Simple stack machine

A virtual machine that executes programs consisting of assembly language instructions.

- The program is a list of instructions with arguments, stored in a continuous block of memory.
- A **stack** is used to store the current state of execution.
- There are eight **registers**, four with a special name:
  - the **program counter** (PC)
  - the **stack pointer** (SP)
  - the **mark pointer** (MP)
  - the **return register** (RR)

Execution

- A step in the execution interprets the instruction pointed to by the program counter.
- Depending on the instruction, the contents of the stack and registers are modified.

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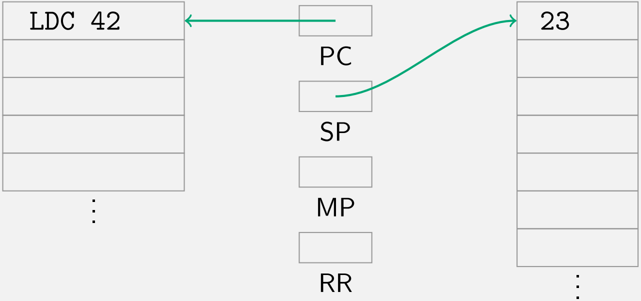
- Depending on the instruction, the contents of the stack and registers are modified.

Example: `LDC` (load constant)

$$\begin{aligned} \mathsf{SP}_{\mathsf{post}} &= \mathsf{SP}_{\mathsf{pre}} + 1 && \text{(increment stack pointer)} \\ \mathsf{M}_{\mathsf{post}}\,[\mathsf{SP}_{\mathsf{post}}] &= \mathsf{M}_{\mathsf{pre}}\,[\mathsf{PC}_{\mathsf{pre}} + 1] && \text{(place argument on stack)} \\ \mathsf{PC}_{\mathsf{post}} &= \mathsf{PC}_{\mathsf{pre}} + 2 && \text{(adjust program counter)} \end{aligned}$$

Visualizing the execution

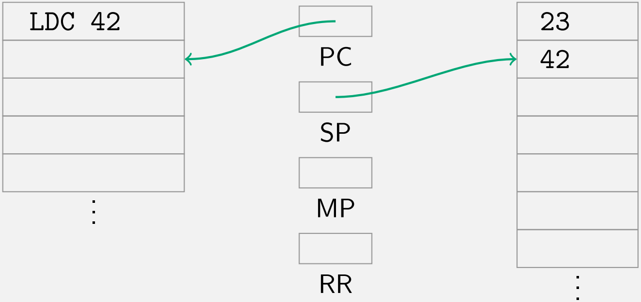
$$
\begin{aligned}
SP_{post} &= SP_{pre} + 1 && \text{(increment stack pointer)} \\
M_{post}[SP_{post}] &= M_{pre}[PC_{pre} + 1] && \text{(place argument on stack)} \\
PC_{post} &= PC_{pre} + 2 && \text{(adjust program counter)}
\end{aligned}
$$



The instruction `LDC 42` takes up two words in memory, but we write it in one cell.

Visualizing the execution

$$
\begin{aligned}
SP_{post} &= SP_{pre} + 1 && \text{(increment stack pointer)} \\
M_{post}\,[SP_{post}] &= M_{pre}\,[PC_{pre} + 1] && \text{(place argument on stack)} \\
PC_{post} &= PC_{pre} + 2 && \text{(adjust program counter)}
\end{aligned}
$$



The instruction `LDC 42` takes up two words in memory, but we write it in one cell.

9.2 Instructions

Instructions

Most instructions can be classified into the following groups:

- load instructions
- store instructions
- jump instructions
- arithmetic and logical operations

Load and store instructions

LDC – load constant

LDR – load from register

LDL – load local

LDS – load from stack

LDLA – load local address

LDA – load via address

STR – store to register

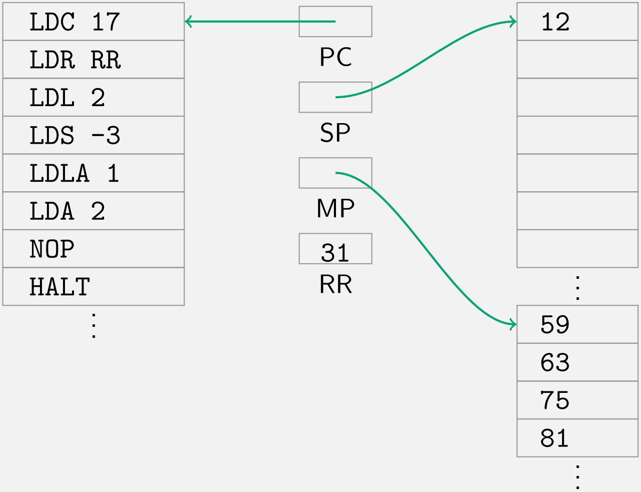
STL – store local

STS – store to stack

SDA – store via address

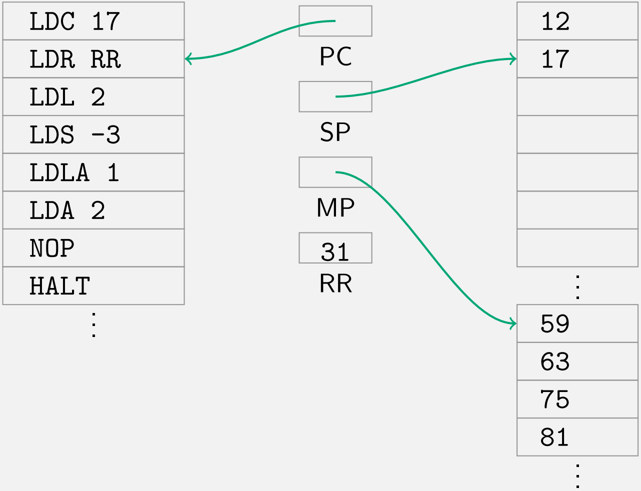
Load instructions

LDC – load constant



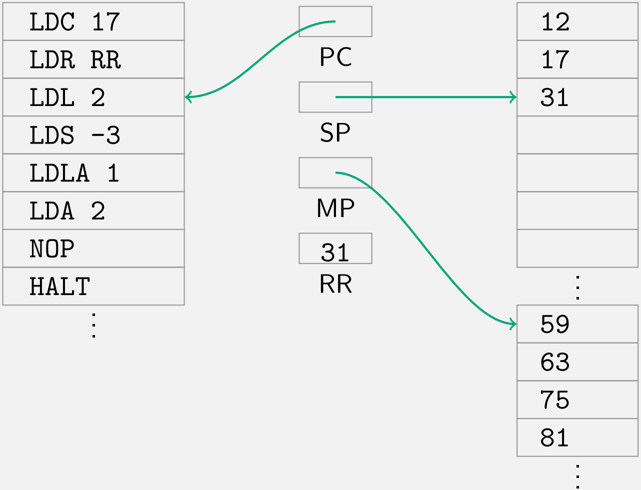
Load instructions

LDR – load from register



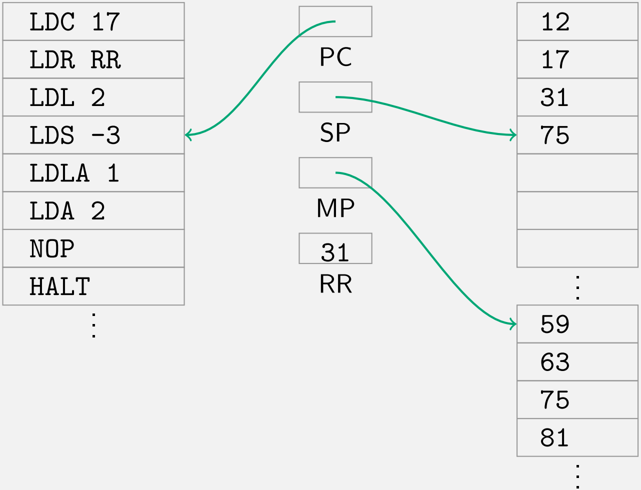
Load instructions

LDL – load local



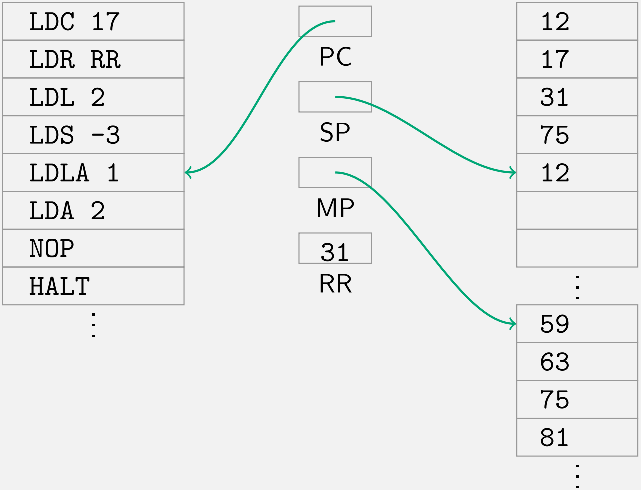
Load instructions

LDS – load from stack



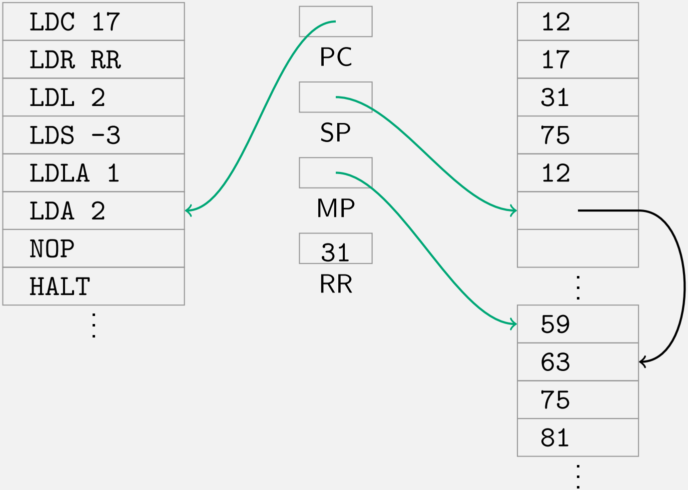
Load instructions

LDLA – load local address



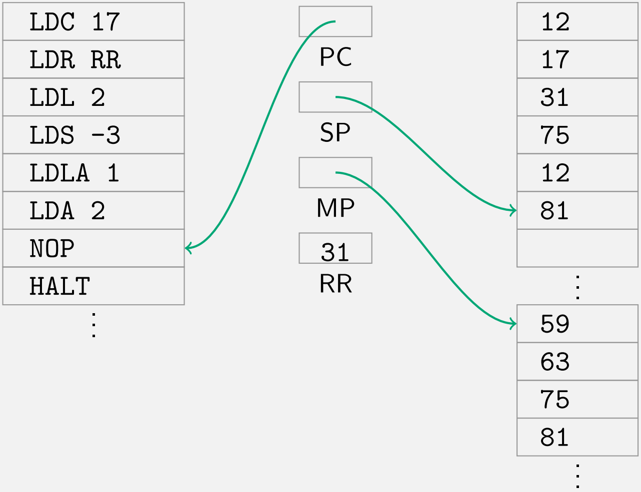
Load instructions

LDA – load via address



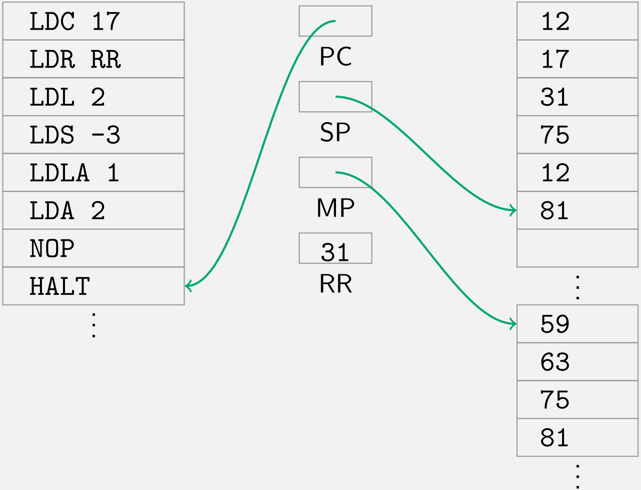
Load instructions

NOP – noop



Load instructions

HALT – halt program



Branch instructions

BRA – branch always (unconditional)

BRT – branch on true (-1)

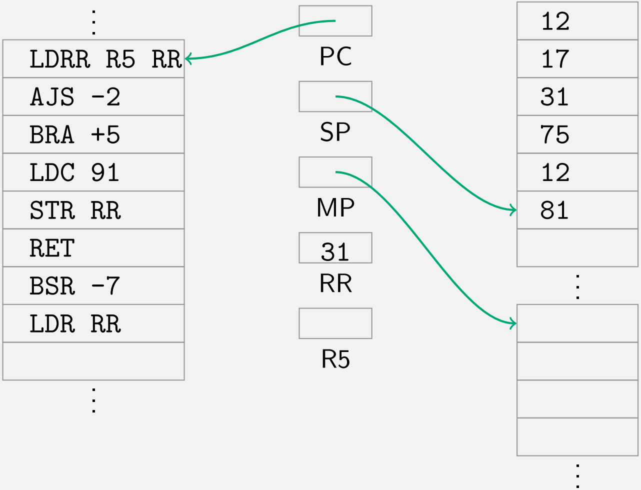
BRF – branch on false (0)

BSR – branch to subroutine (push return address on stack)

RET – return (from subroutine)

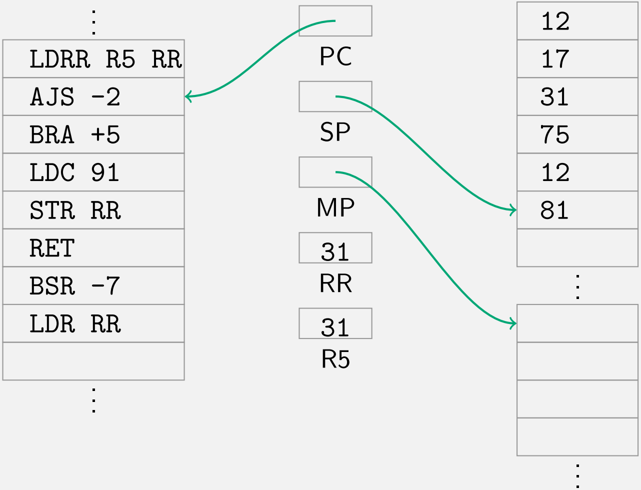
Register and jump instructions

LDRR – load register from register



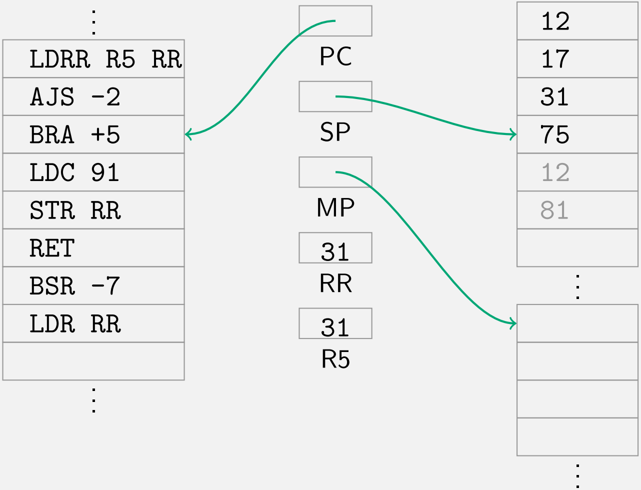
Register and jump instructions

AJS – adjust stack pointer



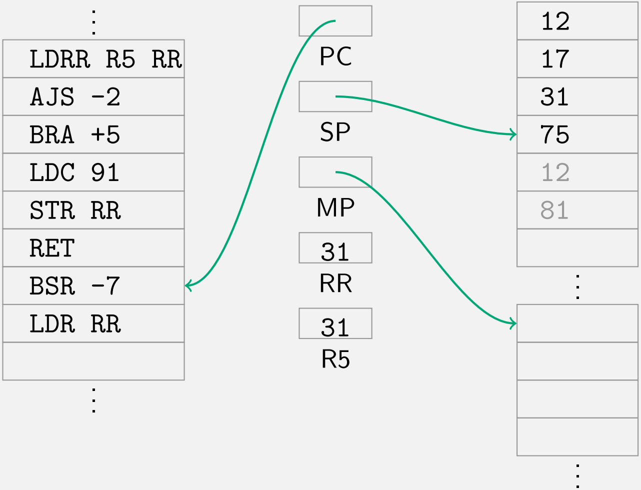
Register and jump instructions

BRA – unconditional branch



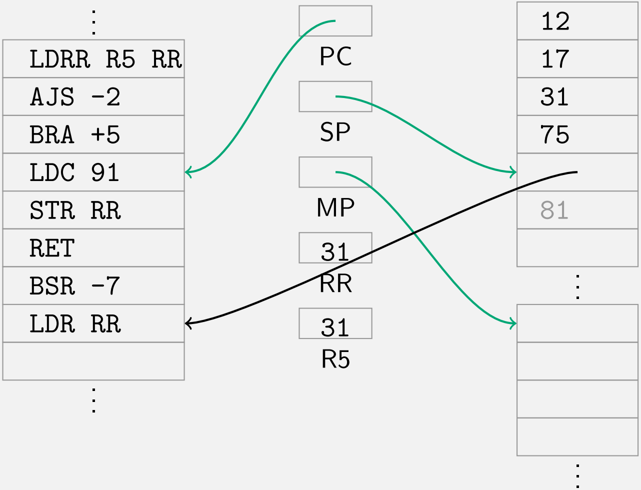
Register and jump instructions

BSR – branch to subroutine



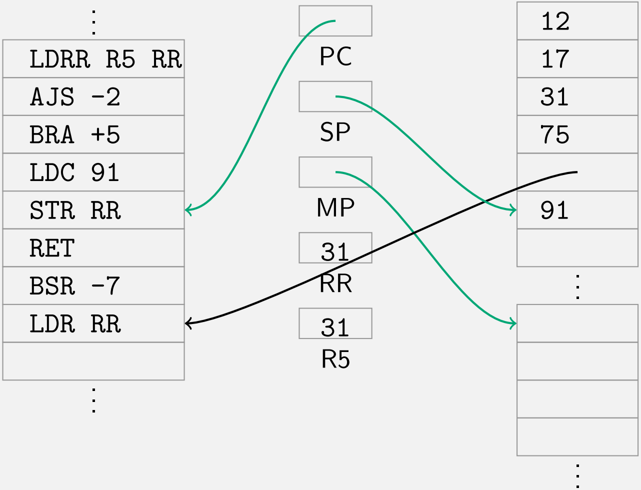
Register and jump instructions

LDC – load constant



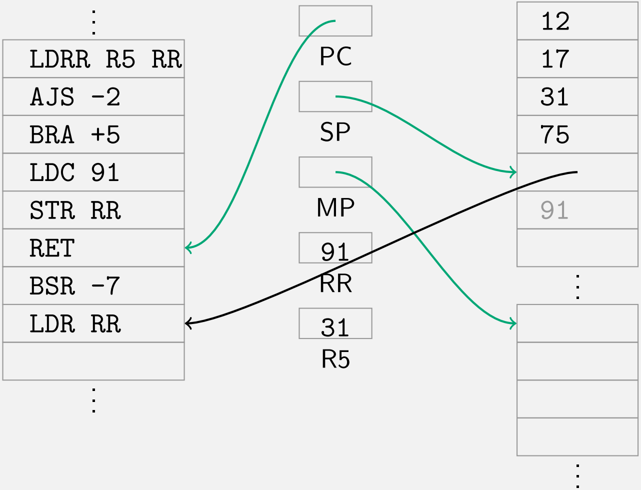
Register and jump instructions

STR – store to register



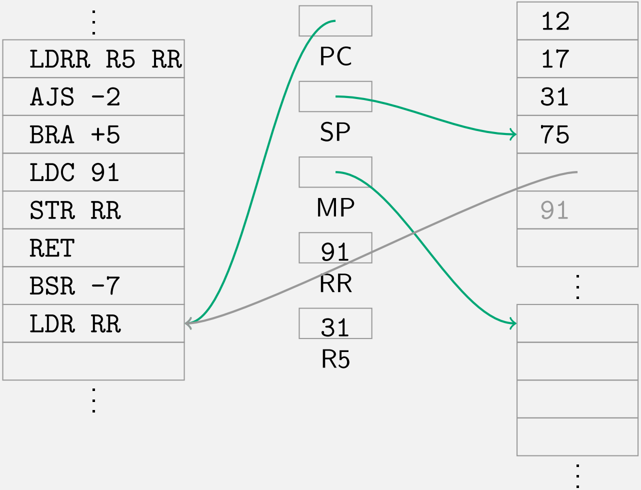
Register and jump instructions

RET – return

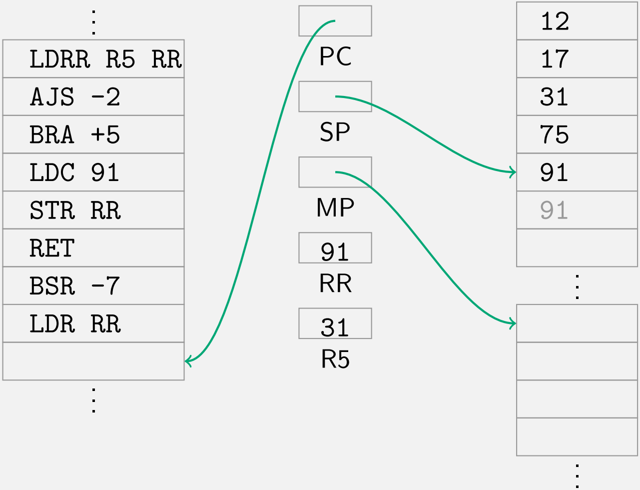


Register and jump instructions

LDR – load from register



Register and jump instructions



Operators

Operators remove stack arguments and put the result back on the stack.

Binary operators			Unary operators
`ADD`	`AND`	`EQ`	`NOT`
`SUB`	`OR`	`NE`	`NEG`
`MUL`	`XOR`	`LT`	
`DIV`		`GT`	
`MOD`		`LE`	
		`GE`	

9.3 Translating programs

Arithmetic expressions

Expression

```
3+4*7+2
```

Arithmetic expressions

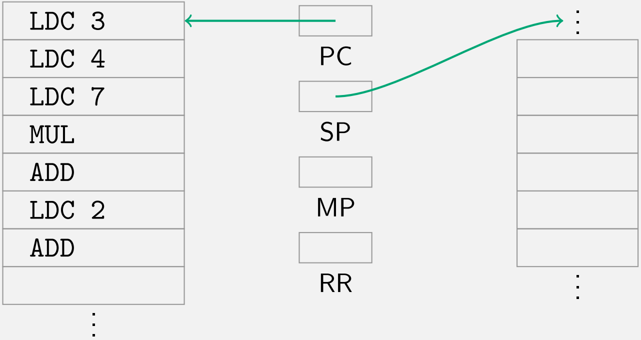
Expression

```
3+4*7+2
```

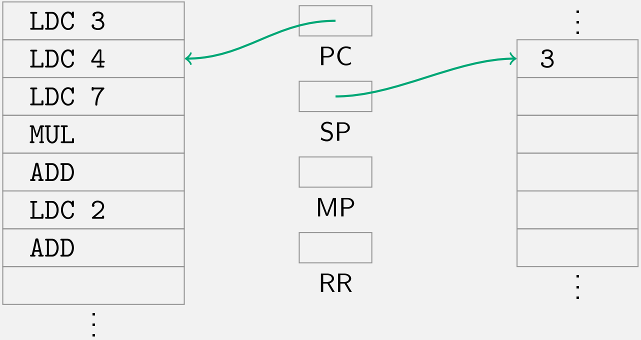
Code

```
LDC 3
LDC 4
LDC 7
MUL
ADD
LDC 2
ADD
```

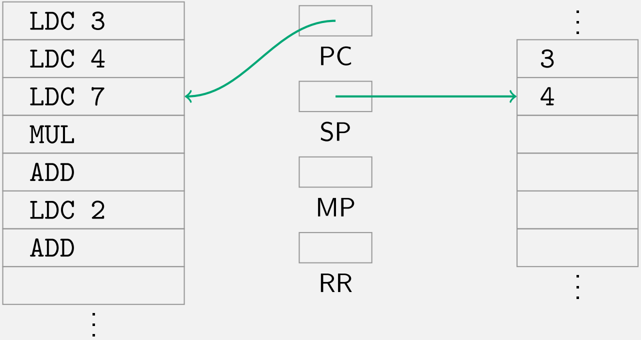
Arithmetic expression example



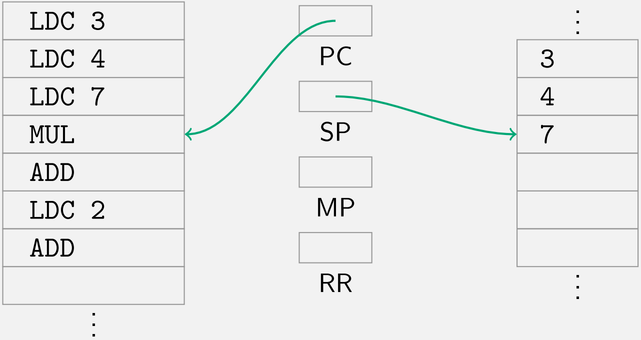
Arithmetic expression example



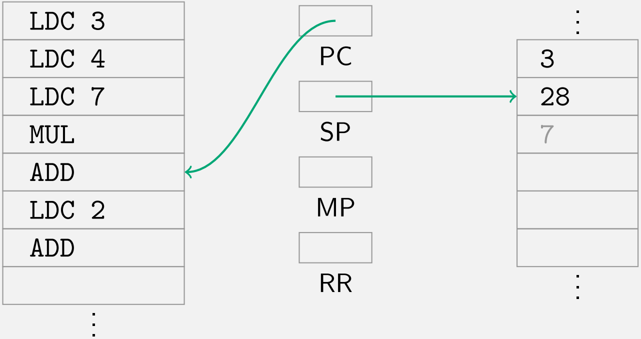
Arithmetic expression example



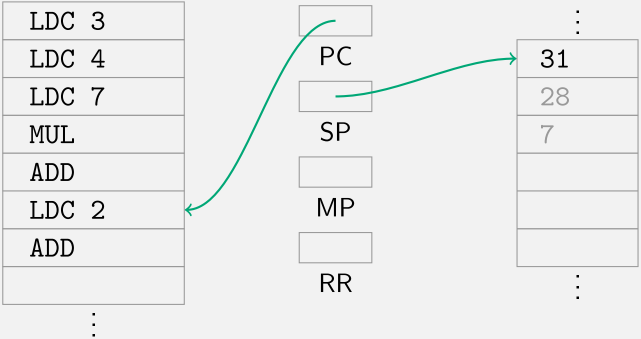
Arithmetic expression example



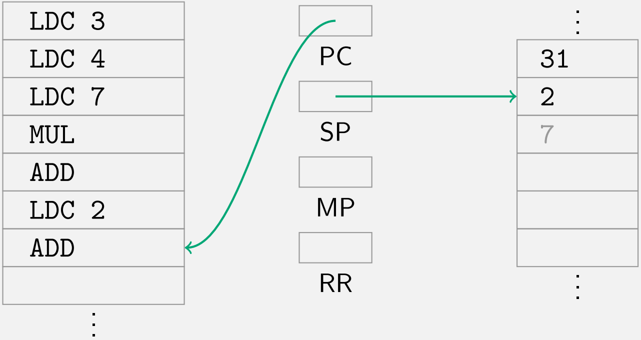
Arithmetic expression example



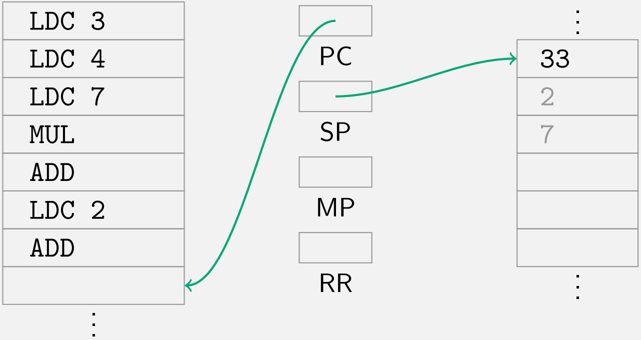
Arithmetic expression example



Arithmetic expression example



Arithmetic expression example



Representing code in Haskell

```
type Code = [Instr]
data Instr = LDC Int
           | LDL Int
           | ADD
           | NEG
           | EQ
           | ...
instrSize :: Instr -> Int
instrSize (LDC n) = 2
instrSize ADD     = 1
...
codeSize :: Code -> Int
codeSize = sum . map instrSize
```

Translating expressions

```
data Expr = Num Int
          | Add Expr Expr
          | Mul Expr Expr
          | Neg Expr
          | Eq  Expr Expr
          | ...

code :: Expr → Code
code (Num n)     = [LDC n]
code (Add e1 e2) = code e1 ++ code e2 ++ [ADD]
code (Mul e1 e2) = code e1 ++ code e2 ++ [MUL]
code (Neg e)     = code e ++ [NEG]
code (Eq e1 e2)  = code e1 ++ code e2 ++ [EQ]
```

Algebra for code generation

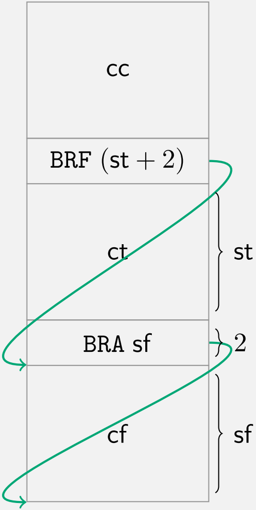
```
data Expr =
    Num Int
  | Add  Expr Expr
  | Neg  Expr
  | Eq   Expr Expr
```

```
code x = foldExpr codeAlg x
  where
  codeAlg :: ExprAlg Code
  codeAlg = ExprAlg
    { num = λn   → [LDC n]
    , add  = λl r → l ++ r ++ [ADD]
    , neg  = λl   → l ++ [NEG]
    , eq   = λl r → l ++ r ++ [EQ]
    }
```

Translating conditional expressions

```
data Expr = ...
          | If Expr Expr Expr
code :: Expr → Code
...
code (If c t f) = cc               ++
                  [BRF (st + 2)] ++
                  ct               ++
                  [BRA sf] ++
                  cf
   where cc = code c
         ct = code t
         cf = code f
         st = codeSize ct
         sf = codeSize cf
```

Translating conditional expressions – contd.



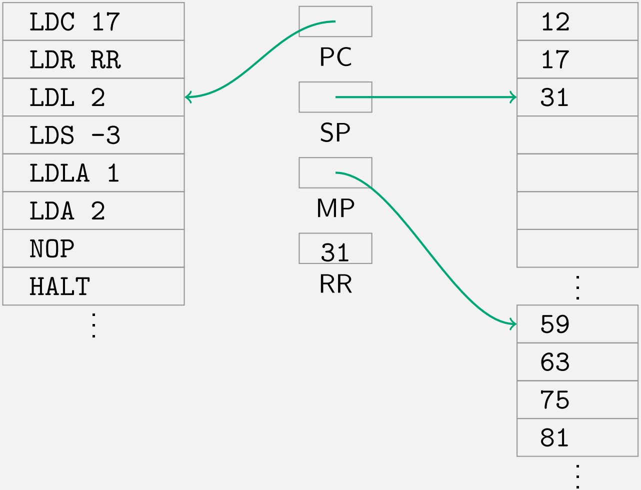
Algebra for code generation: Conditionals

```
data Expr =
      Num Int
    | Add  Expr Expr
    | Neg  Expr
    | Eq   Expr Expr
    | If   Expr Expr Expr
```

```
code x = foldExpr codeAlg x
  where
  codeAlg :: ExprAlg Code
  codeAlg = ExprAlg
    { num = λn      → [LDC n]
    , add  = λl r   → l ++ r ++ [ADD]
    , neg  = λl     → l ++ [NEG]
    , eq   = λl r   → l ++ r ++ [EQ]
    , if   = λc t f →
           let st = codeSize t
               sf = codeSize f
           in  c ++ [BRF (st + 2)] ++ t ++ [BRA sf] ++ f
    }
```

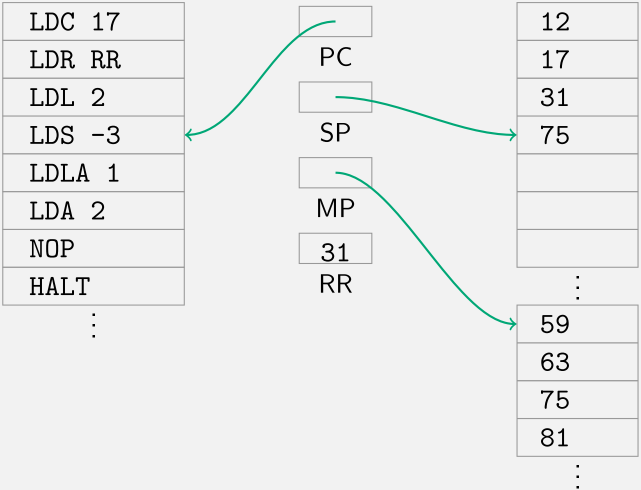
Load instructions

LDL – load local variable



Load instructions

LDL – load local variable



Algebra for code generation: Variables

```
data Expr =
    Num Int
  | Add  Expr Expr
  | Neg  Expr
  | Eq   Expr Expr
  | If   Expr Expr Expr
```

```
code x = foldExpr codeAlg x
  where
  codeAlg :: ExprAlg           Code
  codeAlg = ExprAlg
    {num = λn     →            [LDC n]
    , add  = λl r  →           l  ++ r  ++ [ADD]
    , neg  = λl    →           l  ++ [NEG]
    , eq   = λl r  →           l  ++ r  ++ [EQ]
    , if   = λc t f →
        let st     = codeSize (t  )
            sf     = codeSize (f  )
        in  c   ++ [BRF (st + 2)] ++
            t   ++ [BRA sf] ++ f

    }
```

Algebra for code generation: Variables

```
data Expr =
    Num Int
  | Add  Expr Expr
  | Neg  Expr
  | Eq   Expr Expr
  | If   Expr Expr Expr



  | Var  String
  | Let  String Expr Expr
```

```
code x = foldExpr codeAlg x
  where
  codeAlg :: ExprAlg          Code
  codeAlg = ExprAlg
    {num = λn     →           [LDC n]
    , add  = λl r   →         l  ++ r  ++ [ADD]
    , neg  = λl     →         l  ++ [NEG]
    , eq   = λl r   →         l  ++ r  ++ [EQ]
    , if   = λc t f →
        let st    = codeSize (t  )
            sf    = codeSize (f  )
        in  c   ++ [BRF (st + 2)] ++
            t   ++ [BRA sf] ++ f
    , var =  λs     →
    , leT =  λs d b →

    }
```

Algebra for code generation: Variables

```
data Expr =
    Num Int
  | Add  Expr Expr
  | Neg  Expr
  | Eq   Expr Expr
  | If   Expr Expr Expr




  | Var  String
  | Let  String Expr Expr
```

```
code x = foldExpr codeAlg x
  where
  codeAlg :: ExprAlg        Code
  codeAlg = ExprAlg
    {num = λn      →        [LDC n]
    , add  = λl r  →        l   ++ r   ++ [ADD]
    , neg  = λl    →        l   ++ [NEG]
    , eq   = λl r  →        l   ++ r   ++ [EQ]
    , if   = λc t f →
        let st     = codeSize (t  )
            sf     = codeSize (f  )
        in  c    ++ [BRF (st + 2)] ++
            t    ++ [BRA sf] ++ f
    , var =  λs     →        [LDL ??]
    , leT =  λs d b →

    }
```

Algebra for code generation: Variables

```
data Expr =
    Num Int
  | Add  Expr Expr
  | Neg  Expr
  | Eq   Expr Expr
  | If   Expr Expr Expr
  | Var  String
  | Let  String Expr Expr
```

```
code x = foldExpr codeAlg x
  where
  codeAlg :: ExprAlg (Env → Code)
  codeAlg = ExprAlg
    { num = λn     →          [LDC n]
    , add  = λl r   →          l  ++ r  ++ [ADD]
    , neg  = λl     →          l  ++ [NEG]
    , eq   = λl r   →          l  ++ r  ++ [EQ]
    , if   = λc t f →
        let st    = codeSize (t  )
            sf    = codeSize (f  )
        in  c   ++ [BRF (st + 2)] ++
            t   ++ [BRA sf] ++ f
    , var = λs     → λe → [LDL (e ! s)]
    , leT = λs d b → λe → d e ++ [STL (size e)]
                          ++ b (insert s (size e) e)
    }
```

Algebra for code generation: Variables

```
data Expr =
    Num Int
  | Add  Expr Expr
  | Neg  Expr
  | Eq   Expr Expr
  | If   Expr Expr Expr
  | Var  String
  | Let  String Expr Expr
```

```
code x = foldExpr codeAlg x empty
  where
  codeAlg :: ExprAlg (Env → Code)
  codeAlg = ExprAlg
    { num = λn     → λe → [LDC n]
    , add = λl r   → λe → l e ++ r e ++ [ADD]
    , neg = λl     → λe → l e ++ [NEG]
    , eq  = λl r   → λe → l e ++ r e ++ [EQ]
    , if  = λc t f → λe →
        let st = codeSize (t e)
            sf = codeSize (f e)
        in  c e ++ [BRF (st + 2)] ++
            t e ++ [BRA sf] ++ f e
    , var = λs     → λe → [LDL (e ! s)]
    , leT = λs d b → λe → d e ++ [STL (size e)]
                          ++ b (insert s (size e) e)
    }
```

Expressions vs. statements

We extend our language with statements:

```
data Stmt =
      Assign String Expr
    | If     Expr Stmt Stmt
    | While  Expr Stmt
    | Call   String [Expr]
```

Expressions vs. statements

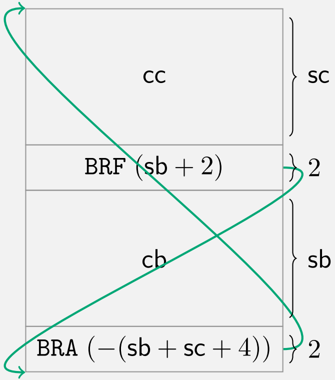
We extend our language with statements:

```
data Stmt =
    Assign String Expr
  | If     Expr Stmt Stmt
  | While  Expr Stmt
  | Call   String [Expr]
```

For many languages, the following invariants hold:

- Expressions always leave a single result on the stack after evaluation.
- Statements do not leave a result on the stack after evaluation.

Translating while loops

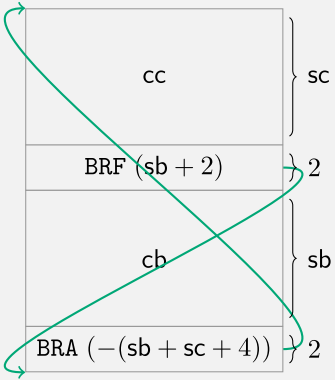


Translating while loops

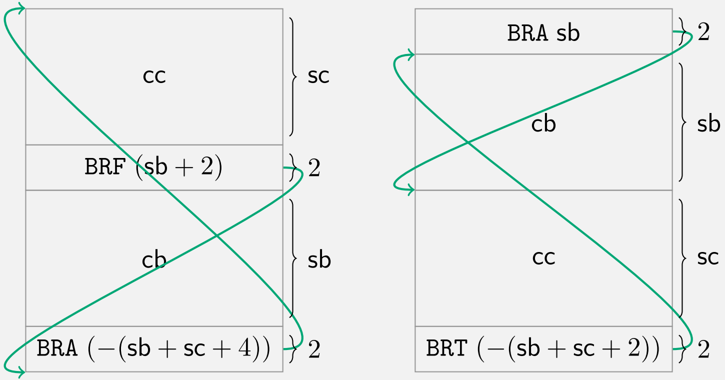
```
data Stmt = ...
          | While Expr Stmt

code :: Stmt → Code
...
code (While c b) = cc                   ++
                   [BRF (sb + 2)]       ++
                   cb                   ++
                   [BRA (−(sb + sc + 4))]
  where cc = code c
        cb = code b
        sc = codeSize cc
        sb = codeSize cb
```

Translating while loops – contd.



Translating while loops – contd.



Translating while loops – contd.

```
data Stmt = ...
          | While Expr Stmt

code :: Stmt → Code
...
code (While c b) = [BRA sb] ++
                   cb       ++
                   cc       ++
                   [BRT (−(sb + sc + 2))]
  where cc = code c
        cb = code b
        sc = codeSize cc
        sb = codeSize cb
```

Algebra type&fold for Statements&Expressions

```
data SEAlg s e = SEAlg
  { add  :: e → e → e
  , num  :: Int → e
  , ifE  :: e → e → e → e
  , ifS  :: e → s → s → s
  , asg  :: String → e → s
  , whl  :: e → s → s
  , cal  :: String → [e] → s
  }
```

```
foldSE :: SEAlg s e → Statement → s
foldSE alg {..} = fs where
  fs (IfS c t f)  = ifS  (fe c) (fs t) (fs f)
  fe (IfE c t f)  = ifE  (fe c) (fe t) (fe f)
  fs (Call v ps)  = cal  v (map fe ps)
  fe (Add x y)    = add  (fe x) (fe y)
  fe (Num n)      = num n
```

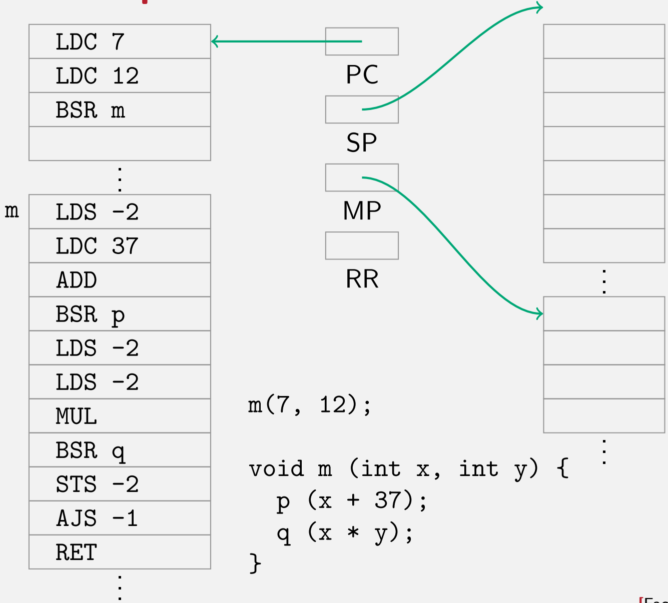
Algebra for code generation

```
data Stmt =
    Assign String Expr
  | If     Expr Stmt Stmt
  | While  Expr Stmt
  | Call   String [Expr]
```

```
code x = foldSE codeAlg x empty
  where
  codeAlg :: SEAlg (Env → Code) (Env → Code)
  codeAlg = SEAlg
    {asg = λs d   e → d e ++ [STL (e ! s)]
    , ifS  = λc t f  e →
            let st  = codeSize (t e)
                sf  = codeSize (f e)
            in  c e ++ [BRF (st + 2)] ++
                t e  ++ [BRA sf] ++ f e
    , whl = λc b   e →
            let sc = codeSize (c e)
                sb = codeSize (b e)
            in  [BRA sb] ++ b e ++ c e ++
                [BRT (−(sb + sc + 2))]
    , cal  = λm ps e →
            concatMap ($e) ps ++ [BSR m]
    , ...}   -- components for Expr
```

9.4 Functions / methods

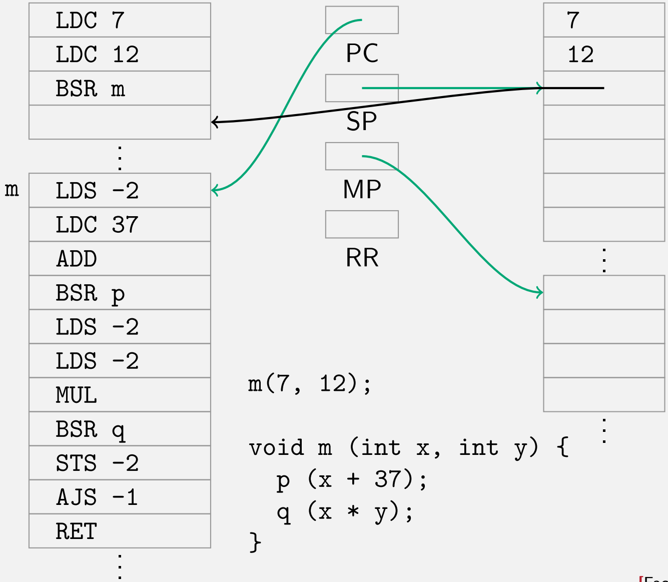
Methods with parameters




```
m(7, 12);

void m (int x, int y) {
  p (x + 37);
  q (x * y);
}
```

Methods with parameters

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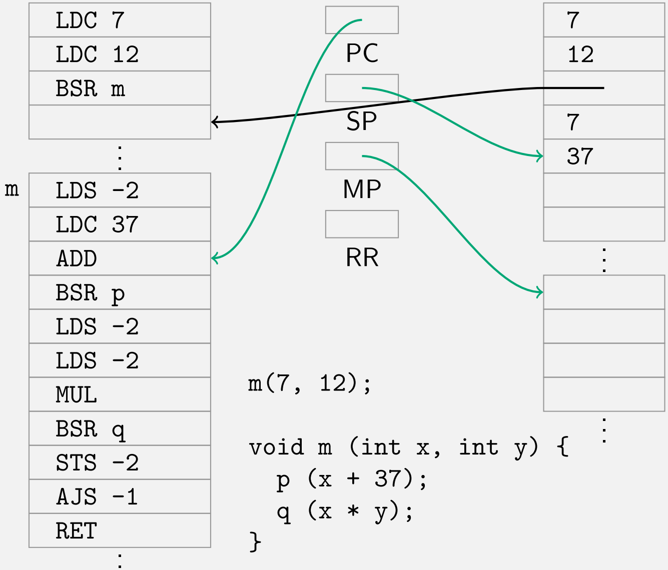
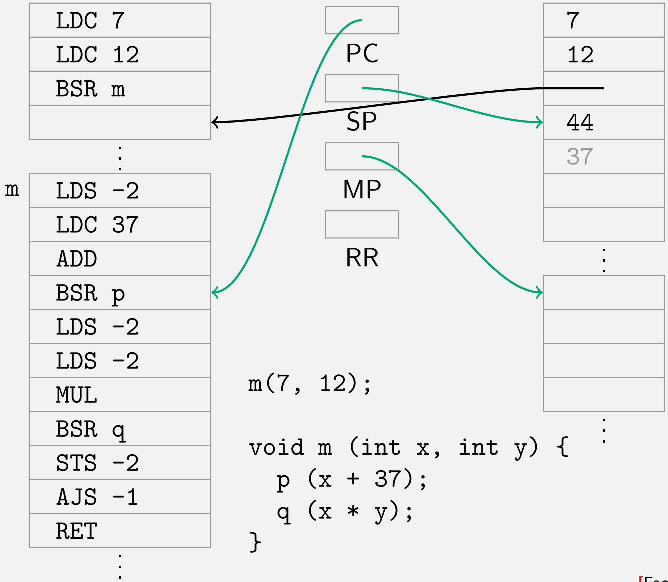


Methods with parameters

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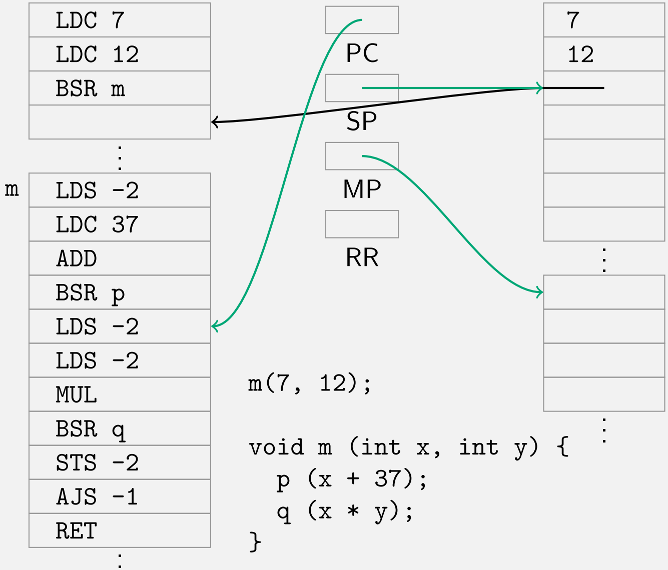


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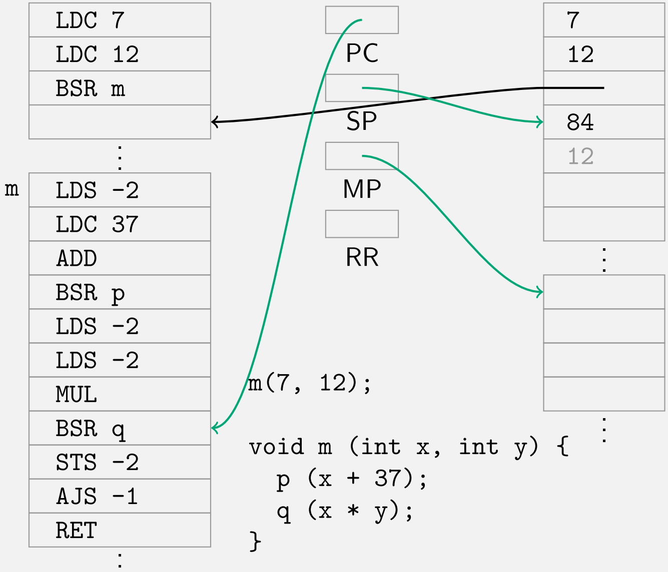


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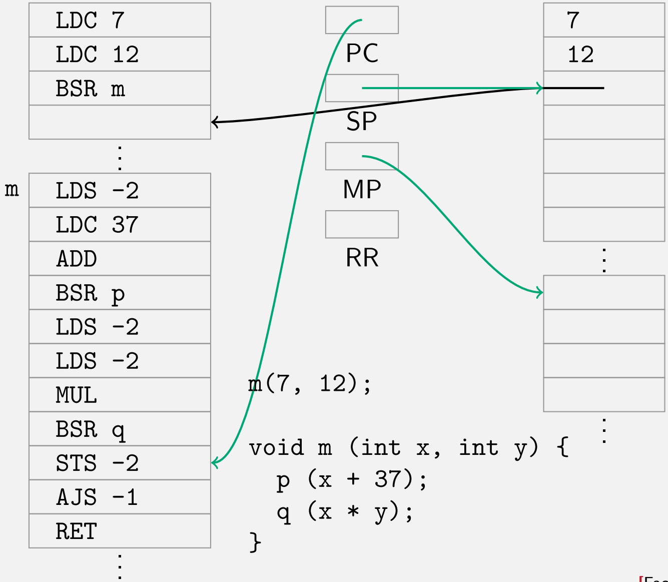


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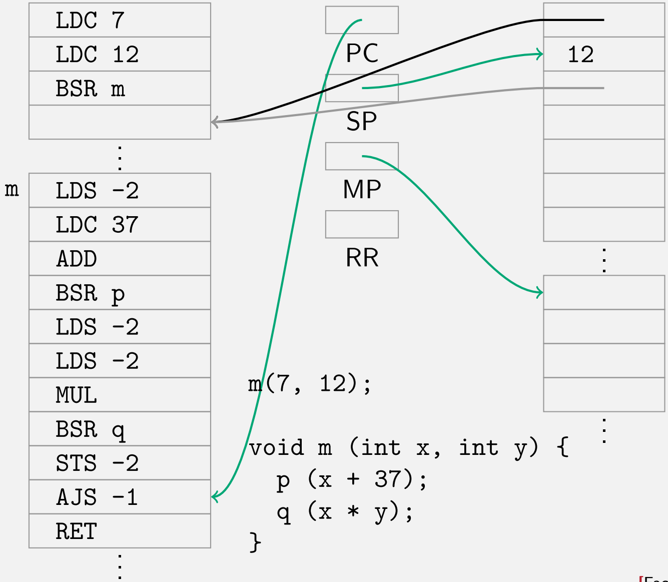


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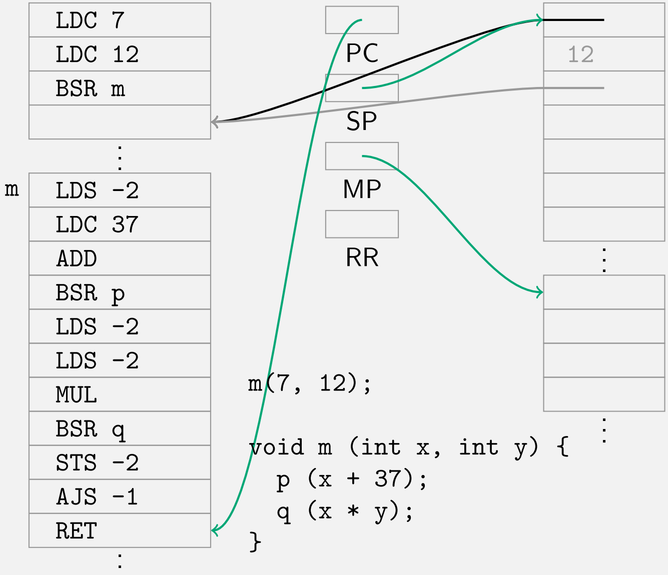


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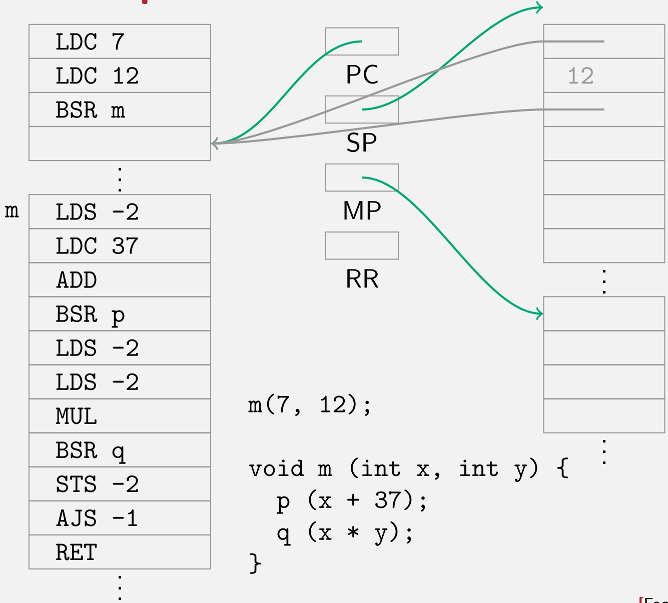


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Method call

- Put parameters on the stack.
- Call `BSR` with the method label.

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- Use parameters: from `LDS` $-(n+d)$ to `LDS` $-(1+d)$, where n is the number of parameters and d is your current offset (this becomes easier with the mark pointer).
- Clean up: `STS` $-n$ followed by `AJS` $-(n-1)$.
- Return: `RET`

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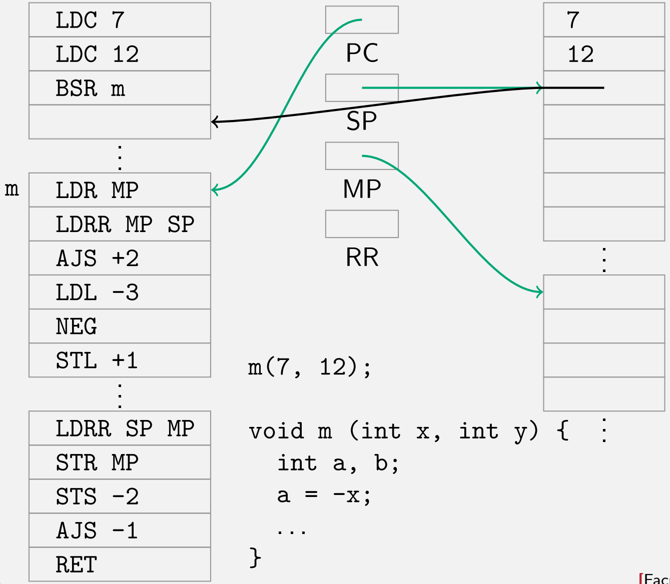
It is also possible, but less common, to let the caller clean up after a method call.

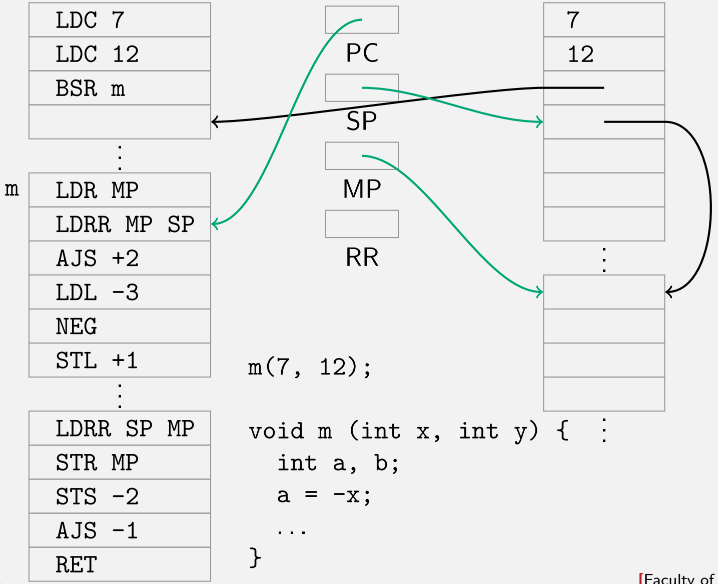
Methods with local variables




```
m(7, 12);

void m (int x, int y) {
  int a, b;
  a = -x;
  ...
}
```

Methods with local variables

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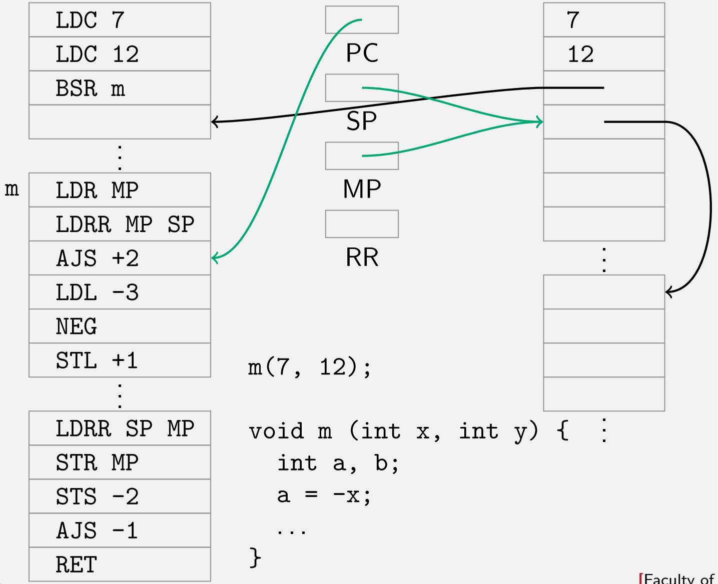
Methods with local variables

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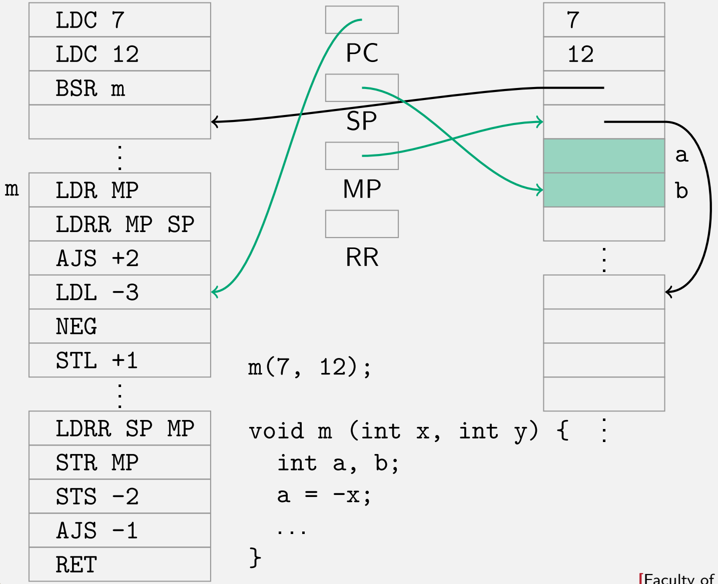


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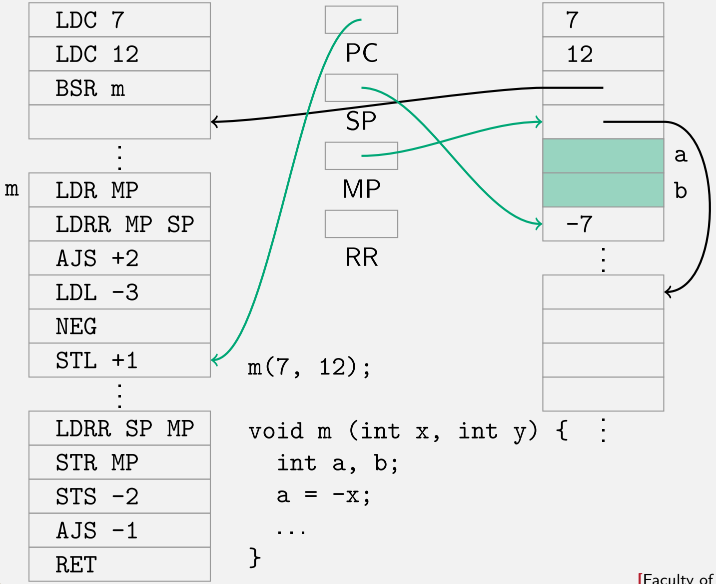


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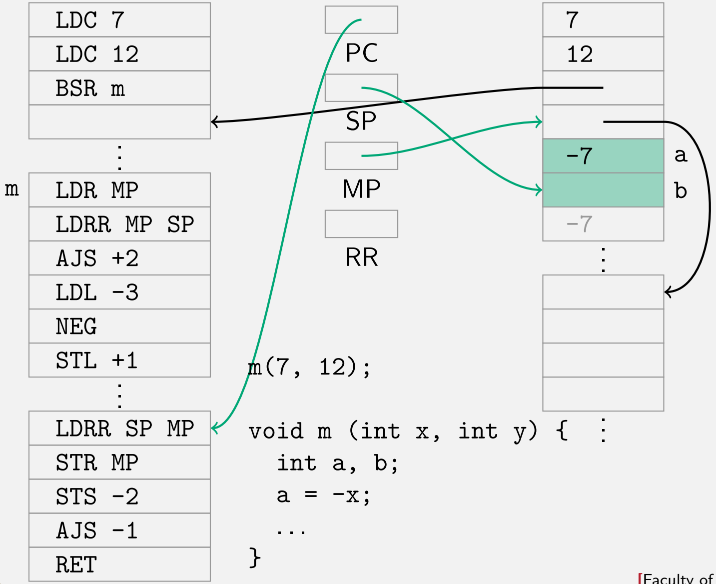


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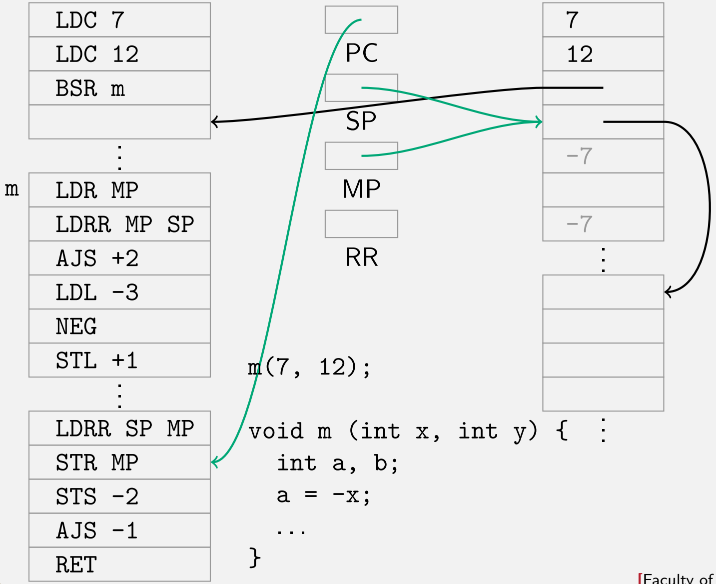


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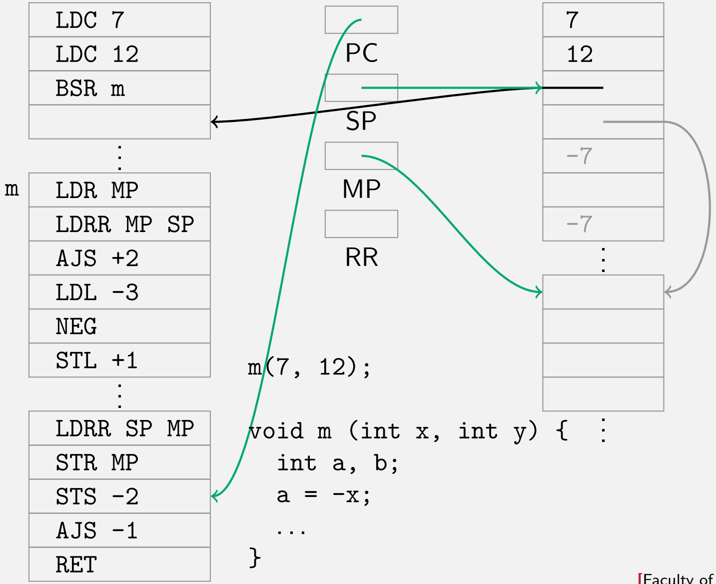


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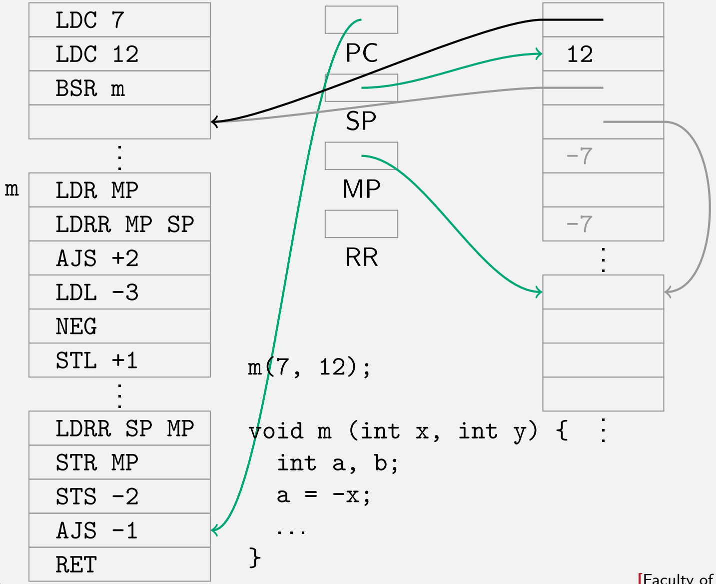


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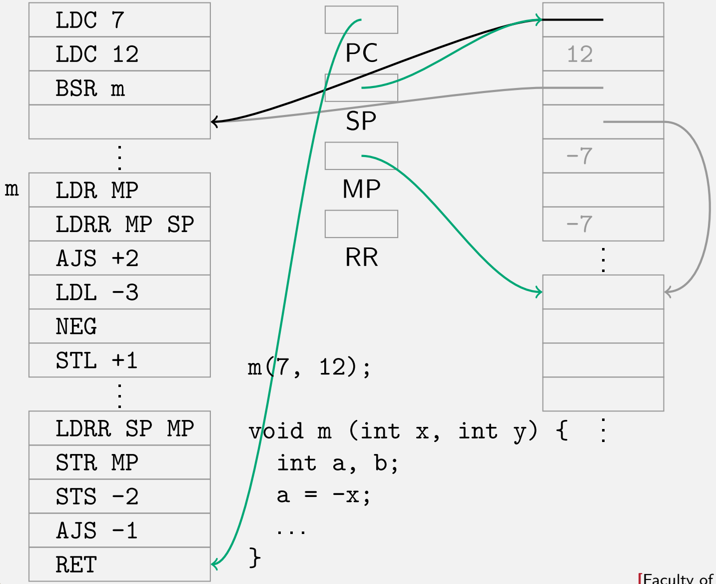


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void m (int x, int y) {
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Method translation with local variables

Method call as before.

Method translation with local variables

Method call as before.

Method definition (n parameters, k local variables)

- Create room for local variables: `LDR MP` to save the mark pointer, `LDRR MP SP` to reset the mark pointer, `AJS` $+k$ to adjust the stack pointer. (Also available as a single instruction `LINK` k.)
- Use parameters: from `LDL` $-(n+1)$ to `LDL` -2.
- Use local variables: from `LDL` $+1$ to `LDL` $+k$.
- Clean up local variables: `LDRR SP MP` to reset the stack pointer, and `STR MP` to restore the mark pointer. (Also available as a single instruction `UNLINK`.)
- Clean up: `STS` $-n$ followed by `AJS` $-(n-1)$.
- Return: `RET`

Methods with return values

Two options.

Methods with return values

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Result on stack

- Leave the result as the final value on the stack.
- Adapt the cleanup code so that this works.

Methods with return values

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Result on stack

- Leave the result as the final value on the stack.
- Adapt the cleanup code so that this works.

Result in register

- Place the result of a method call in a fixed free register (`RR` for example).
- Use the value from there at the call site.