Final Exam Geometric Algorithms, April 18, 2013, 14.00–17.00

Read every question carefully, make sure you understand it, and be sure to answer the question. Read the question again after answering it, as a check whether you really answered the question. Answer the easier questions first, and then the harder ones. Answer questions in sufficient but not too much detail. You may not use the textbook during the exam.

Be sure to put your name on every piece of paper you hand in. Also write down your studentnummer. If you write readable, unambiguous, and technically correct, you get one point for free (in particular, do not write "line" if you mean "line segment" and do not write "Step 1 takes $n \log n$ time" when you mean "Step 1 takes $O(n \log n)$ time"). The other nine points can be earned by answering the questions correctly. Good luck!

1. (1 point) What is the worst-case (or expected worst-case) running time of the algorithms for (use k for the output size whenever appropriate):

   (a.) computing the common intersection of a set of n half-planes in the plane explicitly?

   (b.) computing the arrangement of a set of n lines in the plane?

   (c.) computing the furthest site Voronoi diagram of a set of n point sites in the plane?

   (d.) performing a 3-dimensional range query on a kd-tree storing n points in 3-space?

2. (1 point) Suppose we have a planar subdivision in doubly-connected edge list representation (DCEL), and for every vertex v we have:
   $\mathit{IncidentEdge}(v) = \mathit{Twin}(\mathit{Prev}(\mathit{Twin}(\mathit{Prev}(\mathit{IncidentEdge}(v)))))$. Describe the subdivision in words and draw an example that illustrates your description well.

3. (1 points) Let T be a 3-dimensional range tree storing n points in 3-space. Suppose the main tree uses the z-coordinate to split the point set, while its associated structures have main trees that use the y-coordinate and associated structures that use the x-coordinate. Suppose we perform a 3-dimensional range reporting query on T. Let the number of points in the query range be denoted with k.

   (a.) What is the number of nodes the query visits in the main tree (on z-coordinate) in the worst case, expressed using n, k, and $O(..)$ notation? Give a short explanation (one or two sentences) of your answer.

   (b.) What is the number of nodes the query visits in all of the "middle" associated structures (on y-coordinate) together, in the worst case, expressed using n, k, and $O(..)$ notation? Give a short explanation (one or two sentences) of your answer.

   (c.) What is the number of nodes the query visits in all of the deepest associated structures (on x-coordinate) together, in the worst case, expressed using n, k, and $O(..)$ notation? Give a short explanation (one or two sentences) of your answer.

4. (1 point) In the plane, there are two non-vertical line segments s_1 and s_2 that intersect. The left endpoint p_1 of s_1 is more to the left than the left endpoint p_2 of s_2, and the right endpoint q_1 of s_1 is more to the right than the right endpoint q_2 of s_2. The slope of the line ℓ_1 containing s_1 is larger than the slope of the line ℓ_2 containing s_2.

   Formulate the above paragraph in its dual form using the usual point-line duality. Then draw a possible situation with s_1^* and s_2^* in the dual plane, and label all points and lines clearly in this drawing.

5. (1 point) In the randomized incremental construction algorithm to compute the Delaunay triangulation, the points are added incrementally in random order. When adding the i-th point p_i, the triangle t in which it falls is first located, and then the point is incorporated in the Delaunay triangulation by connecting it to the vertices of t, and then edges of the triangulation are flipped to make sure that the triangulation with p_i is Delaunay again. We use D_i to denote the Delaunay triangulation of the first i points (in some bounding box or triangle, but you may ignore this).

   Prove that the expected number of edge flips performed to make D_i Delaunay, after connecting p_i to the vertices of t, is constant.

6. (2 points) Given a set S of n non-intersecting line segments in the plane. They also do not have shared endpoints, nor endpoints on interiors of other line segments. Furthermore, no two endpoints lie on the same horizontal line.

   We wish to compute a planar spanning tree that includes all the line segments of S (a connected, acyclic planar embedded graph whose vertices are exactly the $2n$ endpoints of the line segments in S, and which includes these n line segments as edges).

   Give a plane sweep algorithm to solve this problem efficiently. Give the status, describe the status structure, describe the event handling, and analyze the running time.

7. (1 point) Consider the following three problems on a set P of n points in the plane: CH (computing the convex hull of P), EMST (computing the Euclidean Minimum Spanning Tree of P), and ETSP (computing the optimal Euclidean Traveling Salesperson Path).

   State for each of these problems whether they are in P, in NP, and/or NP-hard. To this end, make a 3×3 table with rows for “CH”, “EMST”, and “ETSP”, and columns for “in P”, “in NP”, and “NP-hard”. Write in the nine entries of the table “yes” or “no”. In case an answer cannot be given because science does not know the answer yet, then write a “?” in that entry.

8. (1 point) The lecture on dilation defined the concepts of *spanning ratio* and *dilation* of an embedded planar graph (in this order).

   Compute the spanning ratio of the graph shown in the figure. You may use a calculator and give the answer in decimal notation, but you may also write an expression with square-roots still in it if you don’t have a calculator.

   Bonus question: Compute the dilation of the graph.