

Functional Programming

Assignment 4: Type classes

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In this assignment we'll present a few type classes and ask you to implement some instances of them.

1 Containers

1.1 Functors

Recall the rose tree data structure from the previous assignment:

```
data Rose a = MkRose a [Rose a]
```

Similarly to how we might want to apply a function uniformly to all elements in a list, we might also want to apply a function uniformly to all the elements in a rose tree, or any other container-like data structure for that matter. For this purpose Haskell has a *Functor* type class, exposing a single function *fmap* that generalizes the *map* function:

```
class Functor f where  
  fmap :: (a → b) → f a → f b
```

We see that *fmap* generalizes *map* by giving a *Functor* instance for lists:

```
instance Functor [] where  
  fmap = map
```

Verify that *fmap* and *map* have the same type if we instantiate *f* to *[]*.

Exercise 1. Write a *Functor* instance for the *Rose* data type.

1.2 Monoids

A *monoid* is an algebraic structure over a type *m* with a single associative binary operation $(\diamond) :: m \rightarrow m \rightarrow m$ and an identity element *mempty* :: *m*.

```
class Monoid m where  
  mempty :: m  
  ( $\diamond$ ) :: m → m → m
```

Lists are monoids:

```
instance Monoid [] where
  mempty = []
  (◊)    = (++)
```

Verify that $(++)$ is an associative operation (i.e., that $\forall xs\ ys\ zs.(xs ++ ys) ++ zs \equiv xs ++ (ys ++ zs)$) and that the empty list $[]$ is indeed an identity element with respect to list concatenation $(++)$ (i.e., that $\forall ls.[] ++ ls \equiv ls$ and $\forall ls.ls ++ [] \equiv ls$).

Numbers also form a monoid, both under addition with 0 as the identity element, and under multiplication with 1 as the identity element (verify this). However, we are only allowed to give one instance per combination of type and type class. To overcome this limitation we create some **newtype** wrappers:

```
newtype Sum a = Sum {unSum :: a}
newtype Product a = Product {unProduct :: a}
```

Now we can give two instances:

```
instance Num a => Monoid (Sum a) where
  mempty      = Sum 0
  Sum n1 ◊ Sum n2 = Sum (n1 + n2)
```

Exercise 2. Complete the second instance by writing a *Monoid* instance for numbers under multiplication.

1.3 Foldable

If f is some container-like data structure storing elements of type m that form a monoid, then there is a way of folding all the elements in the data structure into a single element of the monoid m .

```
class Functor f => Foldable f where
  fold :: Monoid m => f m -> m
```

In the case of lists:

```
instance Foldable [] where
  fold = foldr (◊) mempty
```

Exercise 3. Write a *Foldable* instance for *Rose*.

It might be the case that we have a container-like data structure storing elements of type a that do not yet form a monoid, but where we do have a function of type $a \rightarrow m$ that makes them into one. In such situation it would be convenient to have a function $foldMap :: Monoid m \Rightarrow (a \rightarrow m) \rightarrow f a \rightarrow m$ that first injects all the elements of the container into a monoid and then folds them into a single monoidal value.

Exercise 4. Add a default implementation of *foldMap* to the *Foldable* type class, expressed in terms of *fold* and *fmap*.

Exercise 5. Write functions $fsum, fproduct :: (Foldable f, Num a) \Rightarrow f a \rightarrow a$ that compute the sum, respectively product, of all numbers in a container-like data structure.

2 Poker

If we want to implement a poker game, we need to represent playing cards, hands and have way of ranking hands:

```

data Rank = R2 | R3 | R4 | R5 | R6 | R7 | R8 | R9 | R10 | J | Q | K | A
      deriving (Bounded, Enum, Eq, Ord)
data Suit = S | H | D | C
      deriving (Bounded, Enum, Eq, Ord, Show)
data Card = Card {rank :: Rank, suit :: Suit}
type Deck = [Card]

```

2.1 Show

Exercise 6. Define a *Show* instance for *Rank* that shows the ranks as "2", "3", "4", "5", "6", "7", "8", "9", "10", "J", "Q", "K", "A" respectively.

Exercise 7. Give a *Show* instance for *Card* showing *Card* {rank = R2, suit = H} as "2H".

Exercise 8. Define constants *fullDeck*, *piquetDeck* :: *Deck* that give a full 52-card deck and a 32-card Piquet deck (with cards of ranks from 7 up to and including the Ace).

2.2 Ord

A poker hand can be represented as:

```

newtype Hand = Hand {unHand :: [Card]}

```

and the various hand categories as:

```

data HandCategory
    = HighCard      [Rank]
    | OnePair       Rank [Rank]
    | TwoPair       Rank Rank Rank
    | ThreeOfAKind  Rank Rank Rank
    | Straight      Rank
    | Flush         [Rank]
    | FullHouse     Rank Rank
    | FourOfAKind  Rank Rank
    | StraightFlush Rank
      deriving (Eq, Ord, Show)

```

If you are not familiar with the ranking of poker hands then https://en.wikipedia.org/w/index.php?title=List_of_poker_hands&oldid=574969062 would be a good place to refer to.

The fields stored together with each category are used to distinguish between hands of the same category. For example, for a *HighCard* the field of type *[Rank]* contains all five cards in the hand, sorted from high to low. In the case of a *TwoPair* the first *Rank* is that of the high pair, the second *Rank* that of the low pair, and the third *Rank* that of the kicker (the card that isn't part of any of the two pairs).

Convince yourself that the derived *Ord* instance correctly ranks poker hands represented as a *HandCategory*.

We are now going to write a function that converts hands of type *Hand* into hands of type *HandCategory*. First we'll need a few helper functions:

Exercise 9. Write a function *sameSuits* :: *Hand* → *Bool* that returns *True* if all cards in a *Hand* are of the same suit.

Exercise 10. Write a function *isStraight* :: *[Rank]* → *Maybe Rank* that return a *Just* with the highest ranked card, if the *Hand* is a straight (or a straight flush). Note that the Ace can count both as the highest and as the lowest ranked card in a straight!

Exercise 11. Write a function *ranks* :: *Hand* → *[Rank]* that converts a *Hand* into a list of *Ranks*, ordered from high to low.

Exercise 12. Write a function `order :: Hand → [(Int, Rank)]` that converts a `Hand` into a list of Ranks paired with their multiplicity, order from high to low using a lexicographical ordering. For example, the hand `["7H", "7D", "QS", "7S", "QH"]` should be ordered as `[(3, R7), (2, Q)]`.

Exercise 13. Finally, write a function `handCategory :: Hand → HandCategory` that converts a `Hand` into a `HandCategory`.

Exercise 14. Using `handCategory`, write an `Ord` instance for `Hand`.

2.3 Combinatorics

Exercise 15. Write a function `combs :: Int → [a] → [[a]]` that returns all the combinations that can be formed by taking `n` elements from a list.

Exercise 16. Write a function `allHands :: Deck → [Hand]` that returns all combinations of 5-card hands that can be taken from a given deck of cards

2.4 Data.Set

The `Ord` class on `Hand` induces an equivalence relation on poker hands. This can be useful when using functions or data structures that require an `Ord` instance on the data they are working with.

One example of such a data structure is `Data.Set`: a data structure that can only store unique elements. Uniqueness is determined by the equivalence relation induced by an `Ord` instance.

Exercise 17. Write a function `distinctHands :: Deck → Set Hand` that constructs a maximal set of distinct hands from deck. (Hints: You will need the `empty` and `insert` functions from `Data.Set`. Use `foldl'` instead of `foldr` to avoid a stack overflow when applying this function to large decks.)

3 Questions

You do not need to send the answer to these questions. However, it is useful for you to reflect about these topics in order to prepare better for the exam.

Question 1. Do numbers¹ form a monoid under subtraction? If so, give the identity element. Do numbers form a monoid under division?

Does `Bool` form a monoid under conjunction (\wedge)? Does `Bool` form a monoid under the biconditional (\equiv)?

Question 2. Sheldon wants to implement `Rock-paper-scissors-lizard-Spock` in Haskell. He defined a data type:

```
data Gesture = Rock | Paper | Scissors | Lizard | Spock
```

and now wants to define an `Ord` instance for this data type that specifies which of two gestures beats the other.

Explain why this is not a good idea. The answer can be found by carefully reading the documentation of `Data.Ord` or imagining what happens if you sort a list of gestures using such an ordering.

¹If you're now asking yourself: "But what kind of numbers do you mean exactly, Sir?" then please consider both various Haskell types having a `Num` instance (`Integer`, `Rational`, `Float`, ...) as well as various mathematical classes of numbers (\mathbb{N} , \mathbb{Z} , \mathbb{Q} , \mathbb{R} , \mathbb{C} , $\mathbb{R} \setminus \{0\}$, ...).