Lecture 10: Implementing RegExp the hard way

Talen en Compilers 2023-2024, period 2

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Department of Information and Computing Sciences, Utrecht University

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Recap: RegExp

```
<|> :: R → R → R
<+> :: R → R → R
many  :: R → R
many1 :: R → R
option  :: R → R
symbol  :: Char → R
satisfy :: (Char → Bool) → R
type R = Parser Char String
```

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$r_1|r_2$

r_1r_2

r*

r+

r?

c

```
\d \s \S [a-z] ...
```

```
{}|{-?\d+(\.\d+)?}
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(0b)?(0|1)+
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(0b)?(0|1)+
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```
co(bra|d)
```

Recap: RegExp performance

```
head $ matchRegExp "a*aaaba*$" "aaaaaaabaa"

 aaa❌
a aaa❌
aa aaa❌
aaa aaa❌
aaaa aaab aa ✅
```

Today: matching RegExp fast 🚀

⏱️ Goal: `O(length input)` matching time

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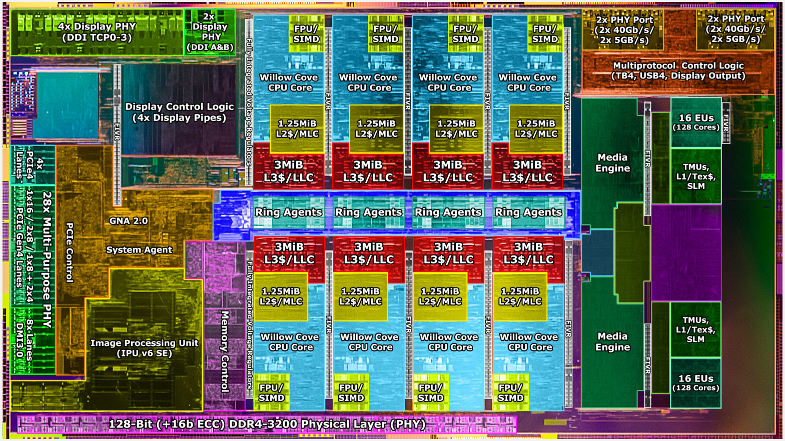
✨ New algorithm

Today: matching RegExp fast 🚀

⏱️ Goal: `O(length input)` matching time

✨ New algorithm from the bottom up

Problem: computers are complicated



4x Display PHY (DDI TCP0-3)
2x Display PHY (DDI A&B)
FPU/SIMD
Willow Cove CPU Core
1.25MiB L2$/MLC
3MiB L3$/LLC
2x PHY Port (2x 40Gb/s/ 2x 5GB/s)
Multiprotocol Control Logic (TB4, USB4, Display Output)
Display Control Logic (4x Display Pipes)
Fully Integrated Voltage Regulators
FIVR
16 EUs (128 Cores)
TMUs, L1/Tex$, SLM
Media Engine
4x PCIe4 Lanes
28x Multi-Purpose PHY
1x16 / 2x8 / 1x8 + 2x4 PCIe Gen4 Lanes
8x Lanes DMI3.0
PCIe Control
GNA 2.0
System Agent
Ring Agents
Image Processing Unit (IPU v6 SE)
Memory Control
128-Bit (+16b ECC) DDR4-3200 Physical Layer (PHY)

10nm SF Tiger Lake 8Core processor (2021) die shot, by @Locuza_ on twitter

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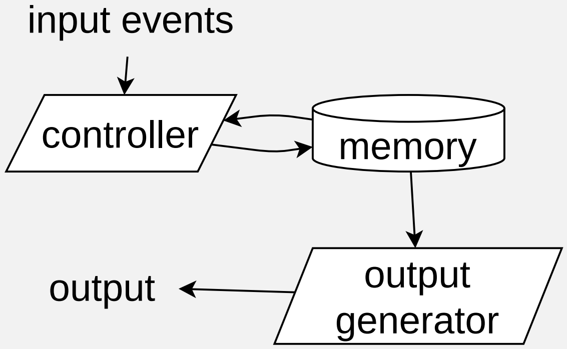
Simplification: Moore Machine



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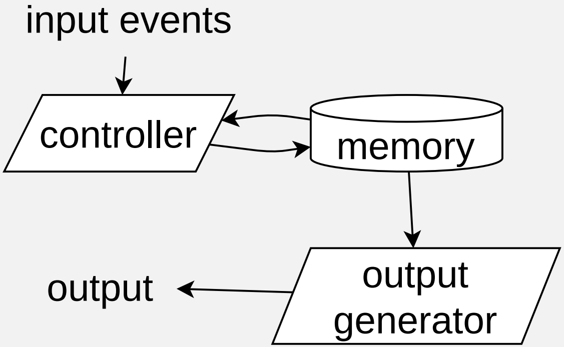
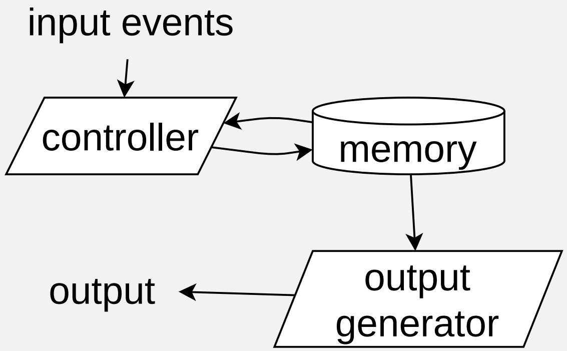


a.k.a. Finite State Machine (FSM)

Simplification: Moore Machine



a.k.a. Finite State Machine (FSM)

v.s.t Finite State Automaton (FSA)

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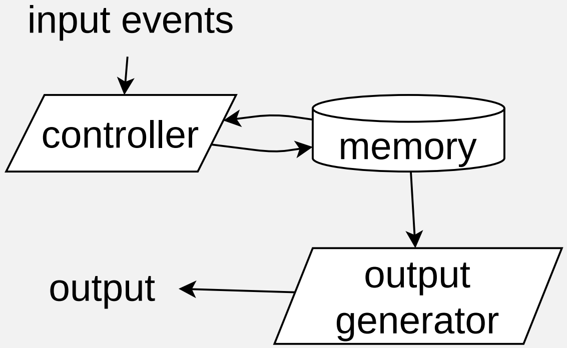
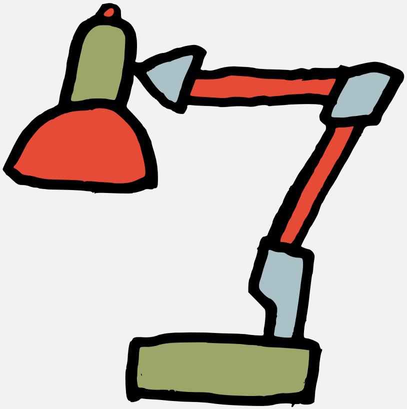


a.k.a. Finite State Machine (FSM)

v.s.t Finite State Automaton (FSA)

a.k.a. Deterministic Finite Automaton (DFA)

Concrete Example



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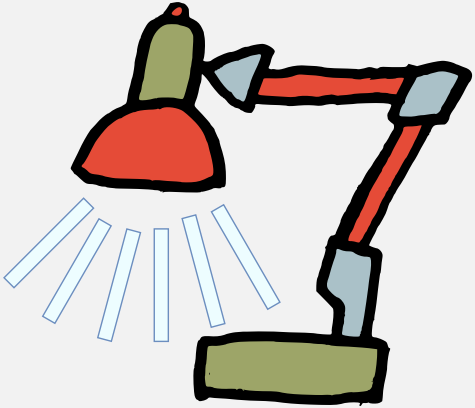

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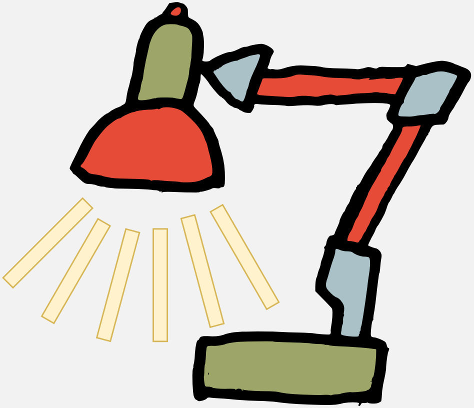

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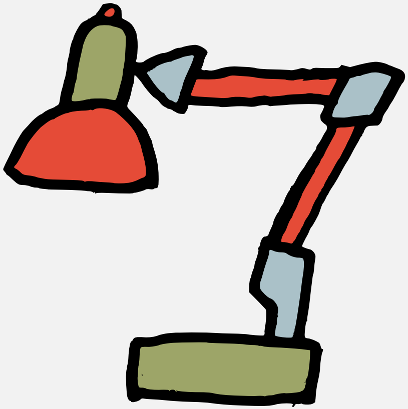

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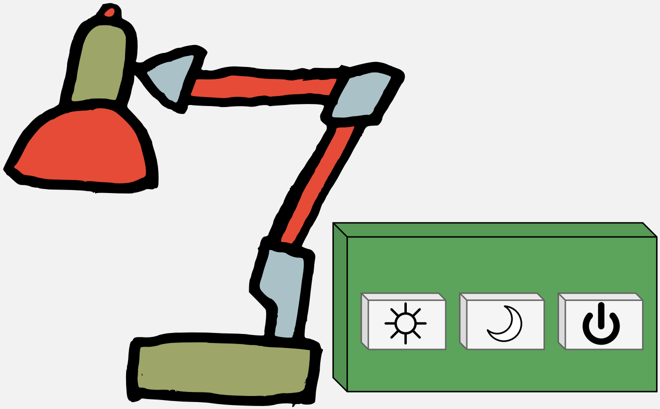

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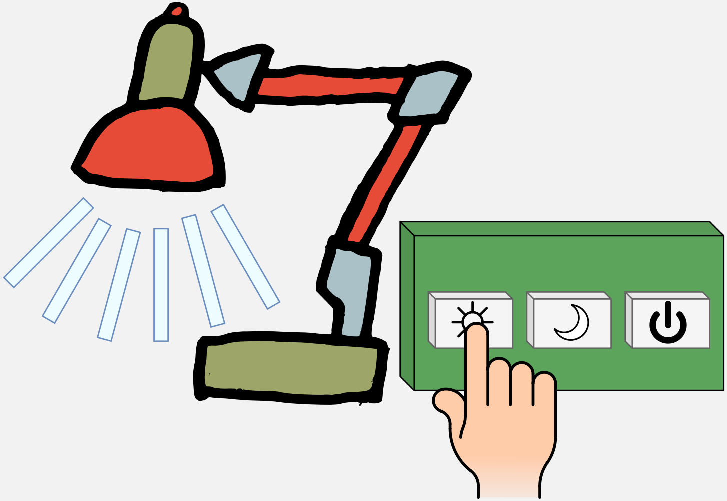

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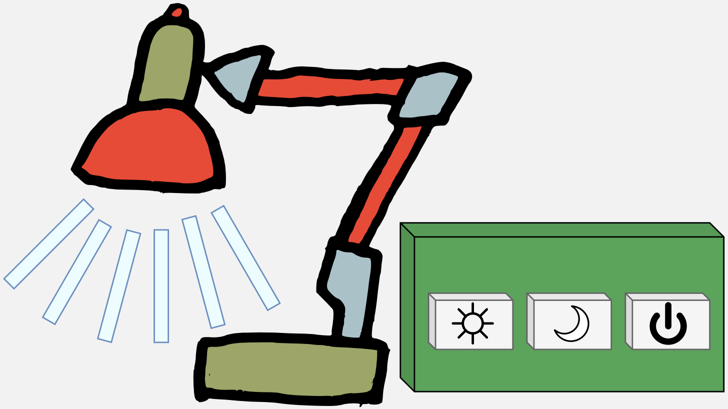

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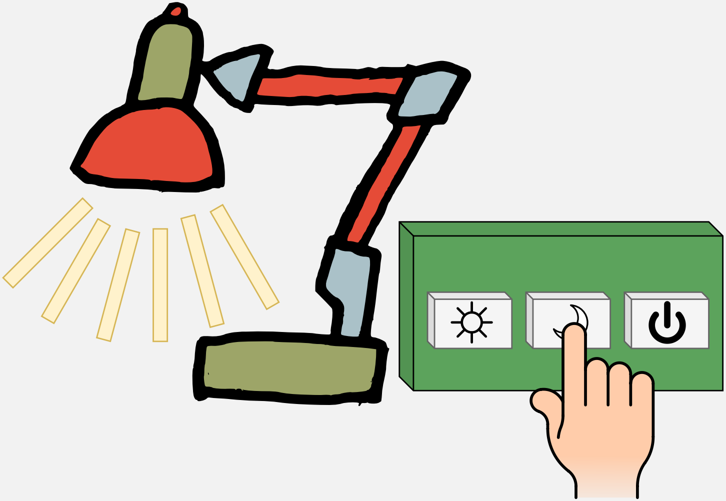

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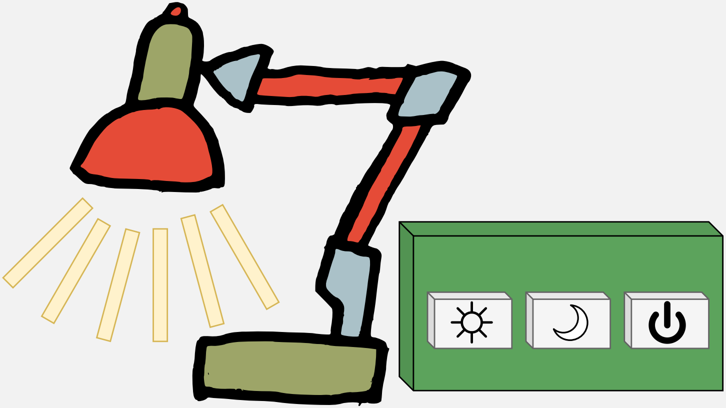

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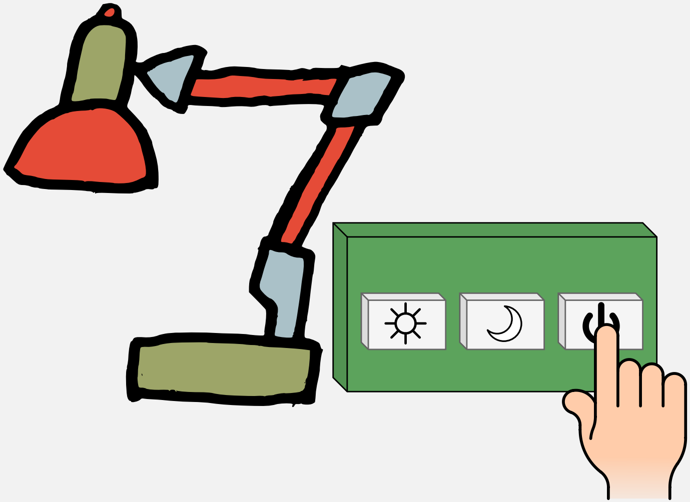

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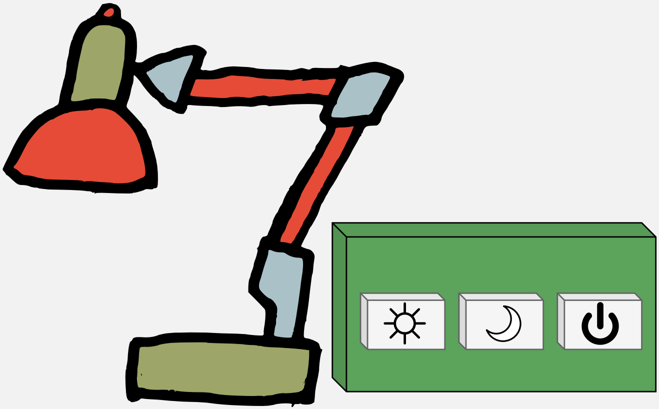

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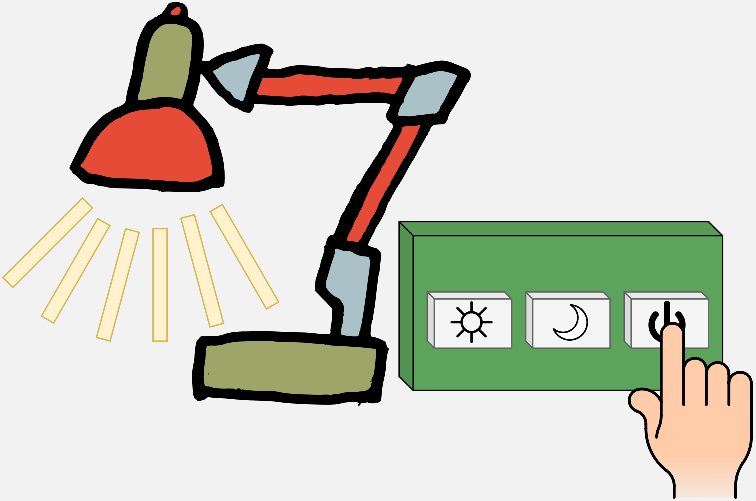
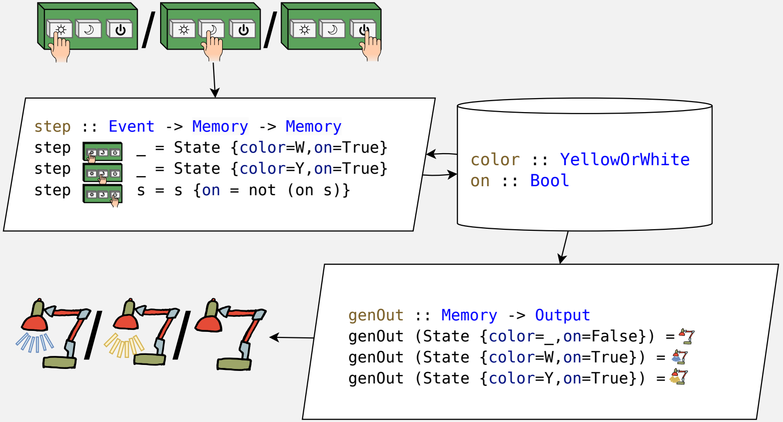


Moore Machine for lamp



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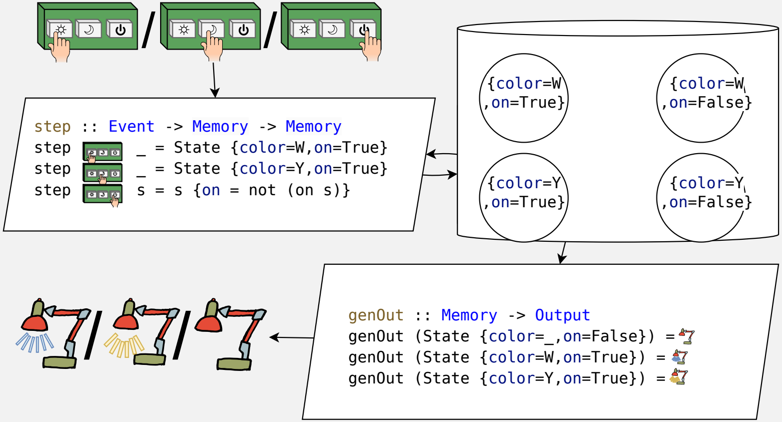

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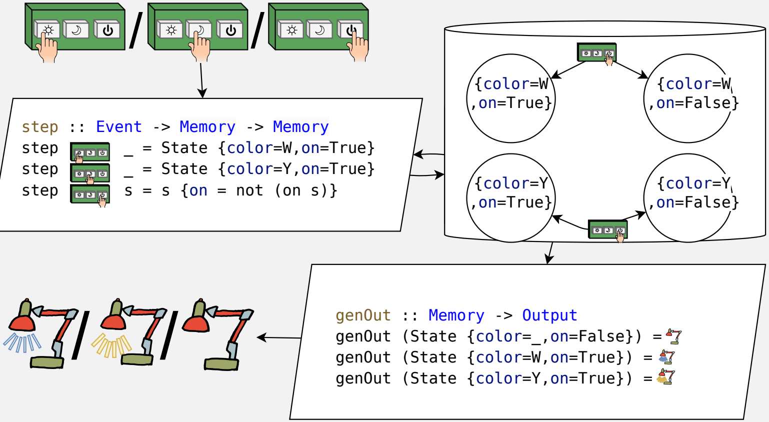

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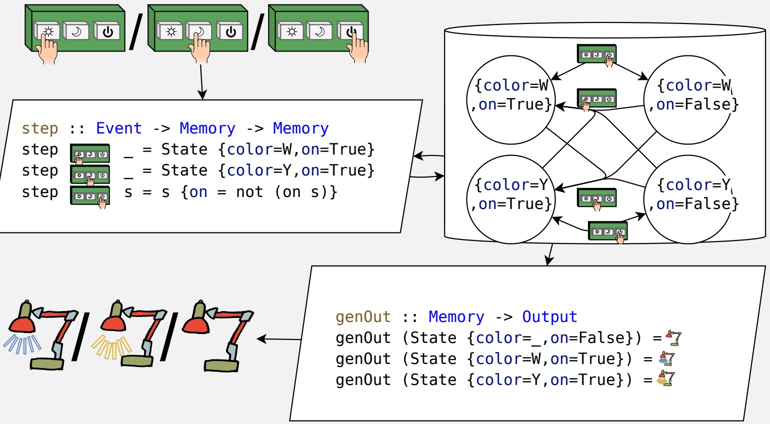

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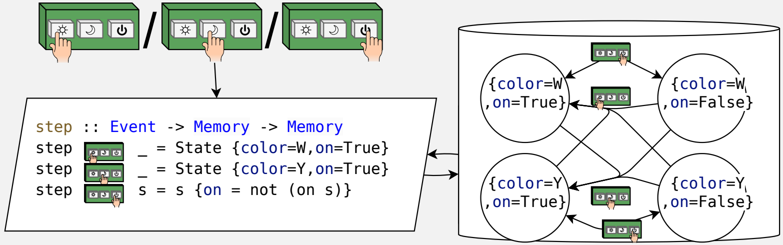

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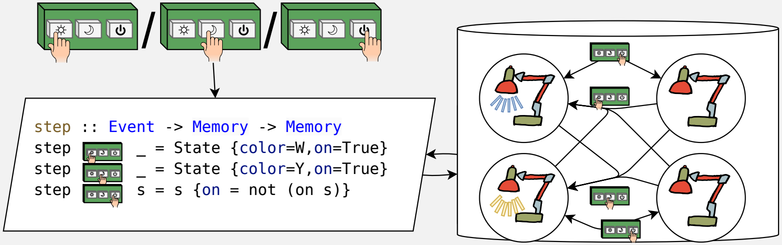

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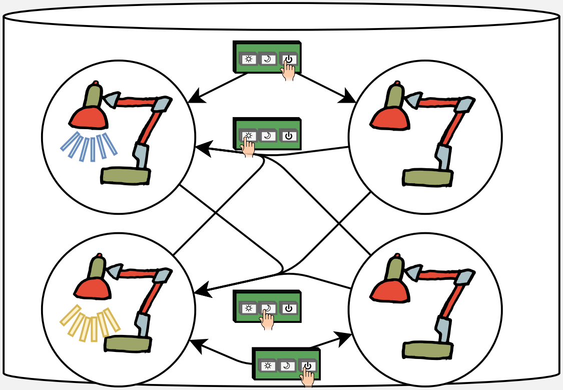

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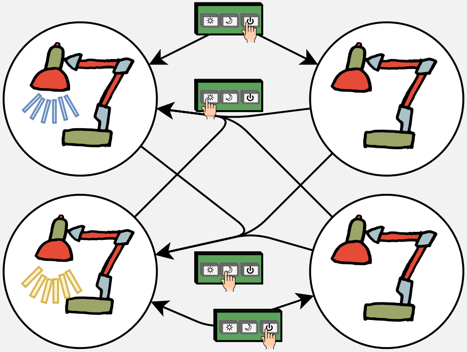
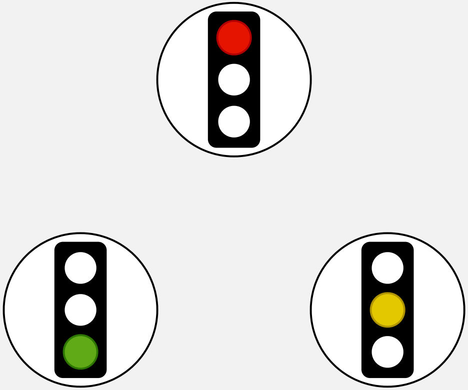


Another Example



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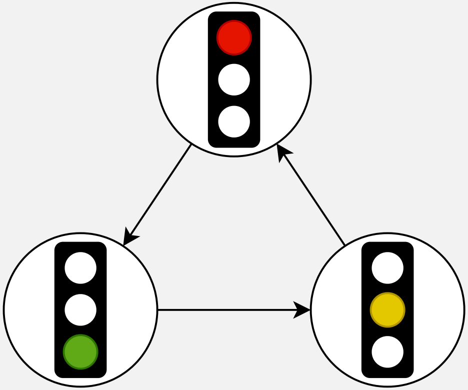

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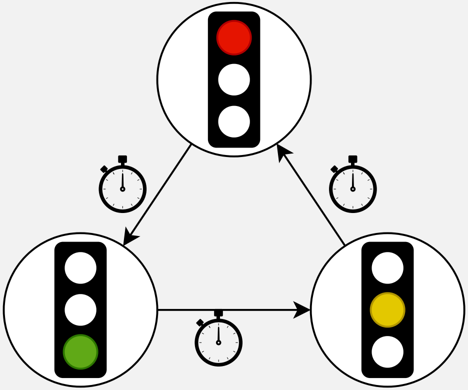

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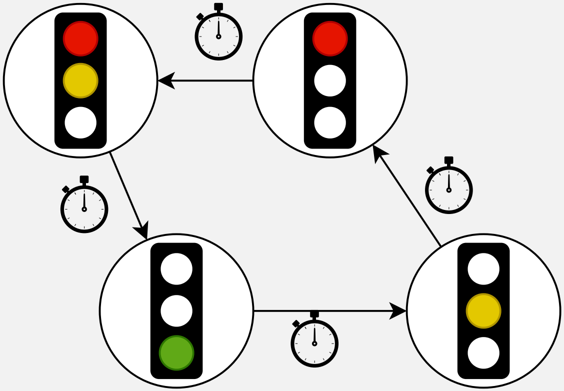

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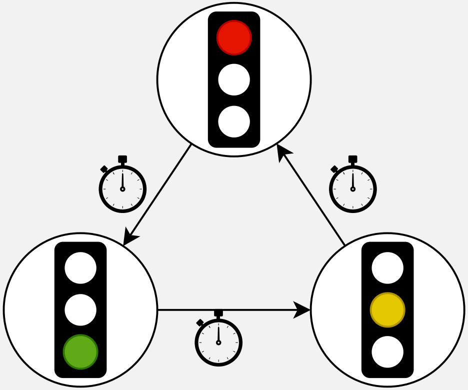

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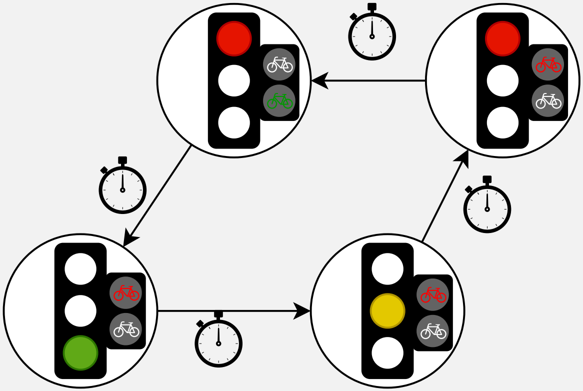
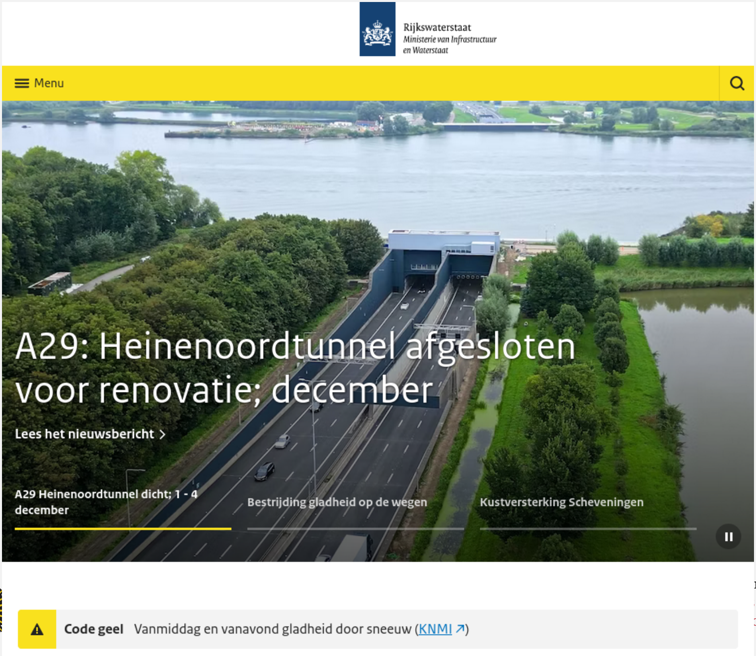


A Big Example


Rijkswaterstaat
Ministerie van Infrastructuur en Waterstaat

Menu

A29: Heinenoordtunnel afgesloten voor renovatie; december

Lees het nieuwsbericht >

A29 Heinenoordtunnel dicht; 1 - 4 december

Bestrijding gladheid op de wegen

Kustversterking Scheveningen

Code geel Vanmiddag en vanavond gladheid door sneeuw (KNMI ↗)

nce
ing
ces]

Moore Machines benefits

- Easy to use
- Easy to modify
- Easy to verify

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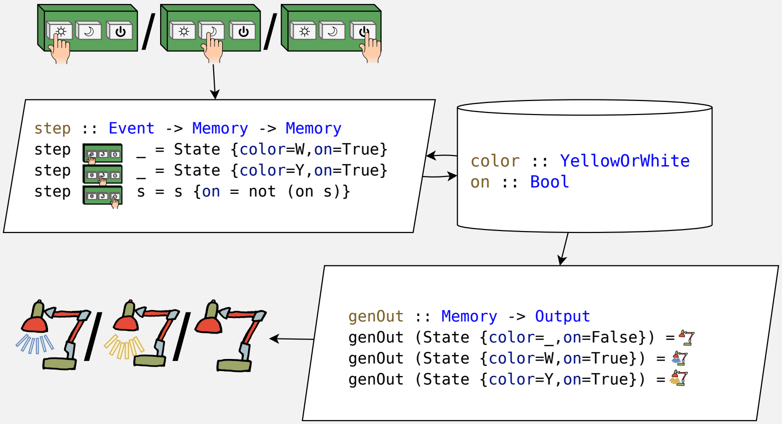
Moore Machines in Haskell



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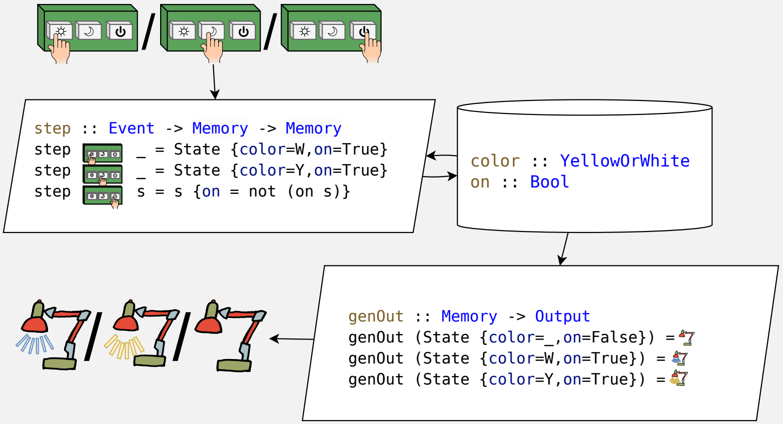

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```haskell
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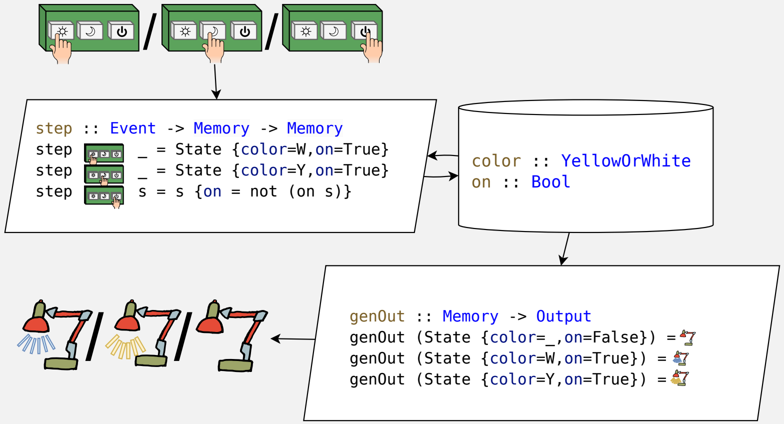

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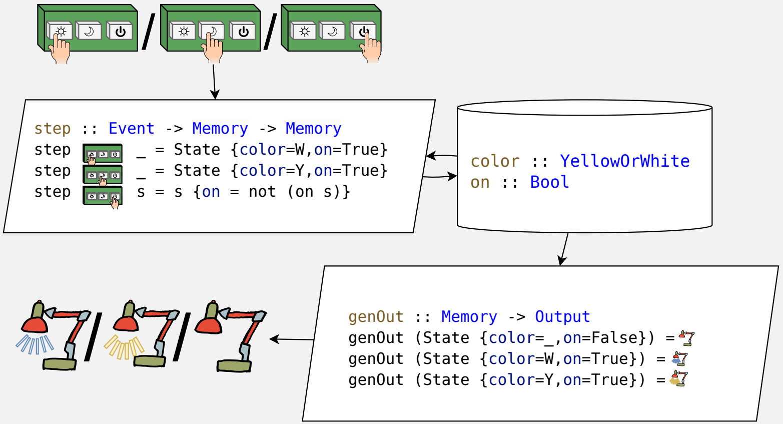

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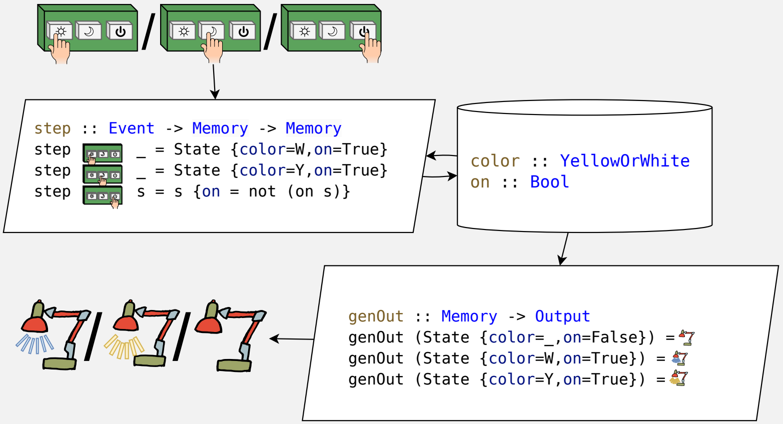

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type DFA symbol state = Moore symbol state Bool
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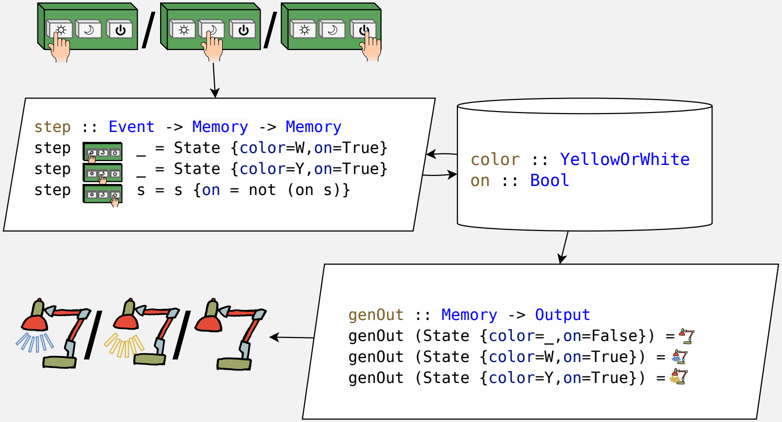

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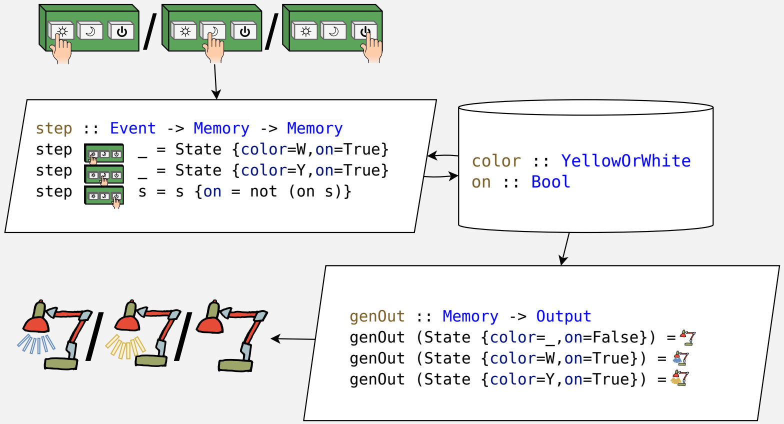

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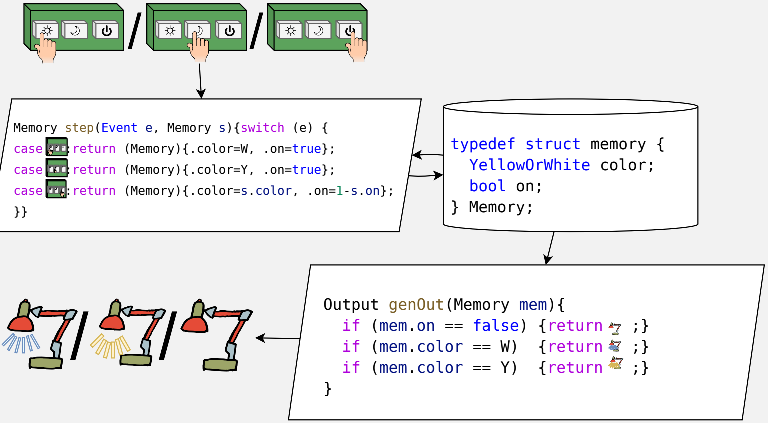
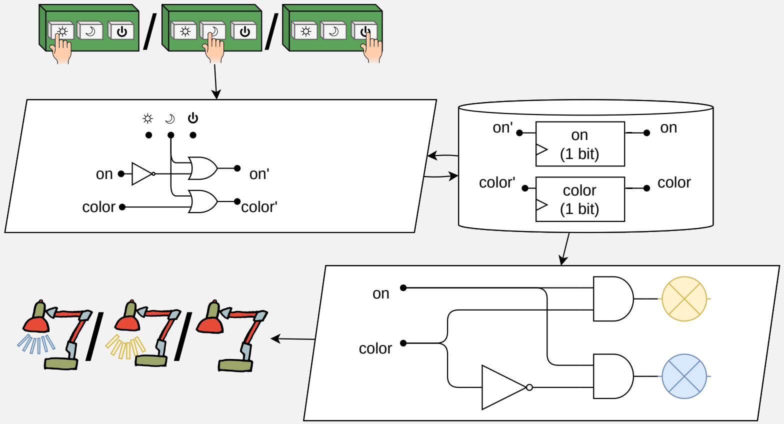


```
Memory step(Event e, Memory s){switch (e) {
case  :return (Memory){.color=W, .on=true};
case  :return (Memory){.color=Y, .on=true};
case  :return (Memory){.color=s.color, .on=1-s.on};
}}
```

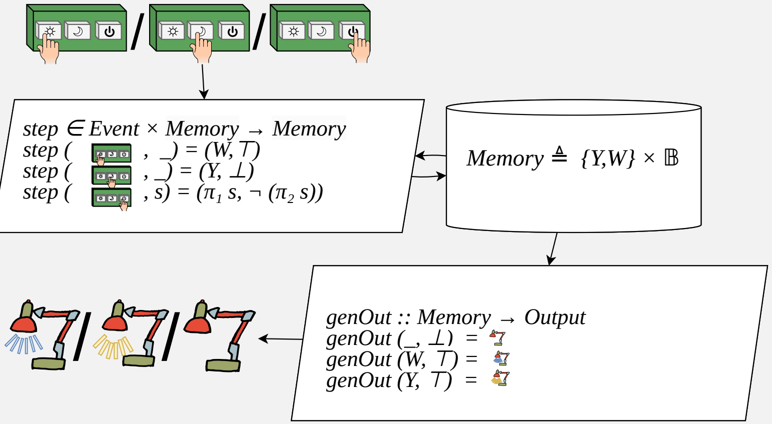
```
typedef struct memory {
  YellowOrWhite color;
  bool on;
} Memory;
```

```
Output genOut(Memory mem){
  if (mem.on == false) {return   ;}
  if (mem.color == W)  {return   ;}
  if (mem.color == Y)  {return   ;}
}
```

Moore Machines in Hardware



Moore Machines in Mathematics



$step \in Event \times Memory \rightarrow Memory$
$step\ (\quad , \_) = (W, \top)$
$step\ (\quad , \_) = (Y, \bot)$
$step\ (\quad , s) = (\pi_1\ s, \neg\ (\pi_2\ s))$

$Memory \triangleq \{Y,W\} \times \mathbb{B}$

$genOut :: Memory \rightarrow Output$
$genOut\ (\_, \bot) =$
$genOut\ (W, \top) =$
$genOut\ (Y, \top) =$

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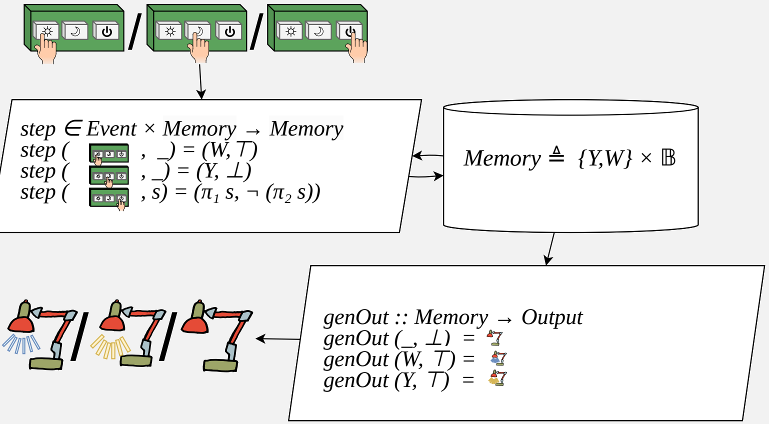
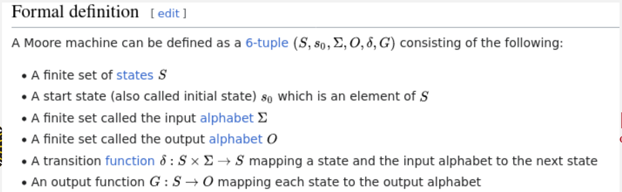
Moore Machines in Mathematics



$step \in Event \times Memory \to Memory$
$step\ (\quad , \_) = (W, \top)$
$step\ (\quad , \_) = (Y, \bot)$
$step\ (\quad , s) = (\pi_1\ s, \neg\ (\pi_2\ s))$

$Memory \triangleq \{Y,W\} \times \mathbb{B}$

$genOut :: Memory \to Output$
$genOut\ (\_, \bot) =$
$genOut\ (W, \top) =$
$genOut\ (Y, \top) =$

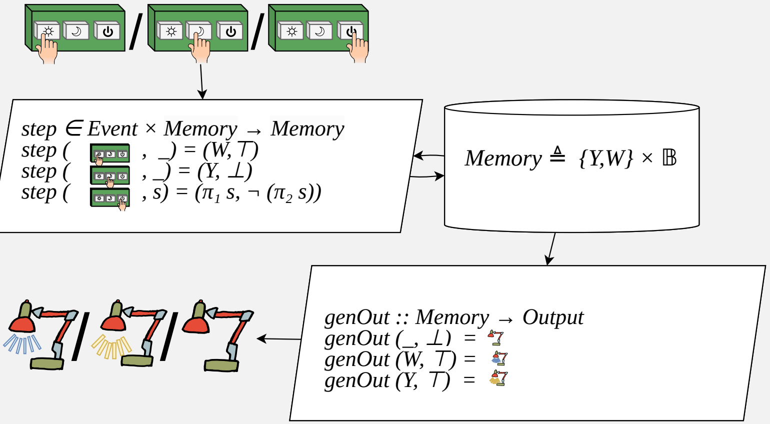


Formal definition [edit]

A Moore machine can be defined as a 6-tuple $(S, s_0, \Sigma, O, \delta, G)$ consisting of the following:

- A finite set of states S
- A start state (also called initial state) s_0 which is an element of S
- A finite set called the input alphabet Σ
- A finite set called the output alphabet O
- A transition function $\delta : S \times \Sigma \to S$ mapping a state and the input alphabet to the next state
- An output function $G : S \to O$ mapping each state to the output alphabet

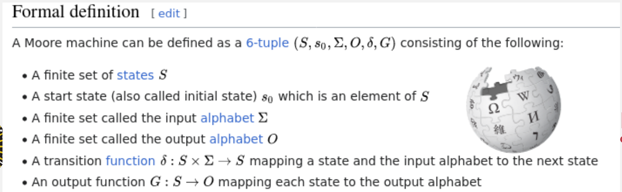
Moore Machines in Mathematics



$$\begin{aligned}
&step \in Event \times Memory \to Memory\\
&step\ (\quad, \_) = (W, \top)\\
&step\ (\quad, \_) = (Y, \bot)\\
&step\ (\quad, s) = (\pi_1\ s, \neg\ (\pi_2\ s))
\end{aligned}$$

$$Memory \triangleq \{Y,W\} \times \mathbb{B}$$

$$\begin{aligned}
&genOut :: Memory \to Output\\
&genOut\ (\_, \bot) =\\
&genOut\ (W, \top) =\\
&genOut\ (Y, \top) =
\end{aligned}$$



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Moore Machines summary

- Easy to use
- Easy to modify
- Easy to verify
- Easy to implement LaTeX

Moore Machines for RegExp Matching

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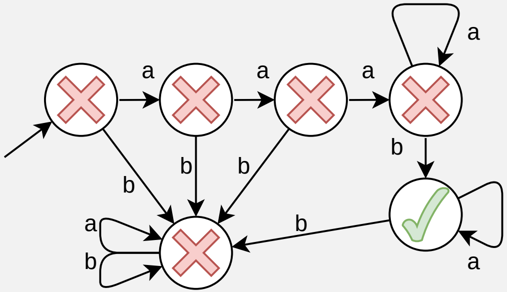

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Moore Machines for RegExp Matching

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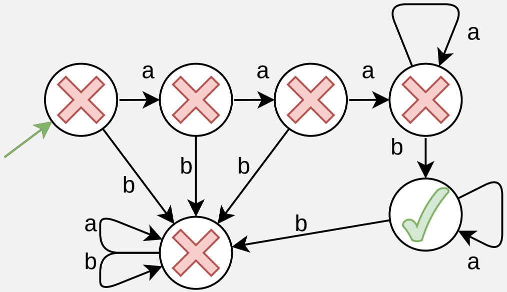

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Moore Machines for RegExp Matching

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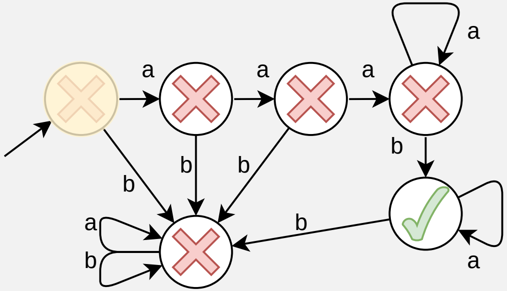

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Moore Machines for RegExp Matching

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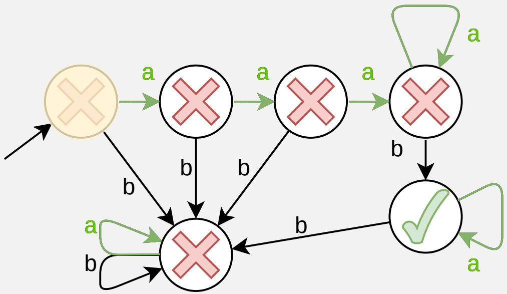

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Moore Machines for RegExp Matching

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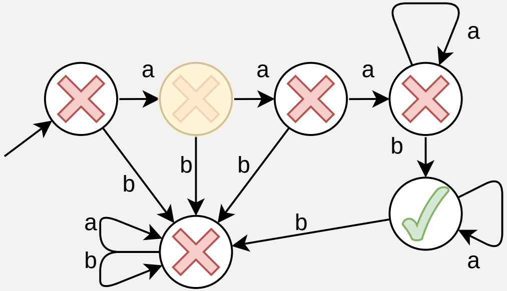

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Moore Machines for RegExp Matching

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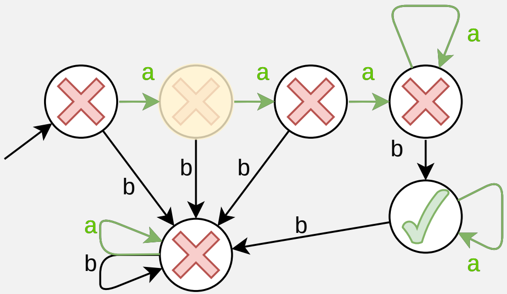

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Moore Machines for RegExp Matching

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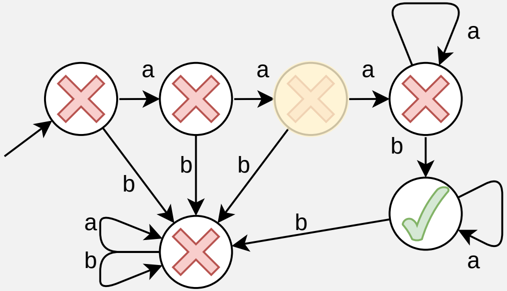

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Moore Machines for RegExp Matching

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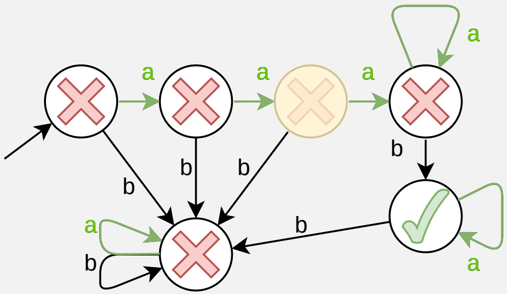

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Moore Machines for RegExp Matching

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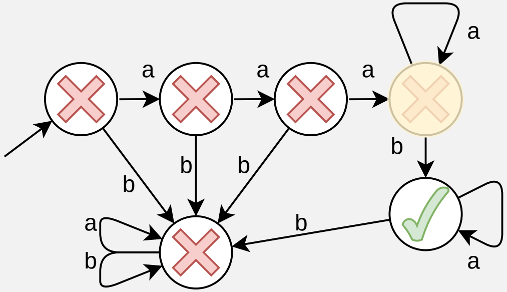

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Moore Machines for RegExp Matching

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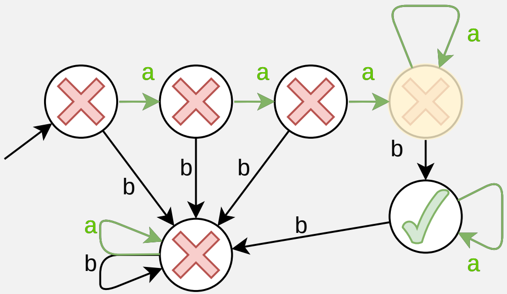

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Moore Machines for RegExp Matching

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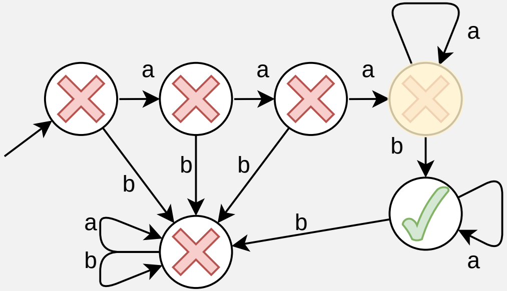

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Moore Machines for RegExp Matching

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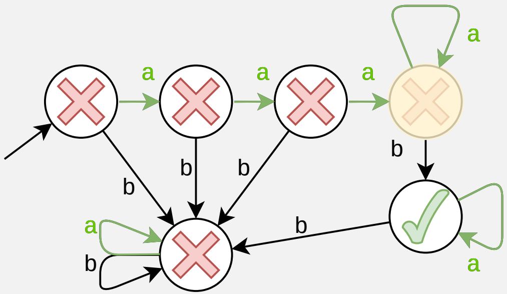

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Moore Machines for RegExp Matching

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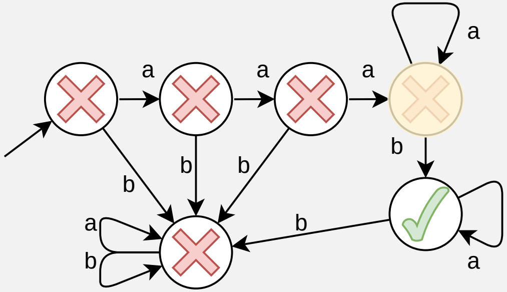

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Moore Machines for RegExp Matching

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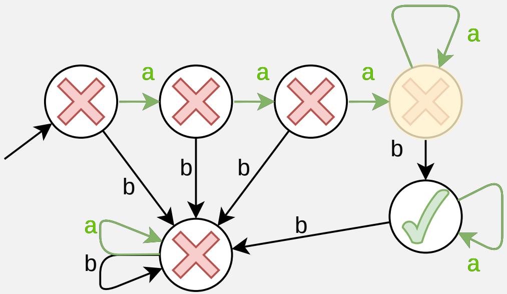

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Moore Machines for RegExp Matching

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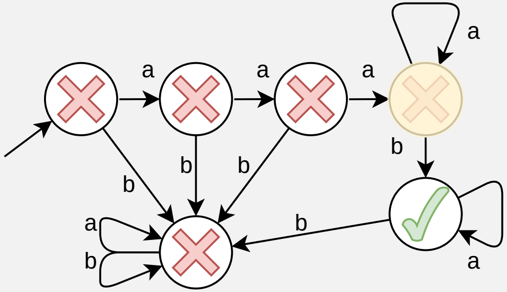

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Moore Machines for RegExp Matching

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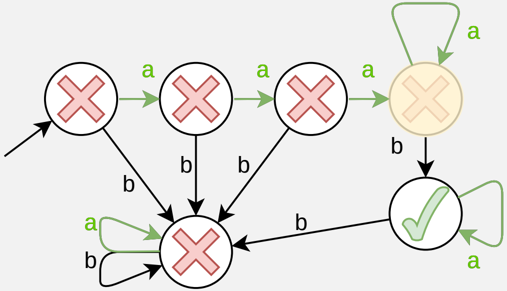

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Moore Machines for RegExp Matching

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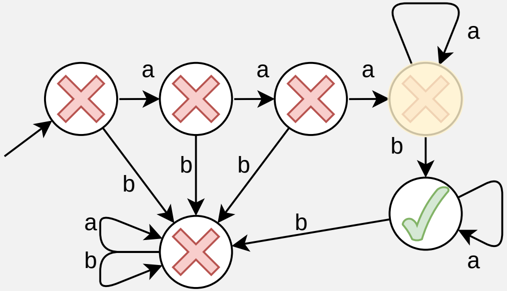

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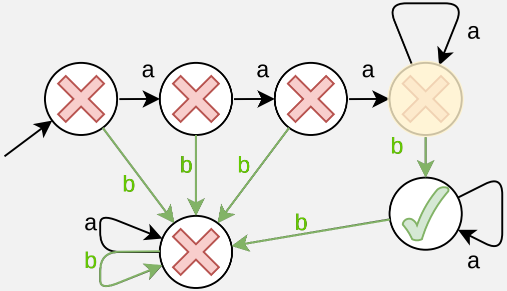

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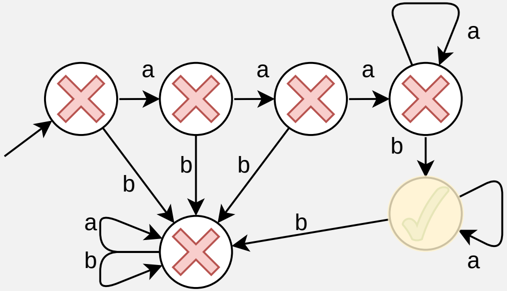

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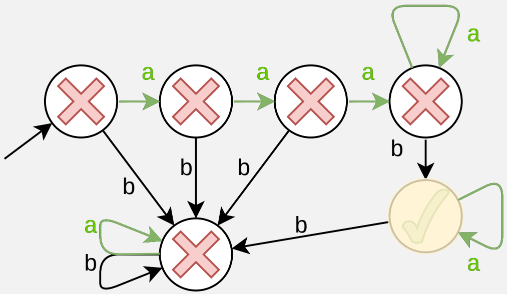

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Moore Machines for RegExp Matching

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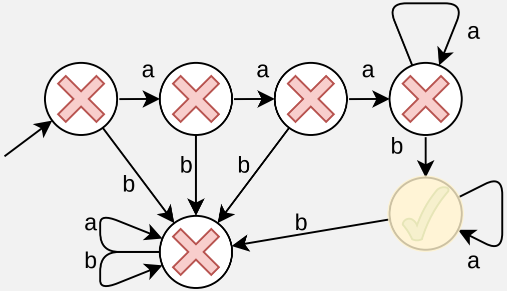

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Moore Machines for RegExp Matching

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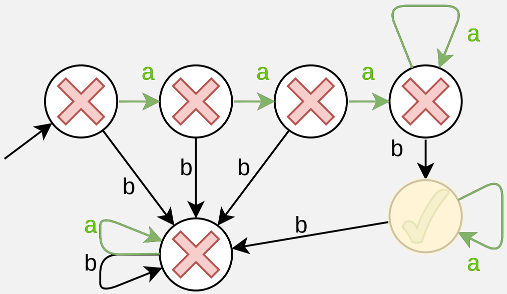

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Moore Machines for RegExp Matching

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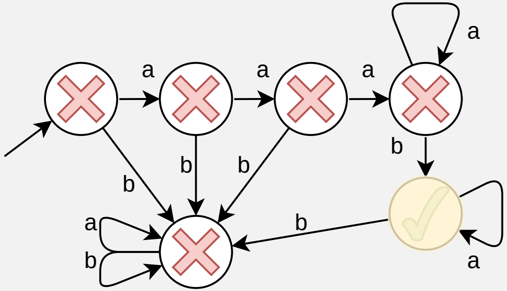

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Moore Machines for RegExp Matching

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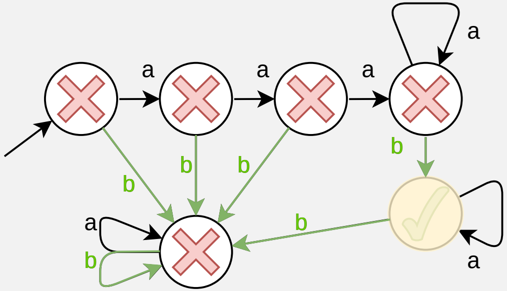

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Moore Machines for RegExp Matching

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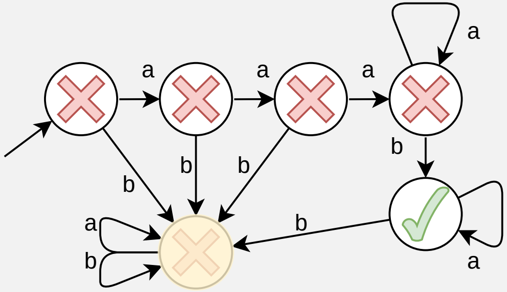

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Moore Machines for RegExp Matching

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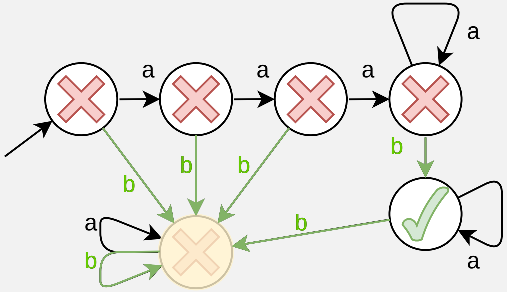

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Moore Machines for RegExp Matching

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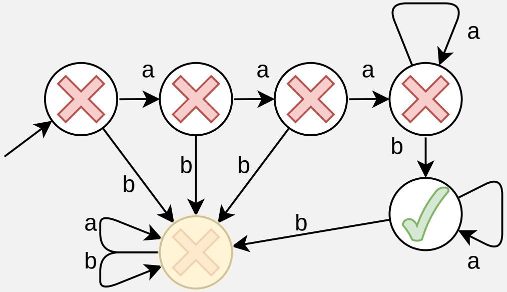

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Moore Machines for RegExp Matching

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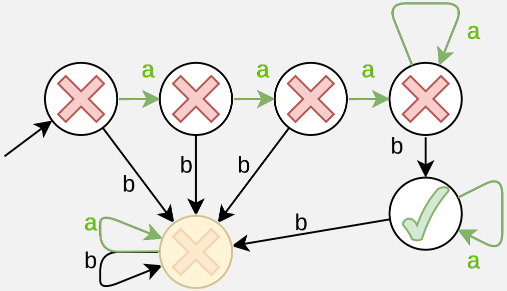

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Moore Machines for RegExp Matching

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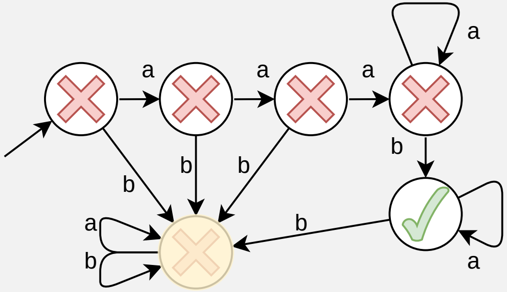

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More Moore Machine Matching

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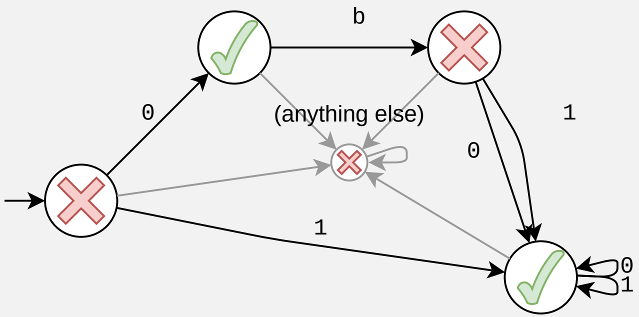

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More Moore Machine Matching

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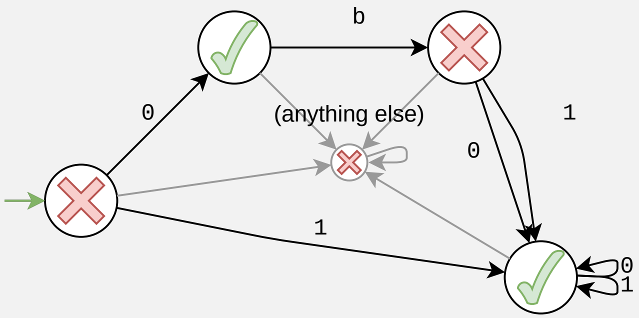

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More Moore Machine Matching

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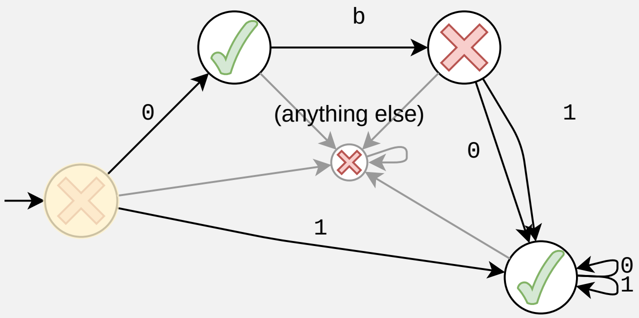

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More Moore Machine Matching

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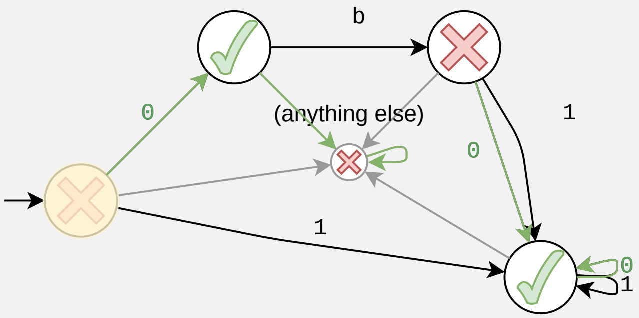

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More Moore Machine Matching

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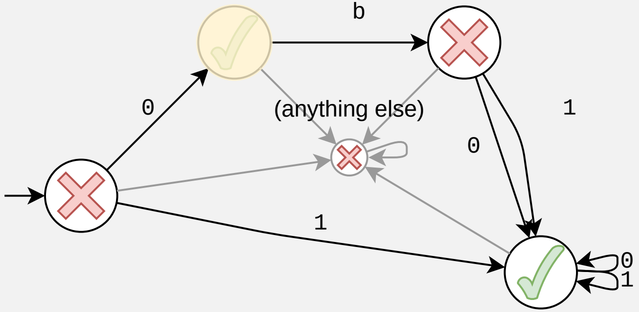

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More Moore Machine Matching

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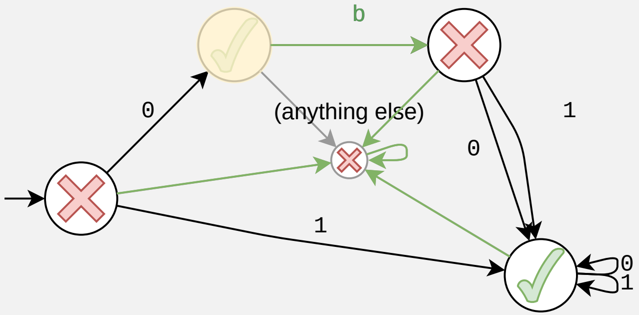

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More Moore Machine Matching

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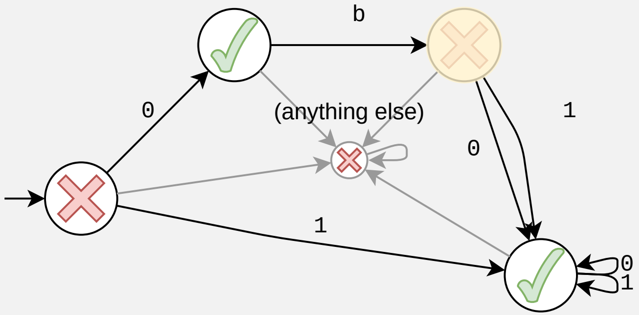

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More Moore Machine Matching

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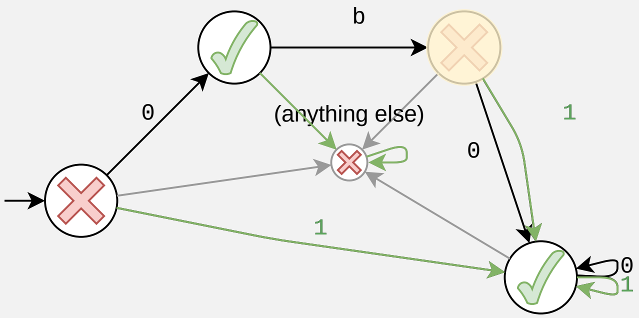

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More Moore Machine Matching

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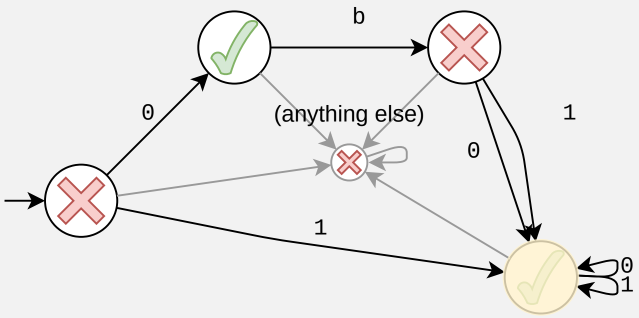

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More Moore Machine Matching

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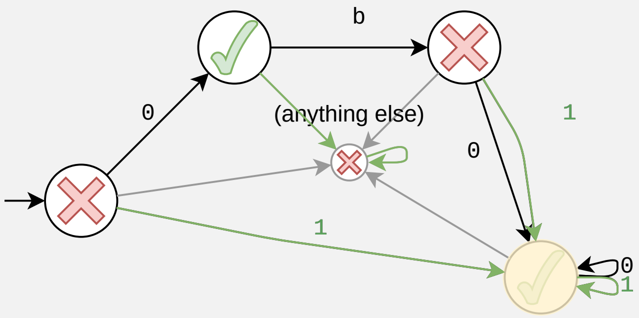

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More Moore Machine Matching

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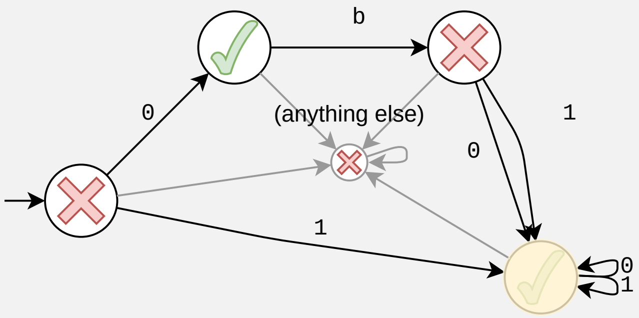

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More Moore Machine Matching

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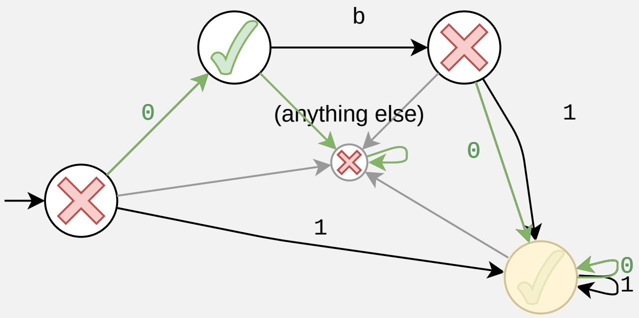

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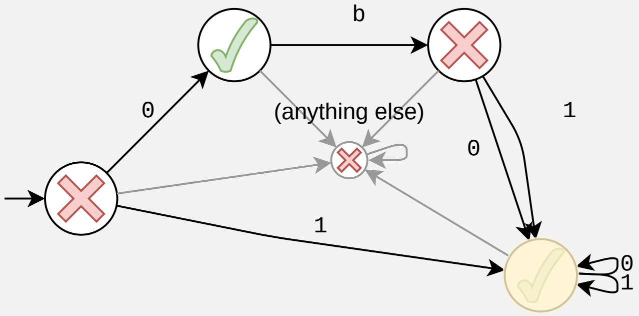

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More Moore Machine Matching

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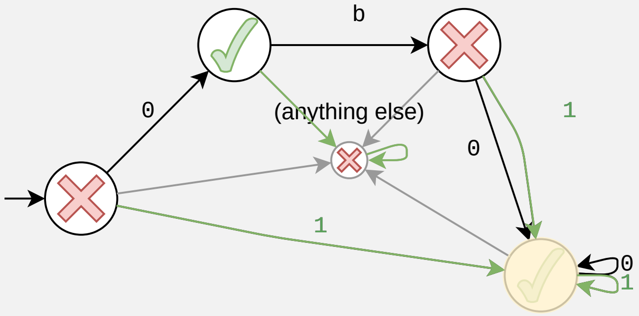

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More Moore Machine Matching

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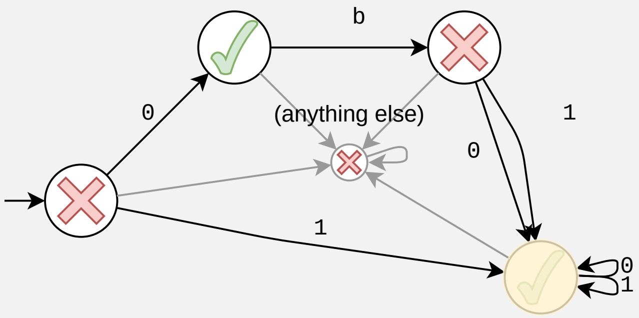

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More Moore Machine Matching

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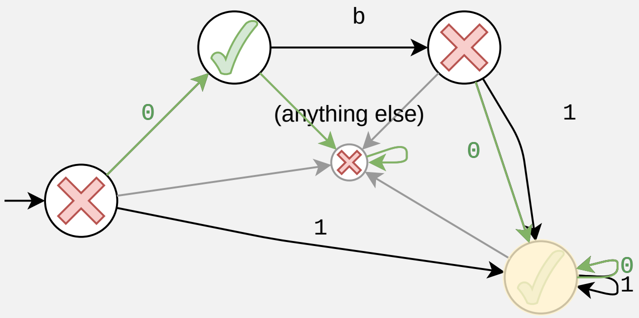

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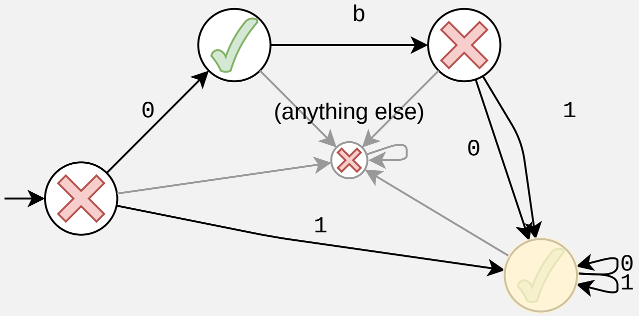

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More Moore Machine Matching

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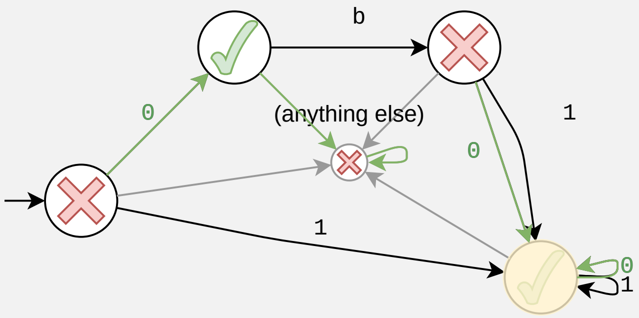

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More Moore Machine Matching

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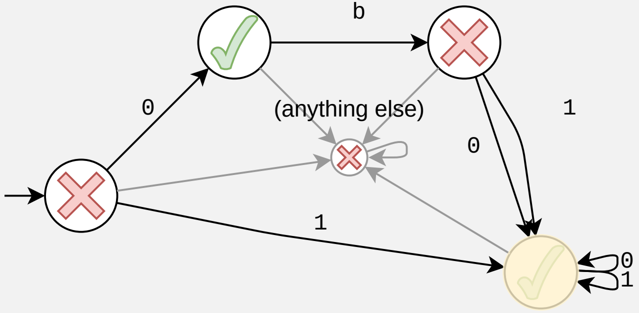

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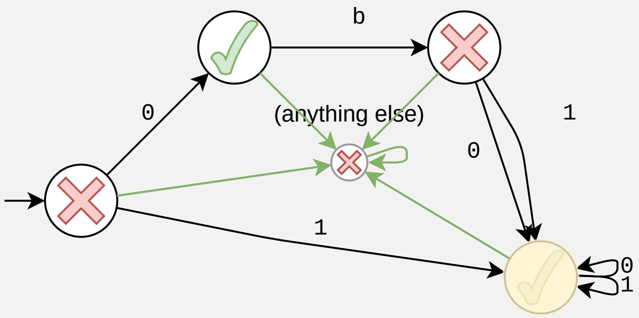
More Moore Machine Matching

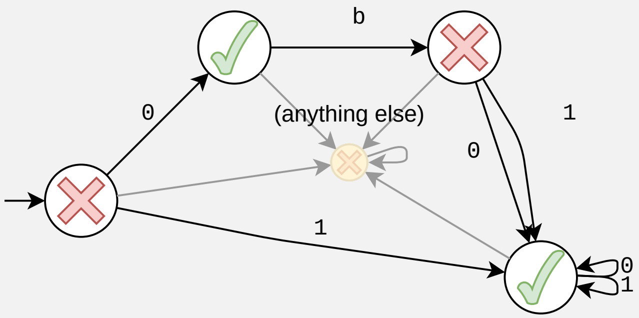
`(0b)?(0|1)+`



`0b110100[,10`

More Moore Machine Matching

`(0b)?(0|1)+`



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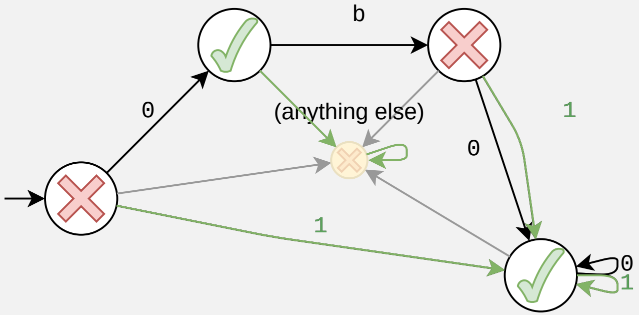
More Moore Machine Matching

`(0b)?(0|1)+`



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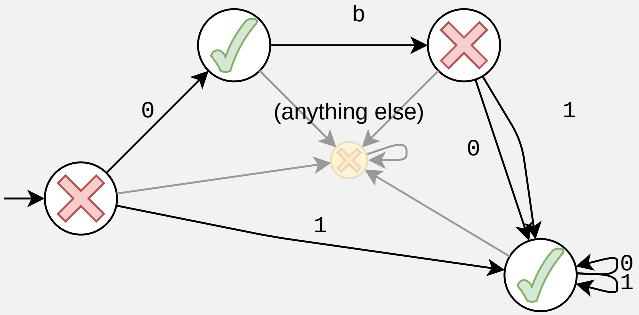
More Moore Machine Matching

`(0b)?(0|1)+`



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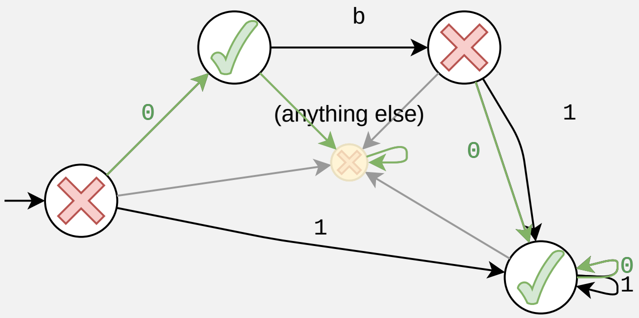

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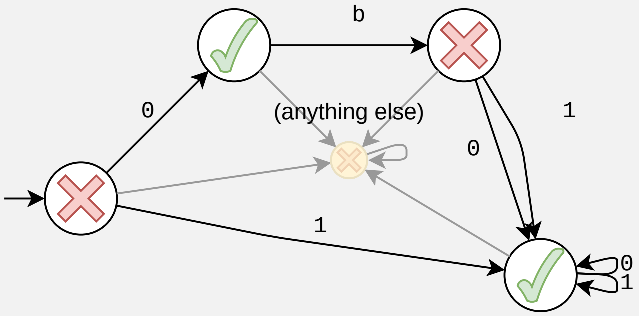
More Moore Machine Matching

`(0b)?(0|1)+`



`0b110100,10|`

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Making Matching Moore Machines

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co(bra|d)
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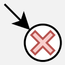
Making Matching Moore Machines

`co(bra|d)`



Making Matching Moore Machines

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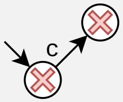

Making Matching Moore Machines

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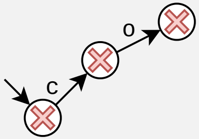

Making Matching Moore Machines

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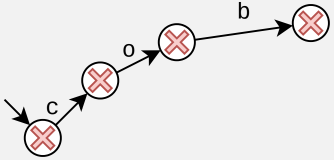

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Making Matching Moore Machines

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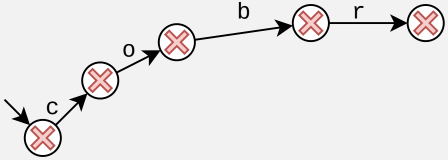

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Making Matching Moore Machines

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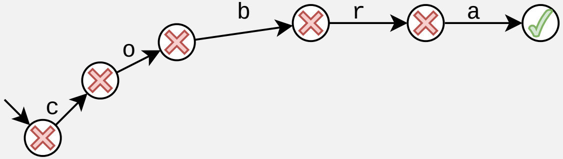

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Making Matching Moore Machines

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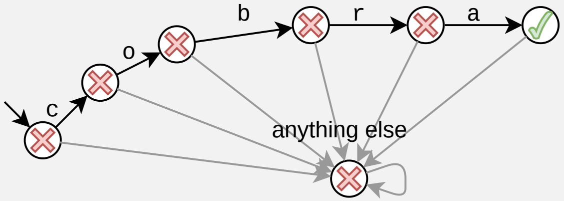
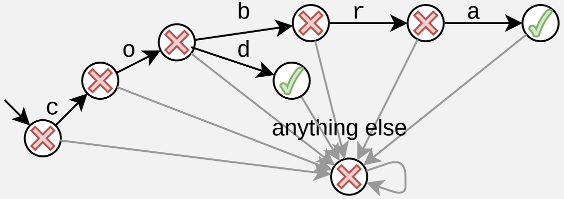


Making Matching Moore Machines

`co(bra|d)`



More Making Matching Moore Machines

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gr(a|e)y|green
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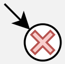
More Making Matching Moore Machines

`gr(a|e)y|green`



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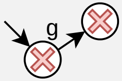

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More Making Matching Moore Machines

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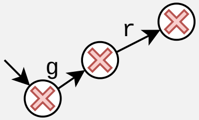

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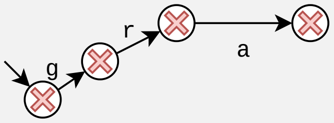

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More Making Matching Moore Machines

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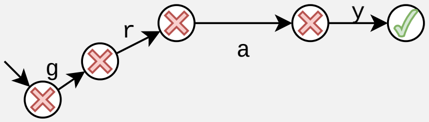

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More Making Matching Moore Machines

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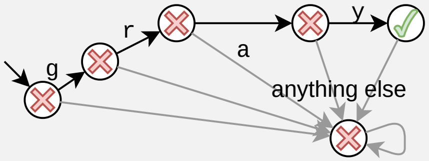

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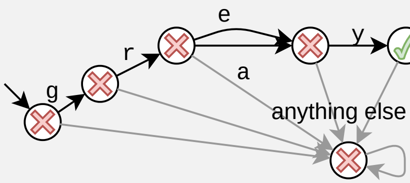

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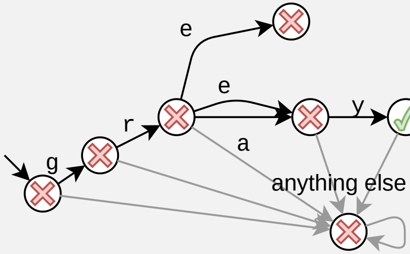

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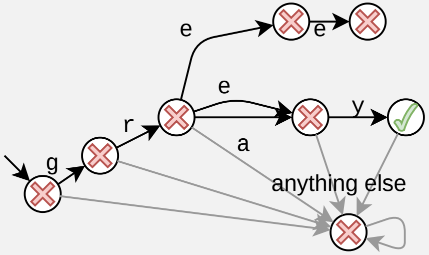

More Making Matching Moore Machines

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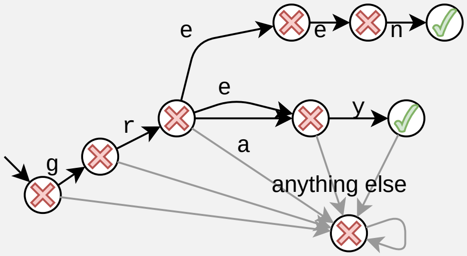

More Making Matching Moore Machines

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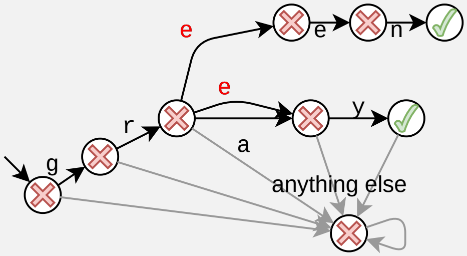

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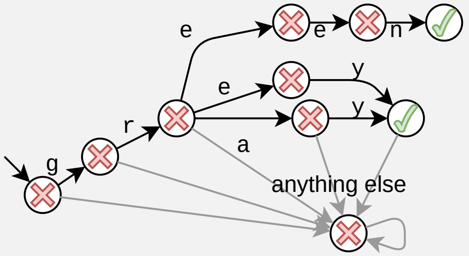
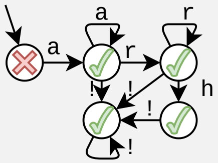
More Making Matching Moore Machines

`gr(a|e)y|green`



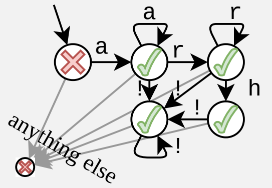
Quiz 1/4

Q: What is wrong with this Moore Machine?



Quiz 1/4

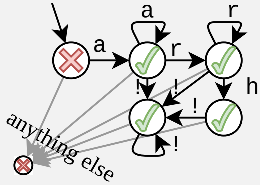
Q: What is wrong with this Moore Machine?



A: Transition Relation must be a function!

Quiz 2/4

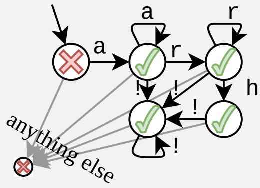
Q: Which strings are matched by this Moore Machine?



1. “arr”
2. “arrgh”
3. “arrh!!!”
4. “arhh!”
5. “aaaaa!!!”
6. “”
7. “arrh me hearties!”

Quiz 2/4

Q: Which strings are matched by this Moore Machine?



1. “`arr`” 👉
2. “`arrgh`”
3. “`arrh!!!`”👉
4. “`arhh!`”
5. “`aaaaa!!!`”👉
6. “”
7. “`arrh me hearties!`”

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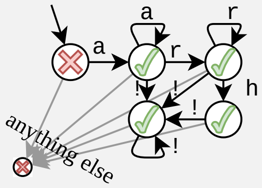
Quiz 3/4

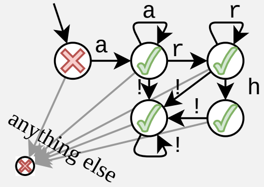
Q: What is the RegExp for this Moore Machine?



1. `arrh!`
2. `aa+r+h!+`
3. `aa*r*h?!*`
4. `a+r*h?!*`
5. `a*r?h!*?`
6. `a*r?h?(!+)?`

Quiz 3/4

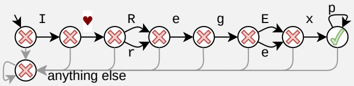
Q: What is the RegExp for this Moore Machine?

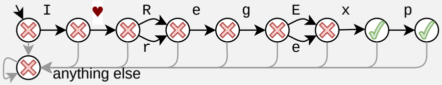


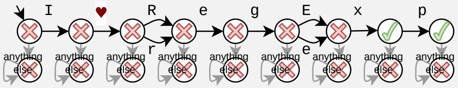
1. `arrh!`
2. `aa+r+h!+`
3. `aa*r*h?!*`👉
4. `a+r*h?!*`👉
5. `a*r?h!*?`
6. `a*r?h?(!+)?`👉

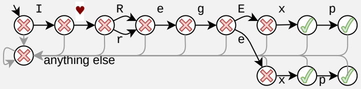
Quiz 4/4

Q: Which Moore Machine matches `I❤(R|r)eg(E|e)xp?` ?

1. 

2. 

3. 

4. 

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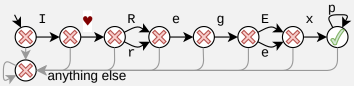
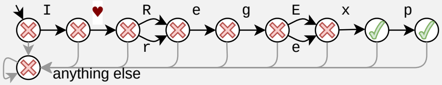
Quiz 4/4

Q: Which Moore Machine matches `I❤️(R|r)eg(E|e)xp?` ?

1. 

2. 

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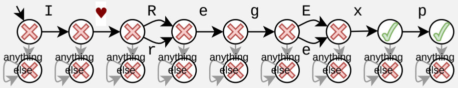
3. 

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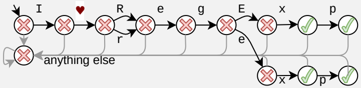
4. 

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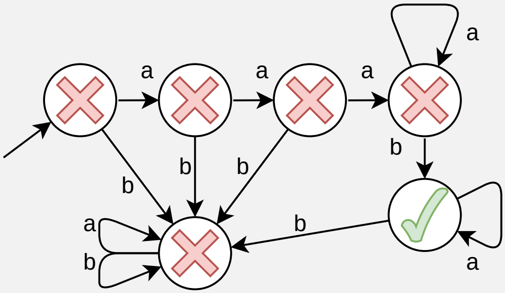
Recap



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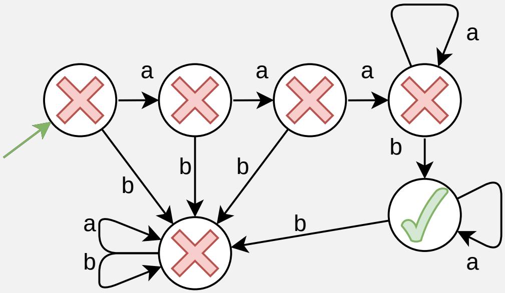
Recap



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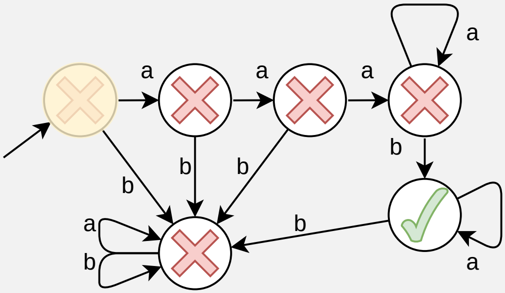
Recap



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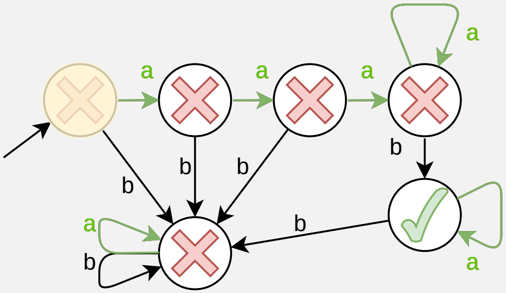

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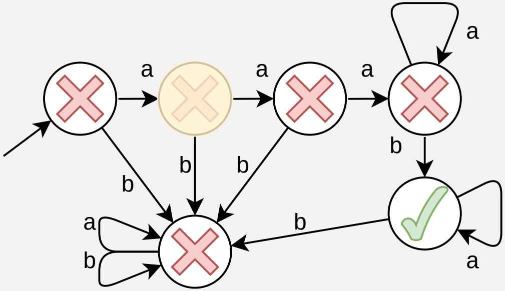
Recap



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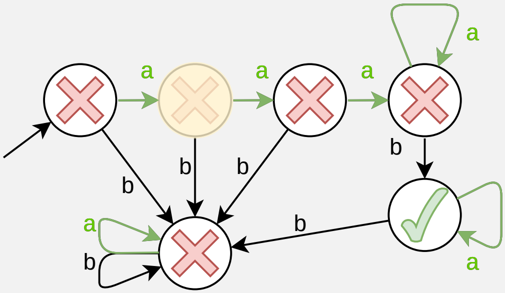
Recap



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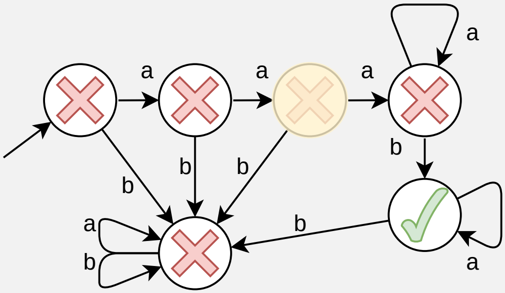

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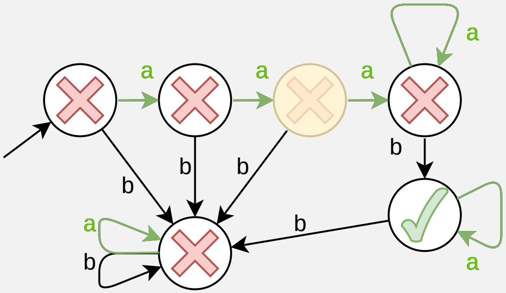

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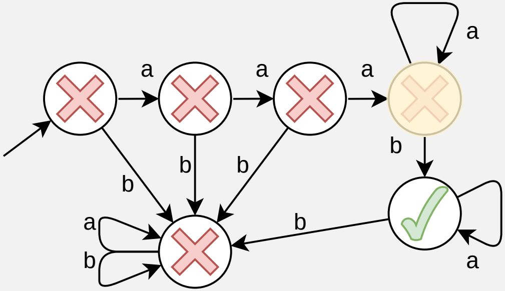
Recap



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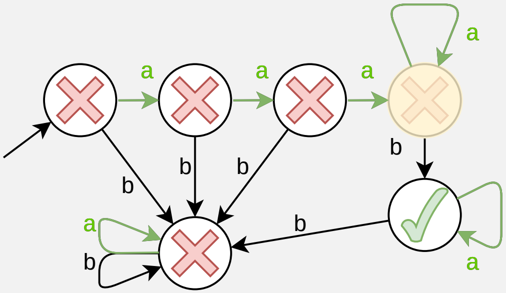
Recap



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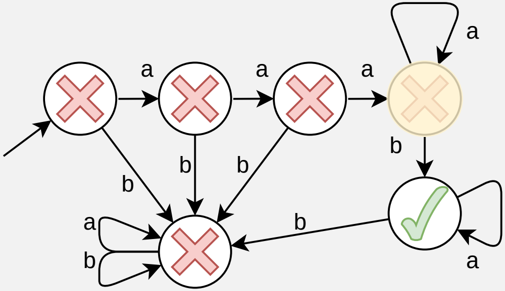
Recap



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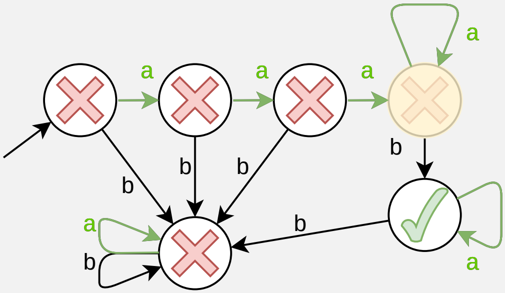
Recap



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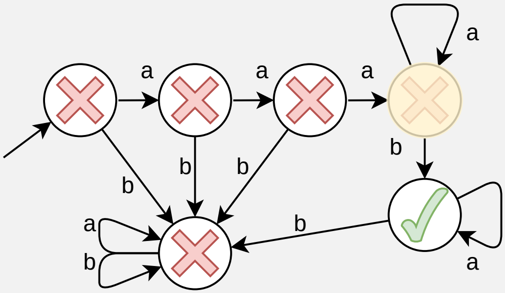

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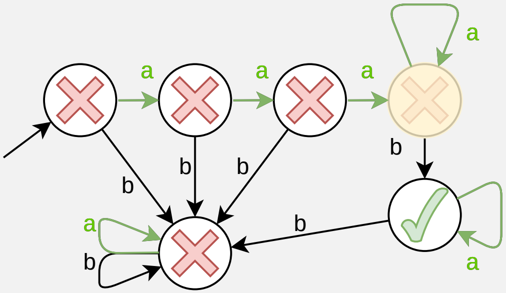

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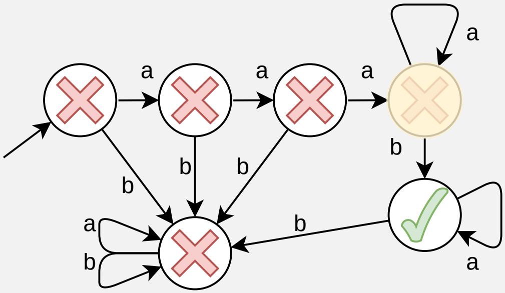
Recap



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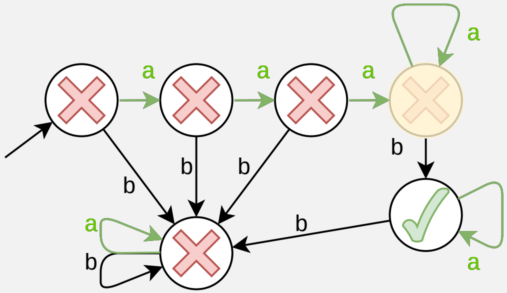
Recap



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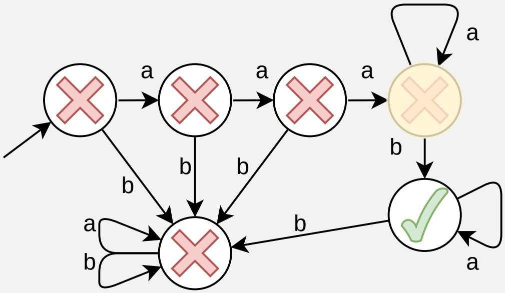
Recap



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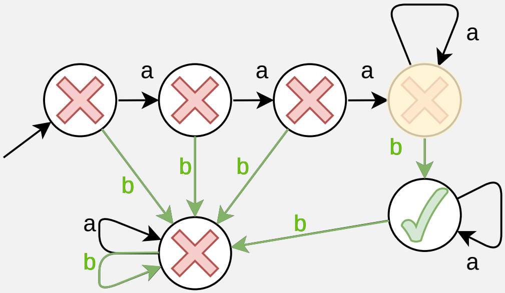
Recap



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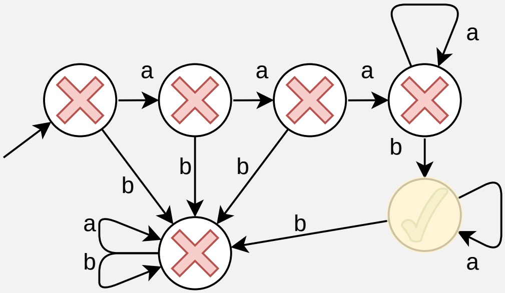
Recap



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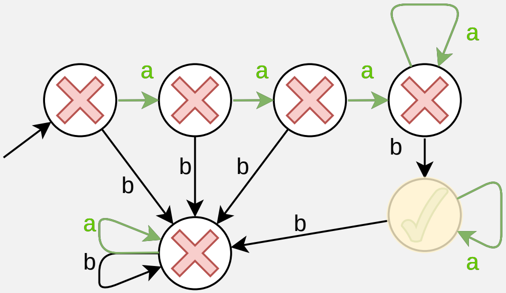
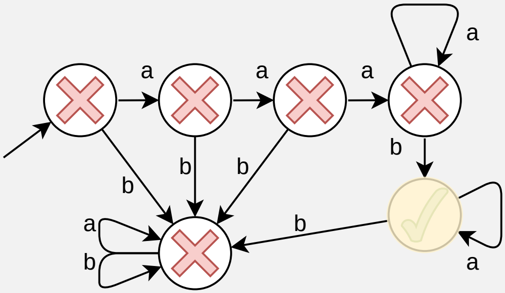
Recap



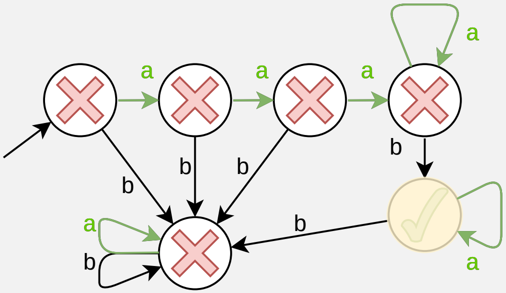
a*aaaba* ~~~~~>

Recap



`a*aaaba*` ~~~~~>

Recap



`a*aaaba*` ~~~~~>

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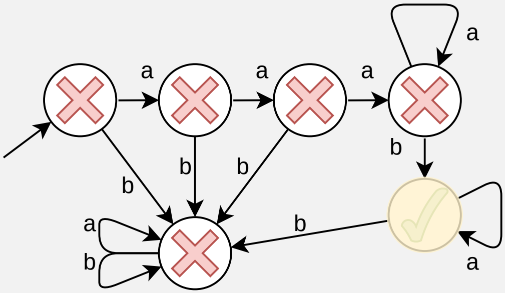
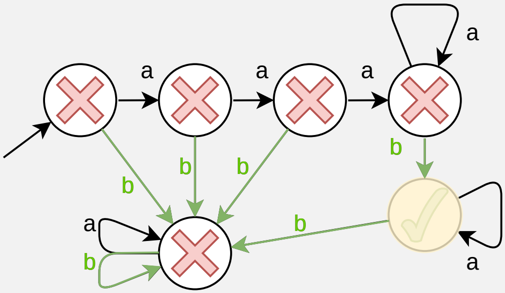
Recap



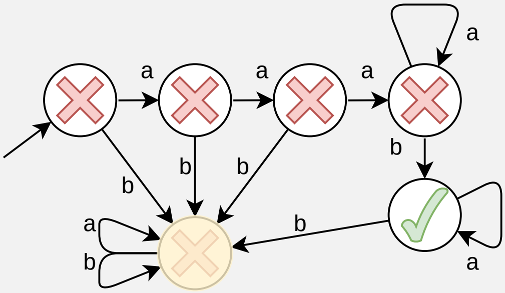
a*aaaba* ~~~~~>

Recap

`a*aaaba*` ~~~~~>

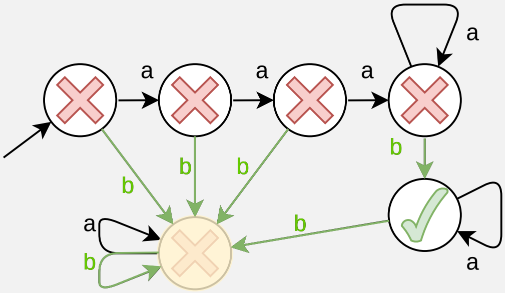


Recap



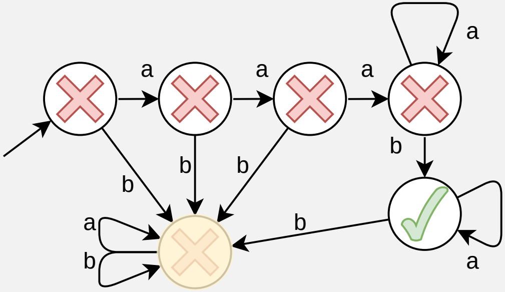
`a*aaaba*` ~~~~~>

Recap



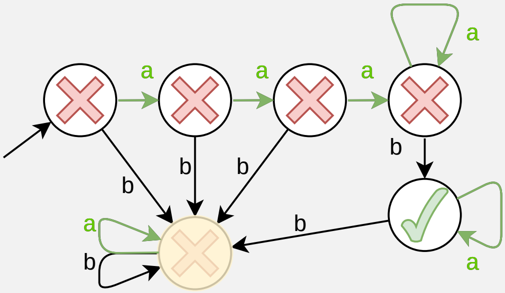
`a*aaaba* ~~~~~>`

Recap



a*aaaba* ~~~~~>

Recap



`a*aaaba*` ~~~~~>

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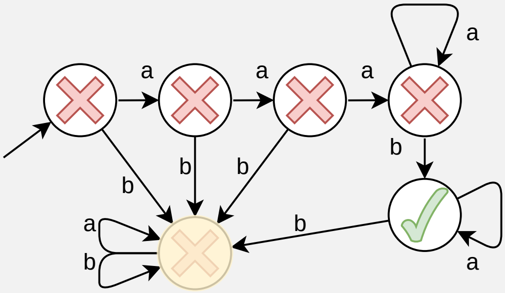
Recap



`a*aaaba*` ~~~~~>

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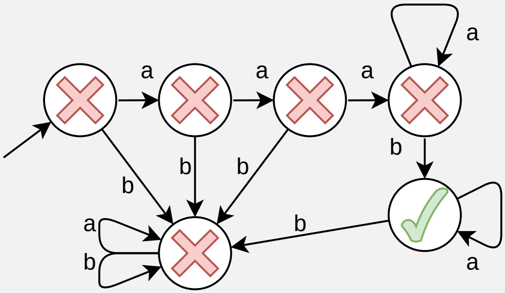
Recap



`a*aaaba*` ~~~~~>

Recap

`a*aaaba*` ~~~~~>

Recap

`a*aaaba* ~~~~~>`

Generally?

Running Moore Machines, generally

```
runMoore :: Moore inp state out → [inp] → state
```

```
runMoore (Moore step genOut s0) = ???
```

Running Moore Machines, generally

```
runMoore :: Moore inp state out → [inp] → state
```

```
runMoore (Moore step _ s0) = ???
```

Running Moore Machines, generally

```
runMoore :: Moore inp state out → [inp] → state
```

```
runMoore (Moore step _ s0) = runFrom s0 where
  runFrom :: state → [inp] → state
  runFrom st sys = ???
```

Running Moore Machines, generally

```
runMoore :: Moore inp state out → [inp] → state
```

```
runMoore (Moore step _ s0) = runFrom s0 where
  runFrom :: state → [inp] → state
  runFrom st [] = ???
  runFrom st (i:is) = ???
```

Running Moore Machines, generally

```
runMoore :: Moore inp state out → [inp] → state
```

```
runMoore (Moore step _ s0) = runFrom s0 where
  runFrom :: state → [inp] → state
  runFrom st [] = st
  runFrom st (i:is) = ???
```

Running Moore Machines, generally

```
runMoore :: Moore inp state out → [inp] → state
```

```
runMoore (Moore step _ s0) = runFrom s0 where
  runFrom :: state → [inp] → state
  runFrom st [] = st
  runFrom st (i:is) = runFrom ??? ???
```

Running Moore Machines, generally

```
runMoore :: Moore inp state out → [inp] → state
```

```
runMoore (Moore step _ s0) = runFrom s0 where
  runFrom :: state → [inp] → state
  runFrom st [] = st
  runFrom st (i:is) = runFrom (step i st) is
```

Running Moore Machines, generally

```
runMoore :: Moore inp state out → [inp] → state
```

```
runMoore (Moore step _ s0) = runFrom s0 where
  runFrom :: state → [inp] → state
  runFrom st [] = st
  runFrom st (i:is) = runFrom (step i st) is
```

🕵 Look familiar?

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Running Moore Machines, generally

```
runMoore :: Moore inp state out → [inp] → state
```

```
runMoore (Moore step _ s0) = runFrom s0 where
  runFrom :: state → [inp] → state
  runFrom st is = foldr step st is
```

Running Moore Machines, generally

```
runMoore :: Moore inp state out → [inp] → state
```

```
runMoore (Moore step _ s0) = runFrom s0 where
  runFrom :: state → [inp] → state
  runFrom = foldr step
```

Running Moore Machines, generally

```
runMoore :: Moore inp state out → [inp] → state
```

```
runMoore (Moore step _ s0) = foldr step s0
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Running Moore Machines, generally

```
runMoore :: Moore inp state out → [inp] → state
```

```
runMoore (Moore step _ s0) = foldr step s0
```

Running Moore Machines, generally

```
runMoore :: Moore inp state out → [inp] → state
```

```
runMoore (Moore step _ s0) = foldr step s0
```

Running DFAs, generally

```
runDFA :: DFA symbol state → [symbol] → state
runDFA = runMoore
```

Running Moore Machines, generally

```
runMoore :: Moore inp state out → [inp] → state
```

```
runMoore (Moore step _ s0) = foldr step s0
```

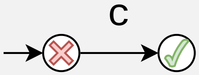
Running DFAs, generally

```
runDFA :: DFA symbol state → [symbol] → state
runDFA = runMoore
```

```
matchesDFA :: DFA symbol state → [symbol] → Bool
matchesDFA dfa = genOutput dfa . runDFA dfa
```

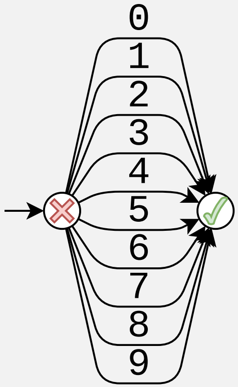
Compiling RegExp to DFA, generally

c



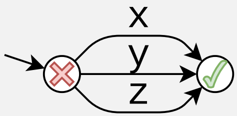
Compiling RegExp to DFA, generally

`\d`



Compiling RegExp to DFA, generally

`[x-z]`

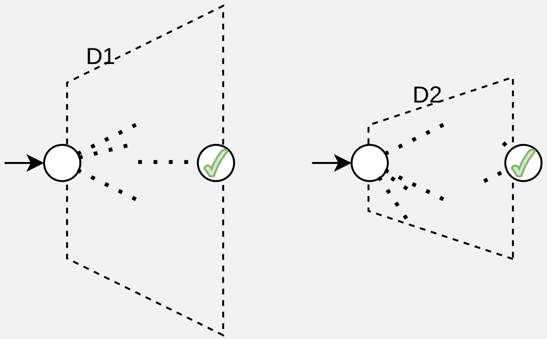


Compiling RegExp to DFA, generally

$\mathtt{r}_1\mathtt{r}_2$

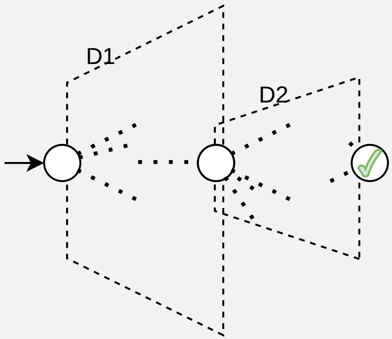
Compiling RegExp to DFA, generally

$\mathtt{r}_1\mathtt{r}_2$



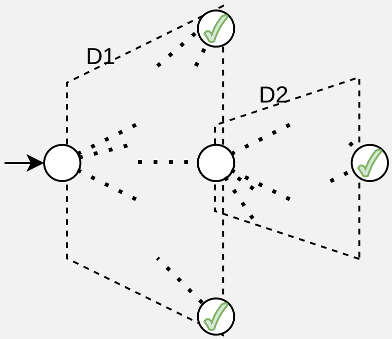
Compiling RegExp to DFA, generally

$r_1 r_2$



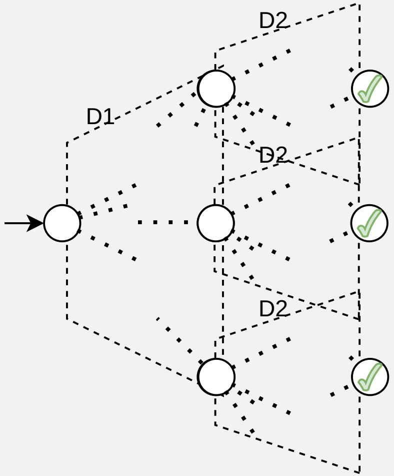
Compiling RegExp to DFA, generally

r_1r_2



Compiling RegExp to DFA, generally

r_1r_2



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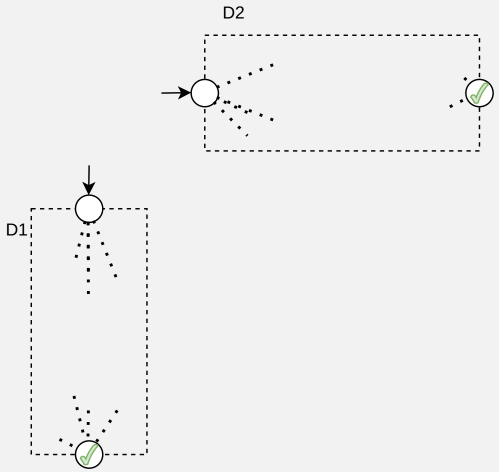
Compiling RegExp to DFA, generally

$\mathtt{r}_1 | \mathtt{r}_2$

Compiling RegExp to DFA, generally

$r_1 | r_2$



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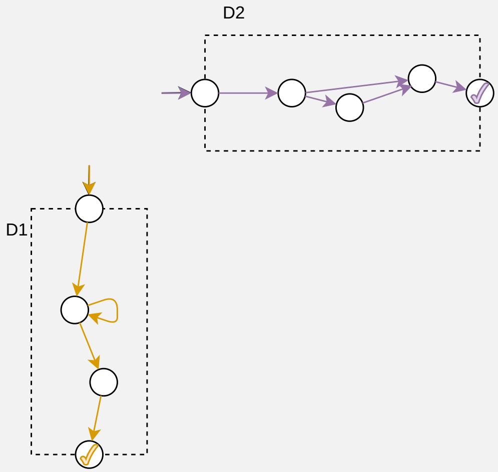
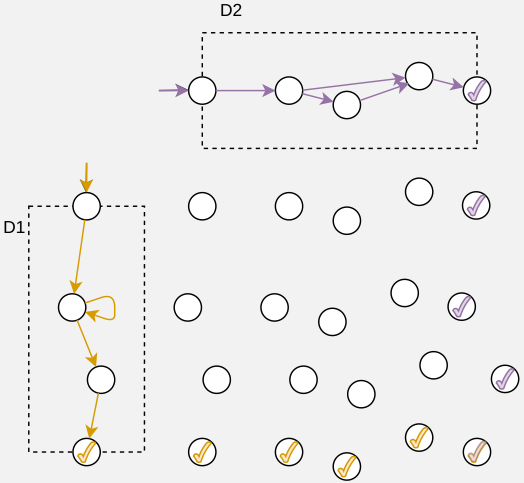
Compiling RegExp to DFA, generally

$r_1 | r_2$



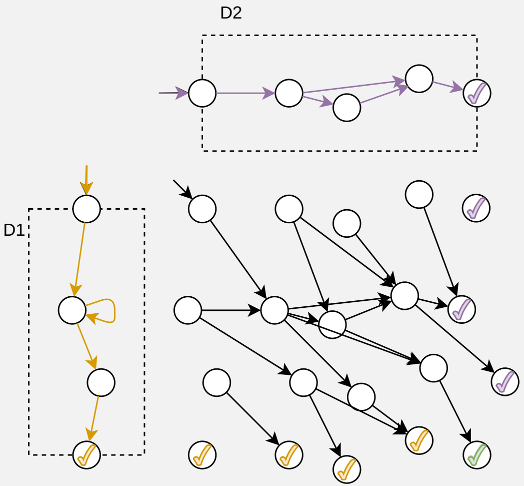
Compiling RegExp to DFA, generally

$r_1 | r_2$



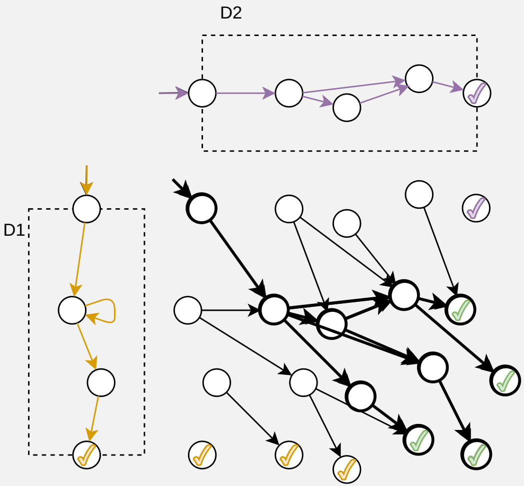
Compiling RegExp to DFA, generally

`r`$_1$ | `r`$_2$



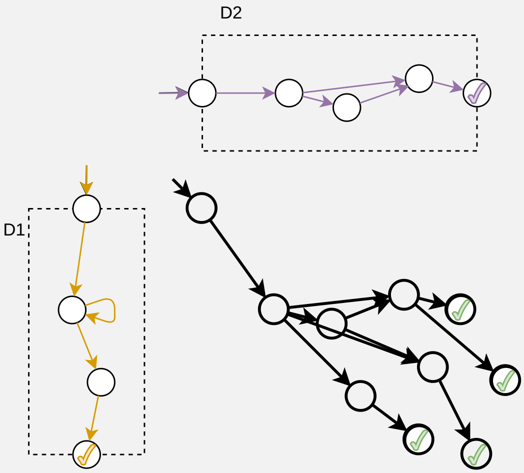
Compiling RegExp to DFA, generally

$r_1 | r_2$



Compiling RegExp to DFA, generally

`r`$_1$`|r`$_2$



Compiling RegExp to DFA, generally

`r+`

Compiling RegExp to DFA, generally

`r+`

🧠

Compiling RegExp to DFA, generally

`r+`

❓

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Compiling RegExp to DFA, generally

`r*`

❓

Compiling RegExp to DFA, generally

`r?`

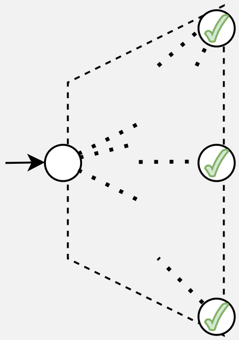
❓

Compiling RegExp to DFA progress

- ✅ `c`
- ✅ `\d`
- ✅ `[x-z]`
- 💣 r_1r_2
- 💣 $r_1|r_2$
- ❓ `r+`
- ❓ `r*`
- ❓ `r?`

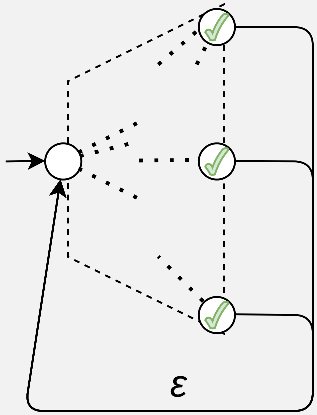
Compiling RegExp to NFAε

`r+`



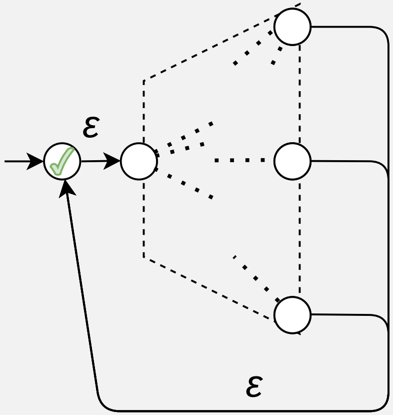
Compiling RegExp to NFAε

`r+`



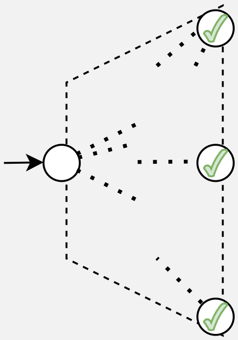
Compiling RegExp to NFAε

`r*`



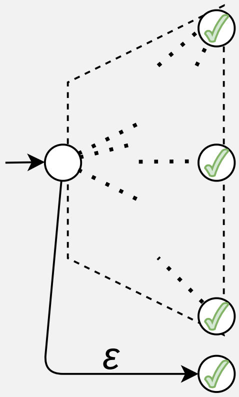
Compiling RegExp to NFAε

`r?`



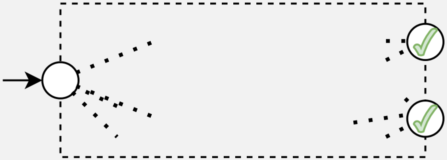
Compiling RegExp to NFAε

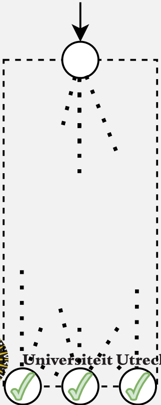
`r?`



Compiling RegExp to NFAε

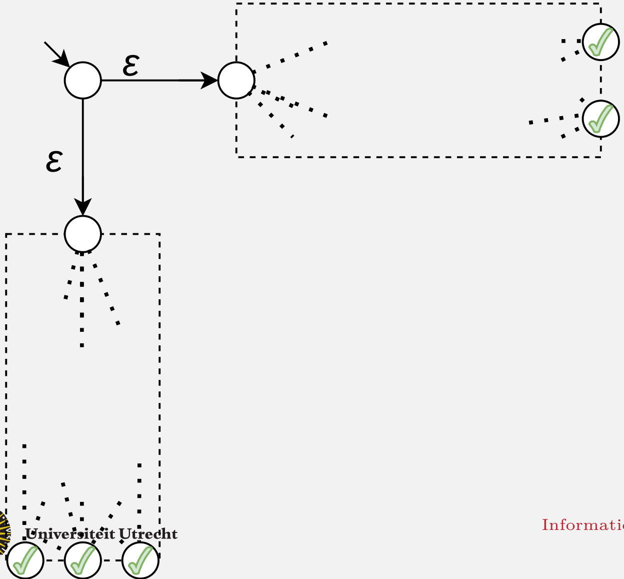
`r`$_1$|`r`$_2$





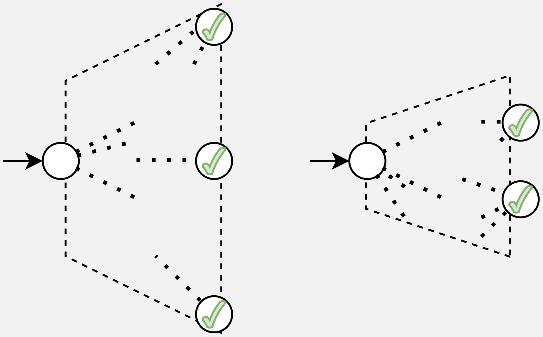
Compiling RegExp to NFAε

$r_1 | r_2$



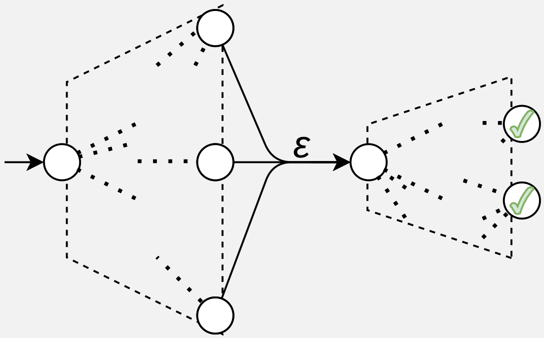
Compiling RegExp to NFAε

$r_1 r_2$



Compiling RegExp to NFAε

r_1r_2



Compiling RegExp to NFAε progress

- ✅ `c`
- ✅ `\d`
- ✅ `[x-z]`
- ✅ $\texttt{r}_1\texttt{r}_2$
- ✅ $\texttt{r}_1|\texttt{r}_2$
- ✅ `r+`
- ✅ `r*`
- ✅ `r?`

Running an NFAε

```
runDFA :: DFA symbol state → [symbol] → state
runDFA = runMoore
```

Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε = ???
```

Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) = ???
```

Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) = ???
```

```
a*aaaba*
```

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Running an NFAε

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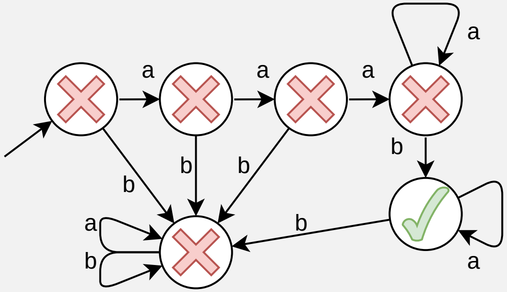
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) = ???
```

a*aaaba*



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Running an NFAε

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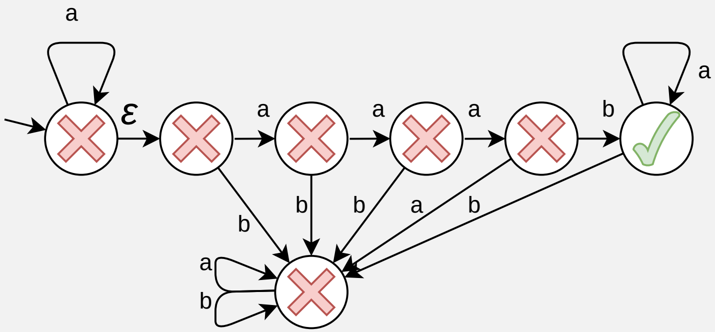
a*aaaba*



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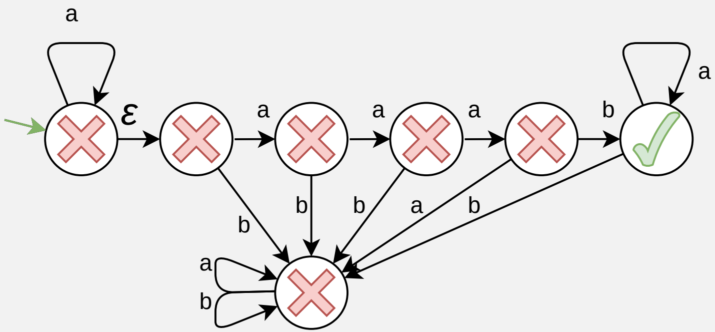
Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) = ???
```

```
a*aaaba*
```

```
aaaaab
```



Running an NFAε

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runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) = ???
```

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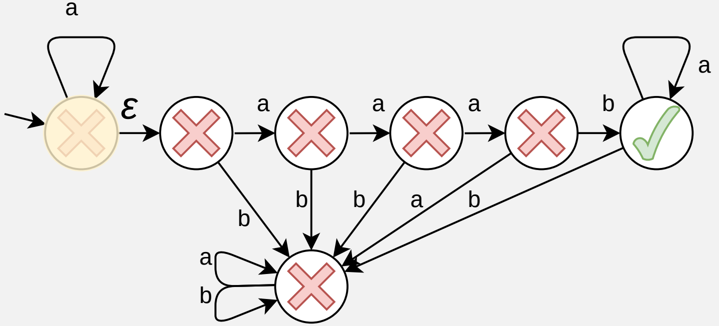
]aaaaab

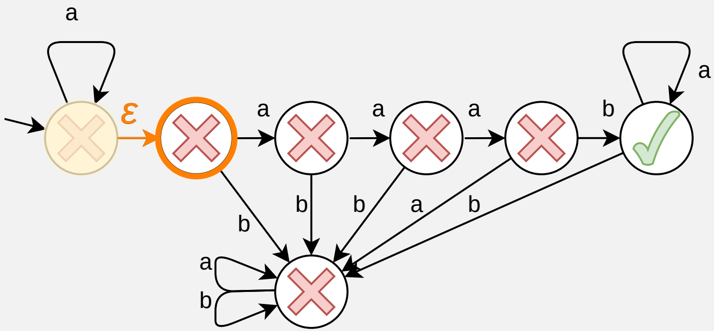


Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) = ???
```

a*aaaba*

!aaaaab



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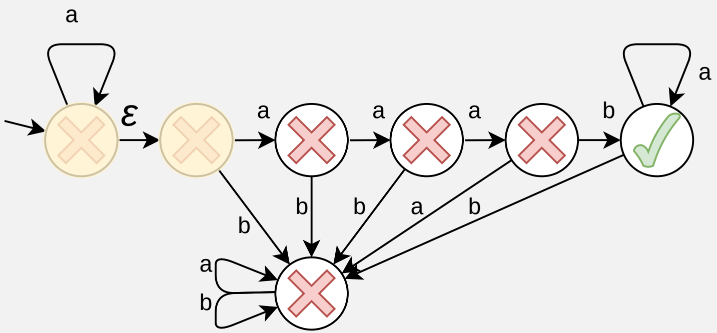
Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) = ???
```

a*aaaba*

|aaaaab



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Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) = ???
```

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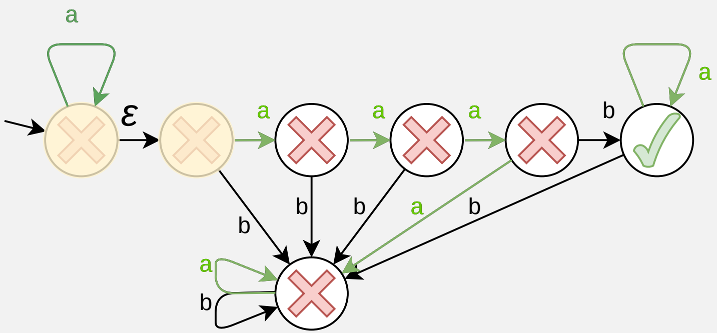
[aaaaab

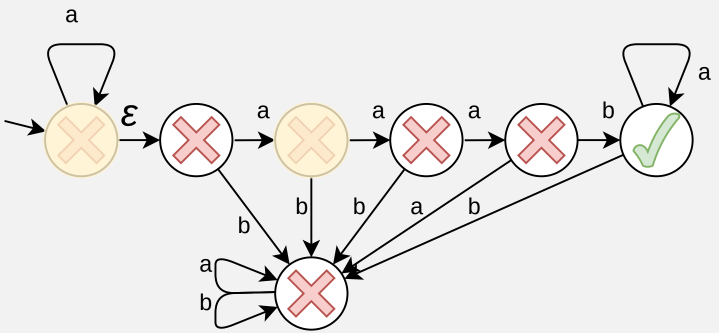


Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) = ???
```

a*aaaba*

a]aaaab



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Running an NFAε

```
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runNFAε (NFAε step εsteps genOut s0) = ???
```

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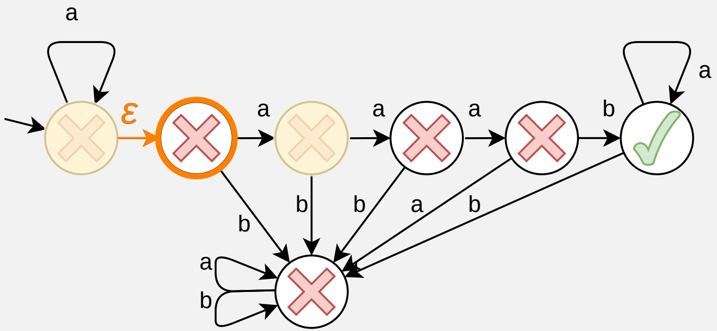
a!aaaab



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Running an NFAε

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runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) = ???
```

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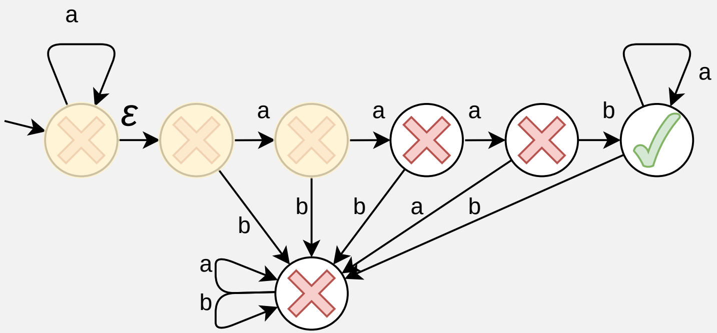
a|aaaab



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Running an NFAε

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runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) = ???
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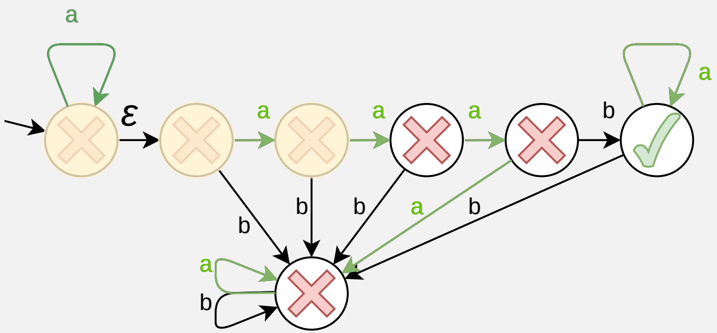
a[aaaab

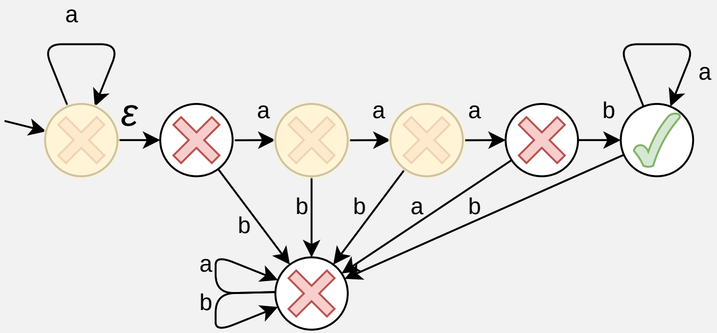


Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) = ???
```

a*aaaba*

aa]aaab



Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) = ???
```

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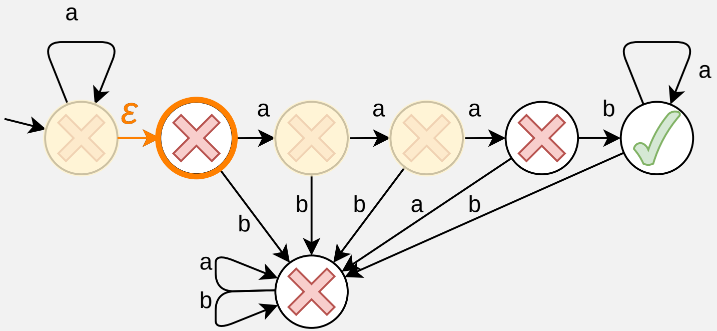
aa!aaab



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Running an NFAε

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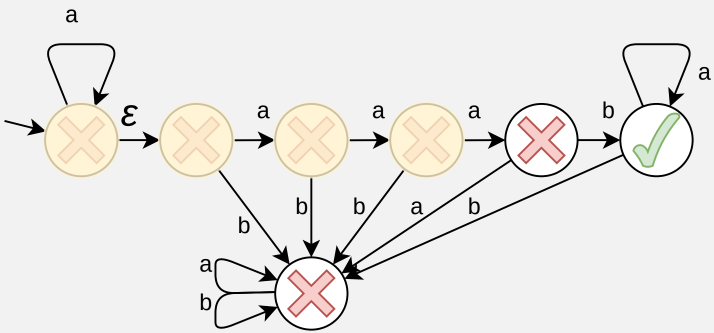

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a*aaaba*

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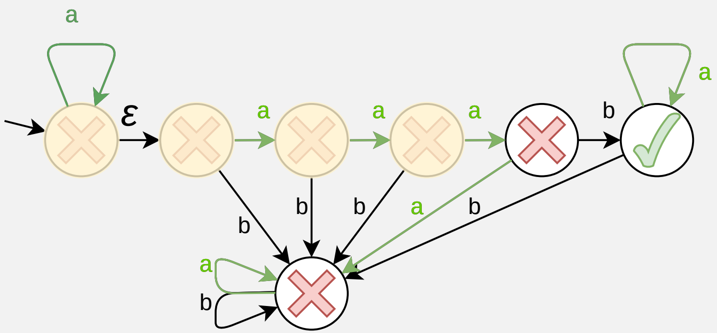

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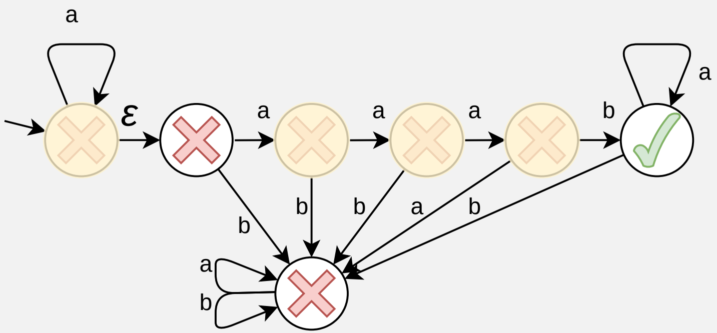
aaa]aab



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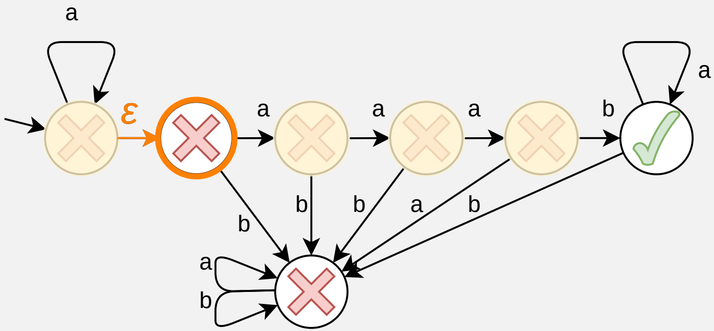
```
a*aaaba*
```

```
aaa!aab
```



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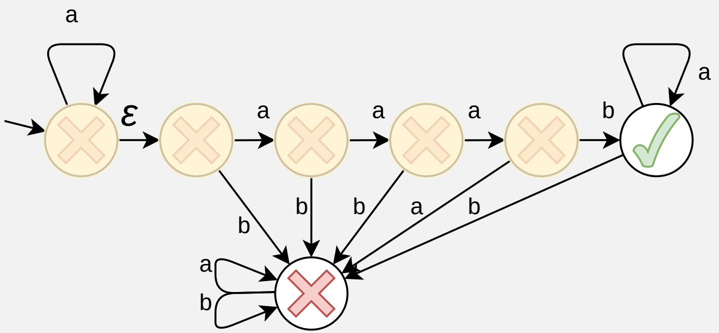
aaa|aab



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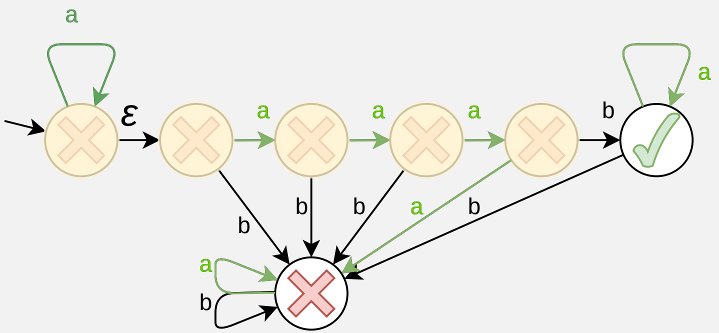

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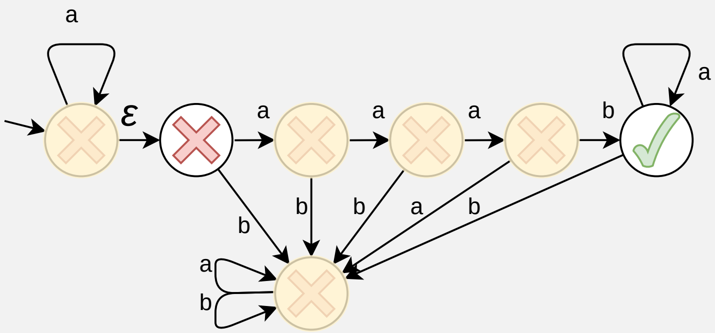

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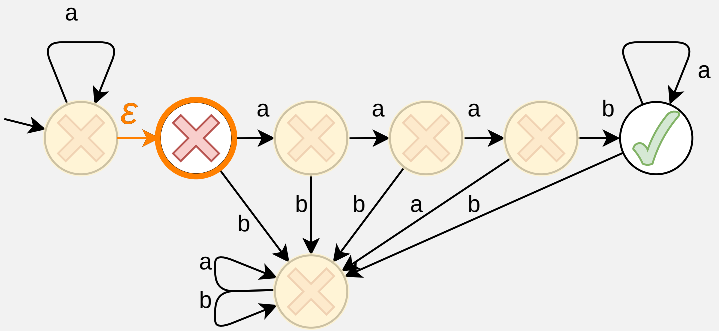
```
a*aaaba*

aaaa!ab
```



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Running an NFAε

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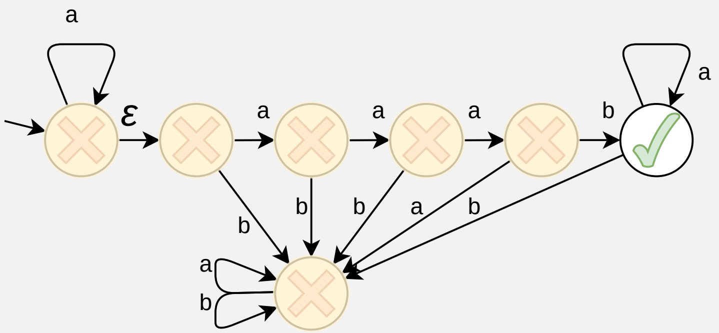
aaaa|ab



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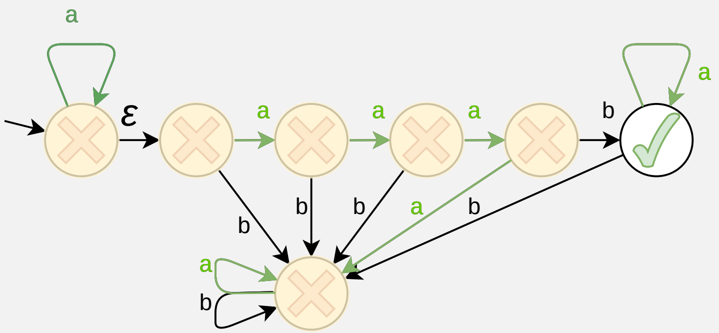

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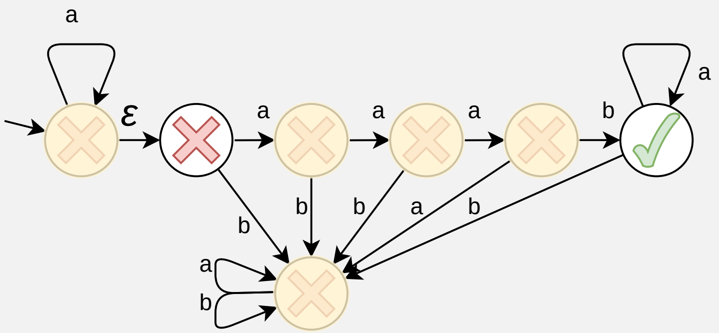

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Running an NFAε

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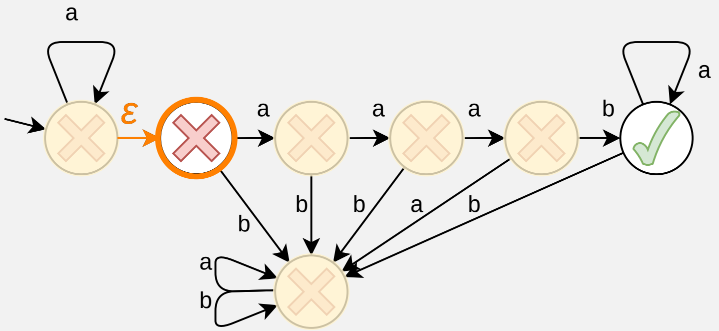
aaaaa!b



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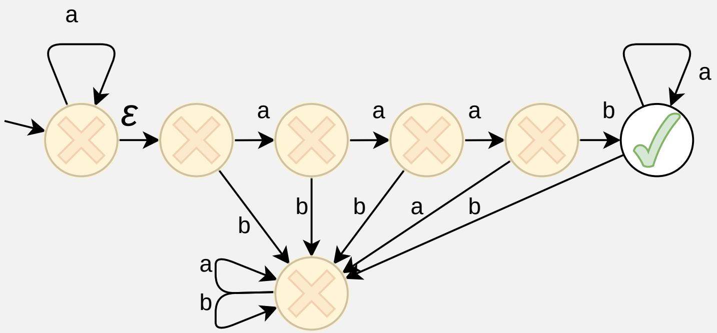

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Running an NFAε

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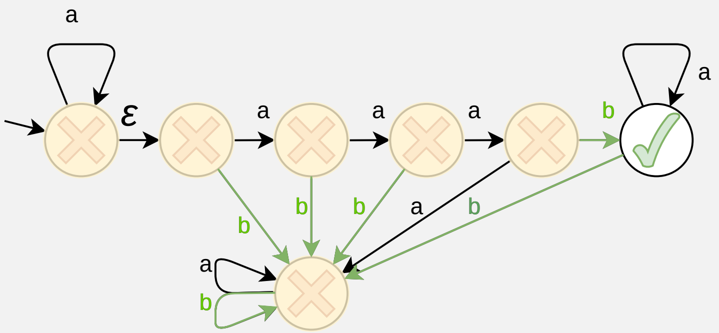

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Running an NFAε

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runNFAε (NFAε step εsteps genOut s0) = ???
```

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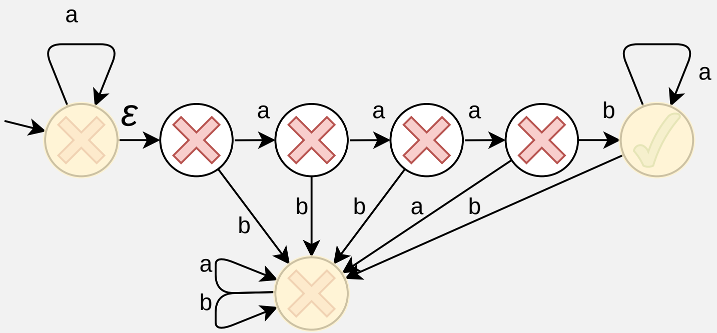

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Running an NFAε

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runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) = ???
```

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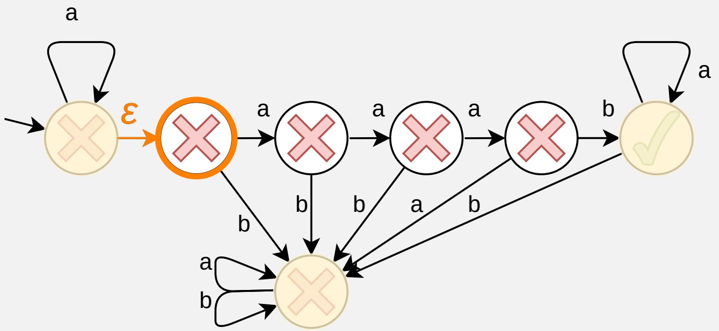

Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) = ???
```

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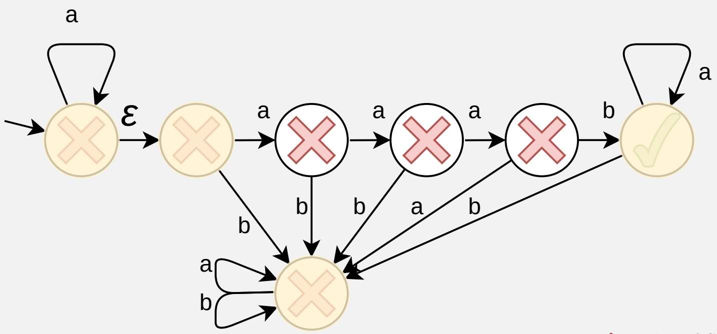

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Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
  foldr ???₁ ???₂
```

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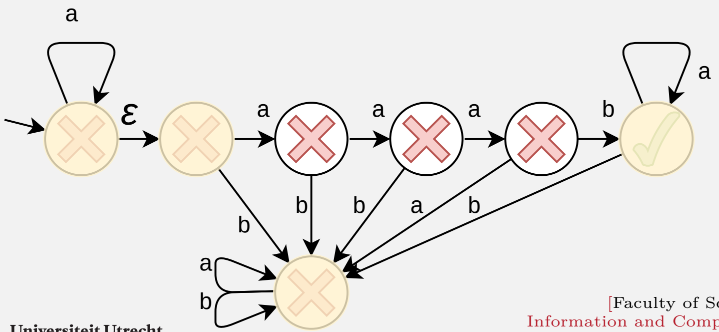

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Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
  foldr ???₁ ???₂
```

```
???₁ :: Set state
???₂ :: symbol → Set state → Set state
```

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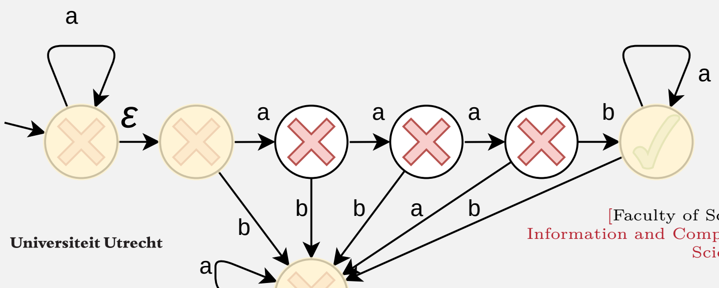

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Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
  foldr ???_1 ???_2
```

```
???_1 :: Set state👉
???_2 :: symbol → Set state → Set state
```

Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
  foldr ???₁ ???₂
```

```
???₁ :: Set state
???₂ :: symbol → Set state → Set state
```

```
runNFAε nfa [] == -- states reachable without input
```

Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
  foldr ???₁ ???₂
```

```
???₁ :: Set state
???₂ :: symbol → Set state → Set state
```

```
runNFAε nfa [] == -- states reachable without input
runNFAε nfa [] == foldr ???₁ ???₂ []
```

Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
  foldr ???₁ ???₂
```

```
???₁ :: Set state
???₂ :: symbol → Set state → Set state
```

```
runNFAε nfa [] == -- states reachable without input
runNFAε nfa [] == ???₁
```

Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
  foldr ???₁ ???₂
```

```
???₁ :: Set state
???₂ :: symbol → Set state → Set state
```

```
???₁ == -- states reachable without input
```

Running an NFAε

```haskell
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
  foldr ???₁ ???₂
```

```haskell
???₁ :: Set state
???₂ :: symbol → Set state → Set state
```

```haskell
???₁ == -- states reachable by ε-transitions only
```

Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
  foldr ???₁ ???₂
```

```
???₁ :: Set state
???₂ :: symbol → Set state → Set state
```

```
???₁ == -- states reachable by 0 or more ε-transitions
```

Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
  foldr ???₁ ???₂
```

```
???₁ :: Set state
???₂ :: symbol → Set state → Set state
```

```
???₁ == reachable εsteps ???₃
reachable :: Set (state,state) → state → Set state
```

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Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
  foldr ???₁ ???₂
```

```
???₁ :: Set state
???₂ :: symbol → Set state → Set state
```

```
???₁ == reachable εsteps (s0 nfa)
reachable :: Set (state,state) → state → Set state
```

Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
  foldr (reachable εsteps (s0 nfa)) ???₂
```

```
???₂ :: symbol → Set state → Set state
```

```
reachable :: Set (state,state) → state → Set state
```

Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
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```
???₂ :: symbol → Set state → Set state
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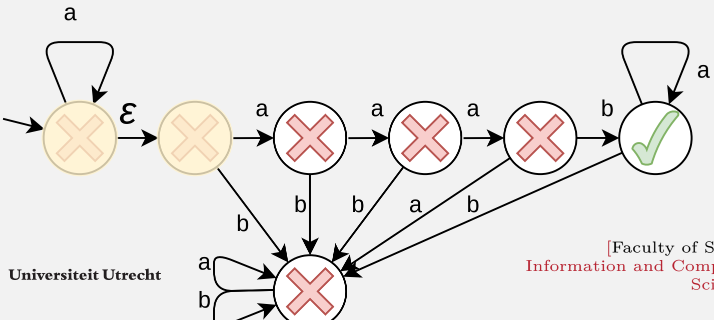
reachable :: Set (state,state) → state → Set state
```

```
|aaaaab
```



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Running an NFAε

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```
???₂ :: symbol → Set state → Set state
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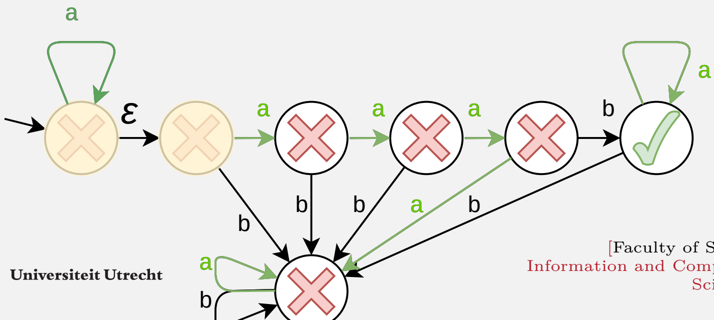
```
reachable :: Set (state,state) → state → Set state
```

```
[aaaaab
```



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```

```
???₂ :: symbol → Set state → Set state
```

```
reachable :: Set (state,state) → state → Set state
```

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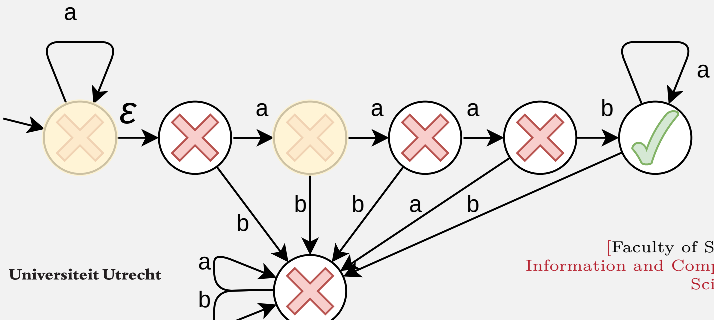

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???₂ :: symbol → Set state → Set state
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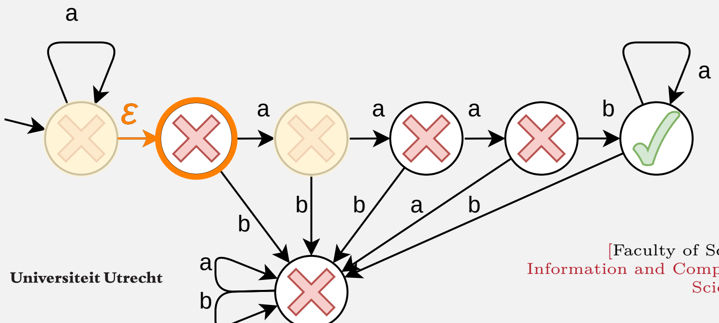
```
reachable :: Set (state,state) → state → Set state
```

```
a!aaaab
```



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Running an NFAε

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???₂ :: symbol → Set state → Set state
```

```
reachable :: Set (state,state) → state → Set state
```

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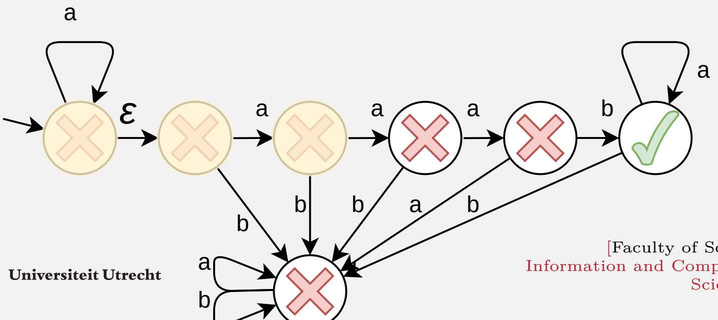

Running an NFAε

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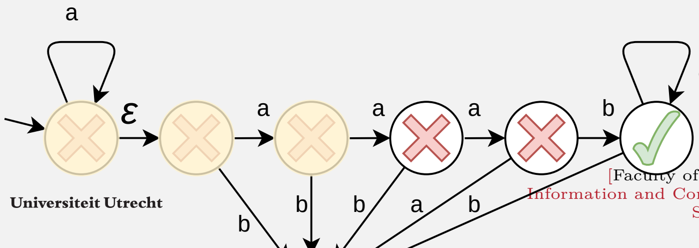
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
  foldr (reachable εsteps (s0 nfa)) ???₂
```

```
???₂ = \sy → ???₅ . ???₄
???₄ :: Set state → Set state
???₅ :: Set state → Set state
```

```
reachable :: Set (state,state) → state → Set state
```

```
a|aaaab
```



Running an NFAε

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```

```
???₄ :: Set state → Set state
???₅ :: Set state → Set state
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```
reachable :: Set (state,state) → state → Set state
```

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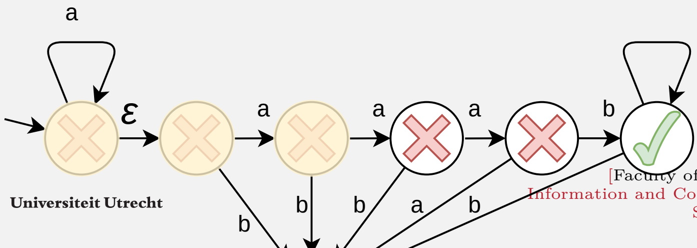

Running an NFAε

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```

```
???₄ :: Set state → Set state👈
???₅ :: Set state → Set state
```

```
reachable :: Set (state,state) → state → Set state
```

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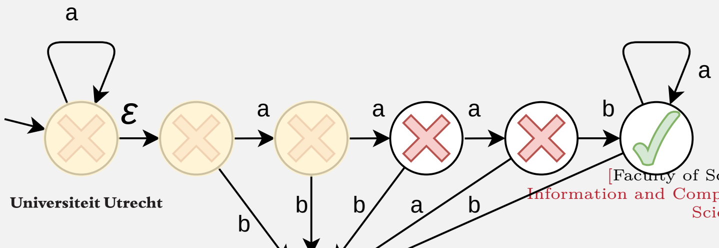

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```

```
???₄ :: Set state → Set state👉
???₅ :: Set state → Set state
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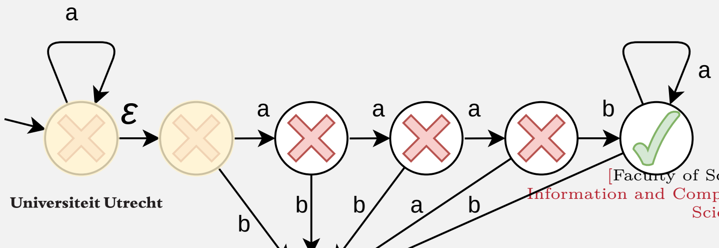
reachable :: Set (state,state) → state → Set state
```

```
|aaaaab
```



Running an NFAε

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```
[aaaaab
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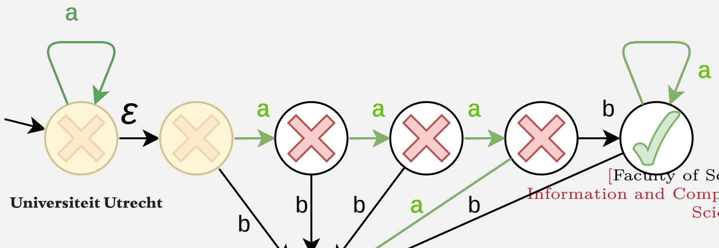


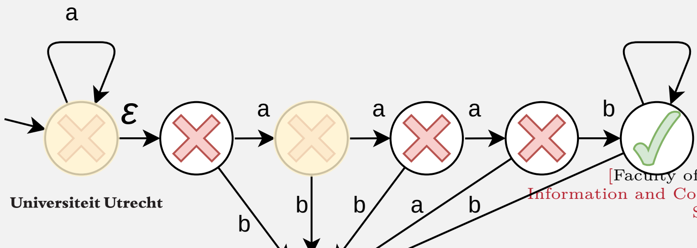
Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
  foldr (reachable εsteps (s0 nfa))
  (\sy → ???₅ . ???₄ )
```

```
???₄ = Set.map (step nfa sy)
???₅ :: Set state → Set state👉
```

```
reachable :: Set (state,state) → state → Set state
```

```
a]aaaab
```



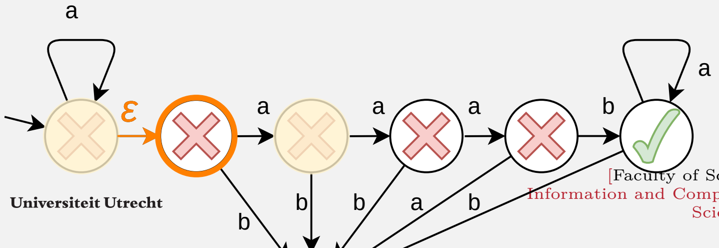
Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
  foldr (reachable εsteps (s0 nfa))
  (\sy → ???₅ . ???₄ )
```

```
???₄ = Set.map (step nfa sy)
???₅ :: Set state → Set state👉
```

```
reachable :: Set (state,state) → state → Set state
```

```
a!aaaab
```



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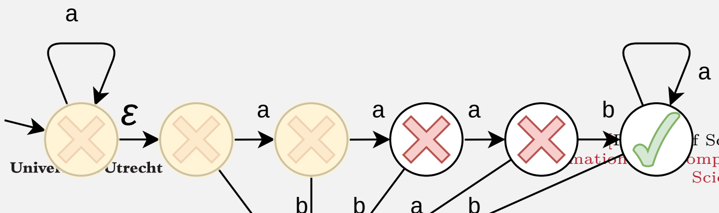
Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
  foldr (reachable εsteps (s0 nfa))
  (\sy → ???₅ . ???₄ )
```

```
???₄ = Set.map (step nfa sy)
???₅ = ???₆ . Set.map (reachable εsteps)
???₆ :: Set (Set state) → Set state
```

```
reachable :: Set (state,state) → state → Set state
```

a|aaaab



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Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
  foldr (reachable εsteps (s0 nfa))
  (\sy → ???₅ . ???₄ )
```

```
???₄ = Set.map (step nfa sy)
???₅ = ???₆ . Set.map (reachable εsteps)
???₆ = Set.unions
```

```
reachable :: Set (state,state) → state → Set state
```

Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
  foldr (reachable εsteps (s0 nfa))
  (\sy → ???5 . ???4 )
```

```
???4 = Set.map (step nfa sy)
???5 = Set.unions . Set.map (reachable εsteps)
```

```
reachable :: Set (state,state) → state → Set state
```

Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
    foldr (reachable εsteps (s0 nfa))
  (\sy → ???₅ . Set.map (step nfa sy))
```

```
???₅ = Set.unions . Set.map (reachable εsteps)
```

```
reachable :: Set (state,state) → state → Set state
```

Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
    foldr (reachable εsteps (s0 nfa))
  (\sy → (Set.unions . Set.map (reachable εsteps))
     . Set.map (step nfa sy))
```

```
reachable :: Set (state,state) → state → Set state
```

Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
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  (\sy → Set.unions . Set.map
    (reachable εsteps . step nfa sy))
```

```
reachable :: Set (state,state) → state → Set state
```

Running an NFAε

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    (reachable εsteps . step nfa sy))
```

```
reachable :: Set (state,state) → state → Set state
```

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Running an NFAε

```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
    foldr (reachable εsteps (s0 nfa))
   (\sy → Set.unions . Set.map
     (reachable εsteps . step nfa sy))
```

```
reachable :: Set (state,state) → state → Set state👈
```

Exercise

Putting together the parts

✅ `runNFAε :: NFAε sy st → [sy] → Set st`

Putting together the parts

- ✅ `runNFAε :: NFAε sy st → [sy] → Set st`
- ✅ `r2n :: RegExp → NFAε Char Label -- (by example)`

Putting together the parts

✅ `runNFAε :: NFAε sy st → [sy] → Set st`

✅ `r2n :: RegExp → NFAε Char Label -- (by example)`

```
matchesRegExp :: RegExp → String → Bool
matchesRegExp r s = any isAccepting $
  runNFAε (r2n r) s
```

Putting together the parts

✅ `runNFAε :: NFAε sy st → [sy] → Set st`

✅ `r2n :: RegExp → NFAε Char Label -- (by example)`

```
matchesRegExp :: RegExp → String → Bool
matchesRegExp r s = any isAccepting $
  runNFAε (r2n r) s
```

Done?

Putting together the parts

✅ `runNFAε :: NFAε sy st → [sy] → Set st`

✅ `r2n :: RegExp → NFAε Char Label -- (by example)`

```haskell
matchesRegExp :: RegExp → String → Bool
matchesRegExp r s = any isAccepting $
  runNFAε (r2n r) s
```

Done?

- No!

Running an NFAε better

```
runNFAε :: NFAε sy      st  → [sy] → Set st
runDFA  :: DFA  sy      st  → [sy] →     st
```

Running an NFAε better

```
runNFAε :: NFAε sy       st  → [sy] → Set st

runDFA  :: DFA  sy (Set st) → [sy] → Set st
```

Running an NFAε better

```
runNFAε :: NFAε sy       st  → [sy] → Set st

runDFA  :: DFA  sy (Set st) → [sy] → Set st

n2d       :: NFAε sy       st
           → DFA  sy (Set st)
```

Running an NFAε better

```
runNFAε :: NFAε sy       st  → [sy] → Set st

runDFA  :: DFA  sy (Set st) → [sy] → Set st

n2d       :: NFAε sy       st
            → DFA  sy (Set st)

runNFAε = runDFA . n2d
```

The subset construction

```
n2d :: NFAε sy st → DFA sy (Set st)
```

The subset construction

```
n2d :: NFAε symbol state → DFA symbol (Set state)
```

The subset construction

```
n2d :: NFAε symbol state → DFA symbol (Set state)
n2d = ???
```

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The subset construction

```
n2d :: NFAε symbol state → DFA symbol (Set state)
```

```
n2d (NFAε step εsteps genOut s0) = Moore
  { s0 = ???₁
  , step = ???₂
  , genOut = ???₃ }
```

The subset construction

```
n2d :: NFAε symbol state → DFA symbol (Set state)
```

```
n2d (NFAε step εsteps genOut s0) = Moore
  { s0 = ???₁
  , step = ???₂
  , genOut = ???₃ }
```

$???_1$ `:: Set state`

$???_2$ `:: symbol → Set state → Set state`

$???_3$ `:: Set state → Bool`

The subset construction

```
n2d :: NFAε symbol state → DFA symbol (Set state)
```

```
n2d (NFAε step εsteps genOut s0) = Moore
  { s0 = ???₁
  , step = ???₂
  , genOut = ???₃ }
```

```
???₁ :: Set state

???₂ :: symbol → Set state → Set state

???₃ :: Set state → Bool

genOut :: state → Bool
```

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The subset construction

```
n2d :: NFAε symbol state → DFA symbol (Set state)
```

```
n2d (NFAε step εsteps genOut s0) = Moore
  { s0 = ???₁
  , step = ???₂
  , genOut = ???₃ }
```

```
???₁ :: Set state

???₂ :: symbol → Set state → Set state

???₃ = any genOut

genOut :: state → Bool
```

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The subset construction

```
n2d :: NFAε symbol state → DFA symbol (Set state)
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```
???₁ :: Set state
```

```
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The subset construction

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n2d (NFAε step εsteps genOut s0) = Moore
  { s0 = ???₁
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  , genOut = any genOut }
```

```
???₁ :: Set state
```

```
???₂ :: symbol → Set state → Set state
```

🕵 Look familiar?

The subset construction

```
n2d :: NFAε symbol state → DFA symbol (Set state)
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n2d (NFAε step εsteps genOut s0) = Moore
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```
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```
runNFAε :: NFAε symbol state → [symbol] → Set state
runNFAε (NFAε step εsteps genOut s0) =
    foldr ???₁ ???₂
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???₁ = reachable εsteps (s0 nfa)
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The subset construction

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```

Recap

- RegExp $\to$ NFAε
- NFAε $\to$ DFA

Recap

- RegExp → NFAε
- NFAε → DFA
- RegExp 🚀

Recap

- RegExp $\rightarrow$ NFAε
- NFAε $\rightarrow$ DFA
- RegExp 🚀 ❓

Performance

If `n = length input` and `m = length regexp`, then...

- $O(nm)$ time

Performance

If `n = length input` and `m = length regexp`, then...

- $O(nm)$ time
- Our 'new' algorithm is not so hot
  - Invented 1959: doi.org/10.1147/rd.32.0114
  - Compiled 1964: doi.org/10.1145/363347.363387

Performance

If `n = length input` and `m = length regexp`, then...

- $O(nm)$ time
- Our 'new' algorithm is not so hot
  - Invented 1959: doi.org/10.1147/rd.32.0114
  - Compiled 1964: doi.org/10.1145/363347.363387
- Best known algorithm (2009):
  - $O(n)$ space
  - $O(nm\frac{\log\log n}{\log^{\frac{3}{2}} n} + n + m)$ time
  - doi.org/10.1007/978-3-642-02927-1_16

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Summary

- RegExp 🚀
- RegExp $\rightarrow$ NFAε
- NFAε $\rightarrow$ DFA
- DFA $\rightarrow$
- Not the only way
  - DFA product (e.g. $\mathtt{r}_1|\mathtt{r}_2$)
  - “Brzozowski derivative”