Lazy evaluation

Functional Programming

From Lecture 1:

Haskell can be defined with four adjectives

- Functional
- Statically typed
- Pure
- Lazy

From Lecture 1:

Haskell can be defined with four adjectives

- Functional
- Statically typed
- Pure
- **Lazy**

Goals

- Understand the lazy evaluation strategy
  - As opposed to strict evaluation
- Understand why lazyness is useful
  - ...
  - Work with infinite structures
- Learn about laziness pitfalls
  - Force evaluation using `seq`

A simple expression

```haskell
square :: Integer -> Integer
square x = x * x

square (1 + 2)
= -- magic happens in the computer
9
```

How do we reach that final value?

Strict or eager or call-by-value evaluation

In most programming languages:

1. Evaluate the arguments completely
2. Evaluate the function call

```
square (1 + 2)
= -- evaluate arguments
square 3
= -- go into the function body
3 * 3
=
9
```

Non-strict or call-by-name evaluation

Arguments are replaced as-is in the function body

```
square (1 + 2)
= -- go into the function body
(1 + 2) * (1 + 2)
= -- we need the value of (1 + 2) to continue
3 * (1 + 2)
=
3 * 3
=
9
```

Does call-by-name make any sense?

In the case of `square`, non-strict evaluation is worse

Is this always the case?

Does call-by-name make any sense?

In the case of `square`, non-strict evaluation is worse

Is this always the case?

```
const x y = x   -- forget about y
```

```
-- Call-by-value
const 5 (1 + 2)
=
const 5 3
=
5
```

```
-- Call-by-name
 const 5 (1 + 2)
=
5
```

Sharing expressions

```
square (1 + 2)
=
(1 + 2) * (1 + 2)
```

Why redo the work for `(1 + 2)`?

Sharing expressions

```
square (1 + 2)
=
(1 + 2) * (1 + 2)
```

Why redo the work for (1 + 2)?

We can share the evaluated result

```
square (1 + 2)
=
Δ * Δ
↑___↑___ (1 + 2)
           = 3
=
9
```

Lazy evaluation

Haskell uses a **lazy** evaluation strategy

- Expressions are not evaluated until needed
- Duplicate expressions are shared

Lazy evaluation never requires more steps than call-by-value

Each of those not-evaluated expressions is called a **thunk**

Does it matter?

Is it possible to get different outcomes using different evaluation strategies?

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Is it possible to get different outcomes using different evaluation strategies?

No and Yes

Does it matter?

- No:

Theorem [Church-Rosser Theorem]

For terminating programs all evaluation strategies produce the same result value.

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- No:

Theorem [Church-Rosser Theorem]

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- Yes:

Does it matter?

- No:

Theorem [Church-Rosser Theorem]

For terminating programs all evaluation strategies produce the same result value.

- Yes:

1. Holds only for terminating programs.
   - What about infinite loops?
   - What about exceptions?

Does it matter?

- No:

Theorem [Church-Rosser Theorem]

For terminating programs all evaluation strategies produce the same result value.

- Yes:

1. Holds only for terminating programs.
   - What about infinite loops?
   - What about exceptions?
2. Performance might be different.
   - As `square` and `const` show

Termination

```
loop x = loop x
```

- This is a well-typed program
- But `loop 3` never terminates

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Question: What does `'const 5 (loop 3)'` evaluate to?

Termination

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loop x = loop x
```

- This is a well-typed program
- But `loop 3` never terminates

Question: What does `'const 5 (loop 3)'` evaluate to?

```
-- Eager
const 5 (loop 3)
=
const 5 (loop 3)
=
...
```

```
-- Lazy
 const 5 (loop 3)
=
 5
```

Observation:

Lazy evaluation terminates more often than eager evaluation.

Question: Why is this useful?

Short-circuiting

```haskell
(&&)        :: Bool -> Bool -> Bool
False && _ = False
True  && x = x
```

- In eager languages, `x && y` evaluates both conditions
  - But if the first one fails, why bother?
  - C/Java/C# include a built-in short-circuit conjunction
- In Haskell, `x && y` only evaluates the second argument if the first one is `True`
  - `False && (loop True)` terminates

Why? Build your own Control structures

```haskell
if_            :: Bool -> a -> a -> a
if_ True  t _ = t
if_ False _ e = e
```

- In eager languages, `if_` evaluates both branches
- In lazy languages, only the one being selected

Why? Build your own Control structures

```
if_             :: Bool -> a -> a -> a
if_ True  t _ = t
if_ False _ e = e
```

- In eager languages, `if_` evaluates both branches
- In lazy languages, only the one being selected

For that reason,

- In eager languages, `if` has to be built-in
- In lazy languages, you can build your own control structures

Why? Separation of Concerns

- Lazyness allows for easier separation of concerns.

```
data Operation = Sum | Product

apply         :: Operation -> [Int] -> Int
apply op xs = case op of
                Sum     -> sumResult
                Product -> productResult
  where
    sumResult     = sum xs
    productResult = product xs
```

Why? Separation of Concerns

- Lazyness allows for easier separation of concerns.

```
minAndMax :: Ord a => a -> [a] -> (a,a)

minimum'    :: Ord a => a -> [a] -> a
minimum' d = fst . minAndMax d
```

Why? Infinite structures

An infinite list of ones:

```
ones :: [Integer]
ones = 1 : ones
```

`ones` is infinite, but everything works fine if we only work with a finite part

```
  take 2 ones
= take 2 (1 : ones)
= 1 : take 1 ones
= 1 : take 1 (1 : ones)
= 1 : 1 : take 0 ones
= 1 : 1 : []
```

A list of all natural numbers

To build an infinite list of numbers, we use recursion

- This kind of recursion is trickier than the usual one

```
nats :: [Integer]
nats = 0 : map (+1) nats

  take 2 nats
= take 2 (0 : map (+1) nats)
= 0 : take 1 (map (+1) nats)
= 0 : take 1 (map (+1) (0 : map (+1) nats))
= 0 : take 1 (1 : map (+1) (map (+1) nats))
= 0 : 1 : take 0 (map (+1) (map (+1) nats))
= 0 : 1 : []
```

A list of all Fibonacci numbers

Remember the usual definition of `fib`,

```
fib 0 = 0
fib 1 = 1
fib n = fib (n-1) + fib (n-2)
```

A list of all Fibonacci numbers

Remember the usual definition of `fib`,

```
fib 0 = 0
fib 1 = 1
fib n = fib (n-1) + fib (n-2)
```

Here is a list containing all Fibonacci numbers:

```
fibs :: [Integer]
fibs = 0 : 1 : zipWith (+) fibs (tail fibs)

fib :: Integer -> Integer
fib n = fibs !! n  -- Take the n-th element
```

A list of all Fibonacci numbers

```
   0  :  1   :  ...
+  1  :  ...
-----------------------------------
   1  :  ...
```

A list of all Fibonacci numbers

```
   0  :  1  :  1   :  ...
+  1  :  1  :  ...
---------------------------------
   1  :  2  :  ...
```

A list of all Fibonacci numbers

```
  0 : 1 : 1 : 2   : ...
+ 1 : 1 : 2 : ...
------------------------------
  1 : 2 : 3 : ...
```

A list of all prime numbers: Sieve of Erastosthenes

An algorithm to compute the list of all primes

- Already known in Ancient Greece

1. Lay all numbers in a list starting with 2
2. Take the first next number p in the list
3. Remove all the multiples of p from the list
   - $2p$, $3p$, $4p\dots$
   - Alternatively, remove n if the remainder with p is 0
4. Go back to step 2 with the first remaining number

Sieve of Erastosthenes

1. Lay all numbers in a list starting with 2

   ```
   primes :: [Integer]
   primes = sieve [2 .. ]   -- an infinite list
   ```

Sieve of Erastosthenes

1. Lay all numbers in a list starting with 2

```
primes :: [Integer]
primes = sieve [2 .. ]   -- an infinite list
```

2. Take the first number p in the list

```
sieve (p:ns) = p : ...
```

3. Remove n if the remainder with p is 0
4. Go back to step 2 with the first remaining number

```
sieve (p:ns)
  = p : sieve [n | n <- ns, n `mod` p /= 0]
```

"Until needed"

How does Haskell know how much to evaluate?

- By default, everything is kept in a thunk
- When we have a case distinction, we evaluate enough to distinguish which branch to follow

```
take 0 _       = []
take _ []      = []
take n (x:xs) = x : take (n-1) xs
```

- If the number is 0 we do not need the list at all
- Otherwise, we need to distinguish `[]` from `x:xs`

Weak Head Normal Form

An expression is in **weak head normal form** (WHNF) if it is:

- A constructor with (possibly non-evaluated) data inside
  - `True` or `Just (1 + 2)`
- An anonymous function
  - The body might be in any form
  - `\x -> x + 1` or `\x -> if_ True x x`
- A function applied to too few arguments
  - `map minimum`

Every time we need to distinguish the branch to follow the expression is evaluated until its WHNF

Weak Head Normal Form

Which of these expressions are in WHNF?

1. `zip [1..]`
2. `Node Leaf 4 (fmap (+1) Leaf)`
3. `map (x:) xs`
4. `height (Node Leaf 'a' (Node Leaf 'b' Leaf))`
5. `\_ b -> b`
6. `map (\x -> x + 1) [1..5]`
7. `(x + 1) : foldr (:) [] [1..5]`

Weak Head Normal Form

Which of these expressions are in WHNF?

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2. `Node Leaf 4 (fmap (+1) Leaf)`
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4. `height (Node Leaf 'a' (Node Leaf 'b' Leaf))`
5. `\_ b -> b`
6. `map (\x -> x + 1) [1..5]`
7. `(x + 1) : foldr (:) [] [1..5]`

answer: 1,2,5,7

Strict versus lazy functions

Note the difference between these two functions

```
  loop 2 + 3
= -- definition of loop
  loop 2 + 3
= -- never-ending sequence
  ...

  const 3 (loop 2)
= -- definition of const
  3
  -- and that's it!
```

Strict versus lazy functions

A function is **strict** on one argument if the result of the function is non-terminating given a non-terminating value for that argument

- `(+)` is strict on its first and second arguments
- `const` is not strict on its second argument, but strict on the first

We represent non-termination by $\perp$ or `undefined`

- We also call $\perp$ a diverging computation
- f is strict if $f\ \perp = \perp$

Some (tricky) questions

What is the result of these expressions?

1. `(\x -> x) True`
2. `(\x -> x) undefined`
3. `(\x -> 0) undefined`
4. `(\x -> undefined) 0`
5. `(\x f -> f x) undefined`
6. `undefined undefined`
7. `length (map undefined [1,2])`

Some (tricky) questions

What is the result of these expressions?

1. `(\x -> x) True              = True`
2. `(\x -> x) undefined         = undefined`
3. `(\x -> 0) undefined         = 0`
4. `(\x -> undefined) 0         = undefined`
5. `(\x f -> f x) undefined     = \f -> f undefined`
6. `undefined undefined         = undefined`
7. `length (map undefined [1,2]) = 2`

Lazy Evaluation vs Performance

Case study: `foldl`

From a long, long time ago…

```
foldl _ v []     = v
foldl f v (x:xs) = foldl f (f v x) xs
```

Case study: `foldl`

From a long, long time ago…

```
foldl _ v []     = v
foldl f v (x:xs) = foldl f (f v x) xs
```

```
foldl (+) 0 [1,2,3]
```

Case study: `foldl`

From a long, long time ago…

```
foldl _ v []     = v
foldl f v (x:xs) = foldl f (f v x) xs
```

```
  foldl (+) 0 [1,2,3]

= foldl (+) (0 + 1) [2,3]
= foldl (+) ((0 + 1) + 2) [3]
= foldl (+) (((0 + 1) + 2) + 3) []
= ((0 + 1) + 2) + 3
```

Case study: `foldl`

```
  foldl (+) 0 [1,2,3]
= ((0 + 1) + 2) + 3
```

Question: What is the problem with this?

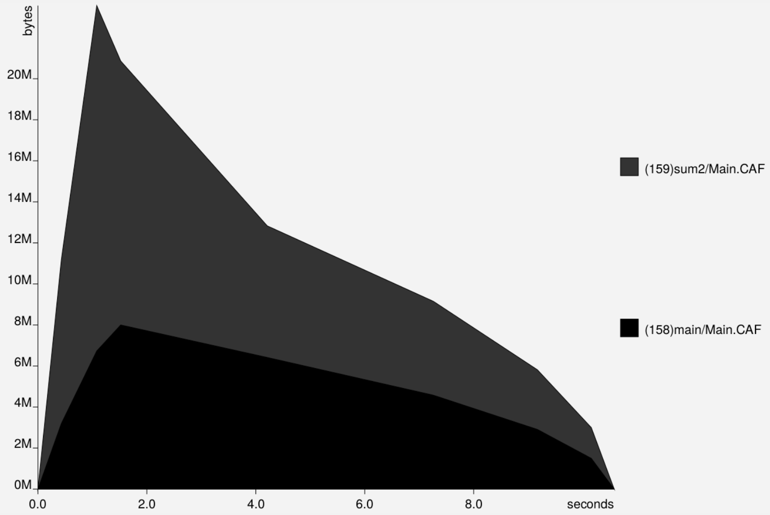
Case study: `foldl`

```
  foldl (+) 0 [1,2,3]
= ((0 + 1) + 2) + 3
```

Question: What is the problem with this?

- Each of the additions is kept in a thunk
  - Some memory need to be reserved!

Case study: `foldl`



Space leaks

Space leak = data structure which grows bigger, or lives longer than expected

- More memory in use means more Garbage Collection
- As a result, performance decreases

The most common source of space leaks are thunks

- Thunks are essential for lazy evaluation
- But they also take some amount of memory

Garbage collection

- Thunks are managed by the run-time system
  - They are created when you need a value
  - But are not reclaimed right after evaluation
- Haskell uses **garbage collection** (GC)
  - Every now and them Haskell takes back all the memory used by thunks which are not needed anymore
  - Pro: we do not need to care about memory
  - Con: GC takes time, so lags can occur
- Most modern languages nowadays use GC
  - Java, Scala, C#, Ruby, Python…
  - Swift uses Automatic Reference Counting (ARC)

Case study: `foldl`

We want to reduce memory usage and speed up the computation.

We force additions before going on

```
  foldl (+) 0 [1,2,3]
= foldl (+) (0 + 1) [2,3]
= foldl (+) 1 [2,3]
= foldl (+) (1 + 2) [3]
= foldl (+) 3 [3]
= foldl (+) (3 + 3) []
= foldl (+) 6 []
= 6
```

Forcing evaluation

Haskell has a primitive operation to force

```
seq :: a -> b -> b
```

A call of the form `seq x y`

- First evaluates `x` up to WHNF
- Then it proceeds normally to compute `y`

Usually, `y` depends on `x` somehow

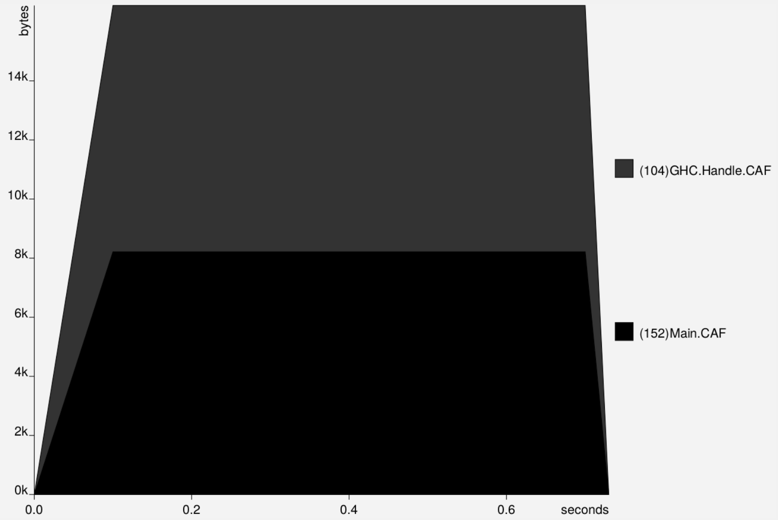
Case study: `foldl`

We can write a new version of `foldl` which forces the accumulated value before recursion is unfolded

```
foldl' _ v []     = v
foldl' f v (x:xs) = let z = f v x
                     in z `seq` foldl' f z xs
```

This version solves the problem with addition

Case study: `foldl`



Strict application

Most of the times we use `seq` to force an argument to a function, that is, strict application

```
($!) :: (a -> b) -> a -> b
f $! x = x `seq` f x
```

Because of sharing, `x` is evaluated only once

More (tricky) questions

What is the result of these expressions?

1. `(\x -> 0) $! undefined`
2. `seq (undefined, undefined) 0`
3. `snd $! (undefined, undefined)`
4. `(\x -> 0) $! (\x -> undefined)`
5. `undefined $! undefined`
6. `length $! map undefined [1,2]`
7. `seq (undefined + undefined) 0`
8. `seq (foldr undefined undefined) 0`
9. `seq (1 : undefined) 0`

More (tricky) questions

What is the result of these expressions?

1. `(\x -> 0) $! undefined = undefined`
2. `seq (undefined, undefined) 0 = 0`
3. `snd $! (undefined, undefined) = undefined`
4. `(\x -> 0) $! (\x -> undefined) = 0`
5. `undefined $! undefined = undefined`
6. `length $! map undefined [1,2] = 2`
7. `seq (undefined + undefined) 0 = undefined`
8. `seq (foldr undefined undefined) 0 = 0`
9. `seq (1 : undefined) 0 = 0`

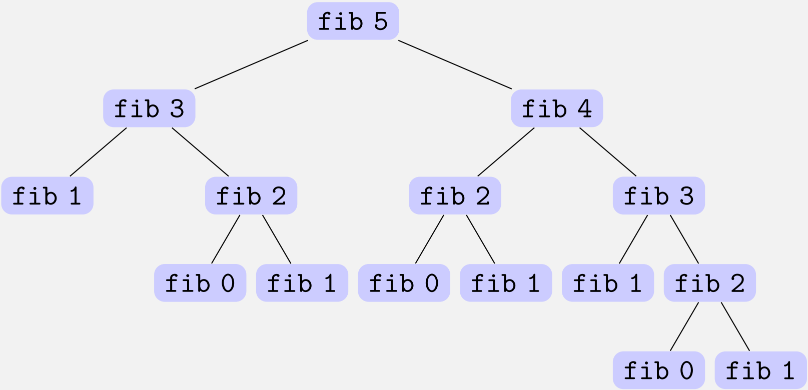
`seq` only evaluates up to WHNF

Case study: Fibonacci numbers

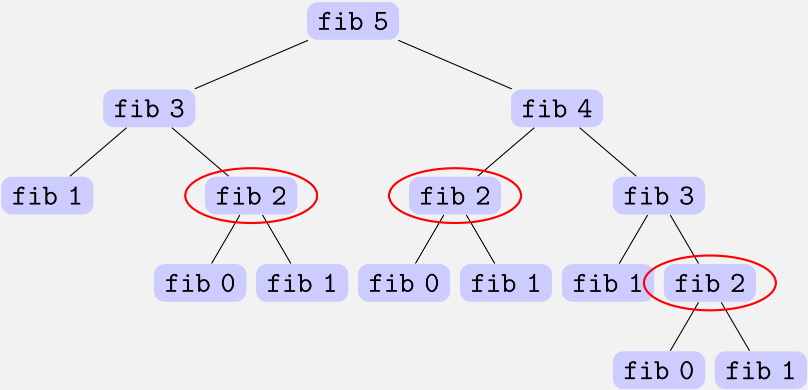
```
fib 0 = 0
fib 1 = 1
fib n = fib (n-1) + fib (n-2)
```

What happens when we ask for `fib 5`?

Case study: Fibonacci numbers



Case study: Fibonacci numbers



Local memoization (aka Dynamic Programming)

Idea: remember the result for function calls

- We build a list of partial results
- Sharing takes care of evaluating only once

```
memo_fib n = go i
  where go i  = fibs !! i
        fibs  = map fib [0 .. ]
        fib 0 = 0
        fib 1 = 1
        fib n = go (n-1) + go (n-2)
```

You can get even faster by using a better data structure

- For example, `IntMap` from `containers`

Summary

- Laziness = evaluate only as much as needed
  - As opposed to the more common eager evaluation
- Evaluation is guided by pattern matching
  - We need WHNF to choose a branch
  - Some arguments may not even be evaluated
- Laziness is tricky when it fails
  - Too many thunks lead to a space leak
  - `seq` is used to force evaluation