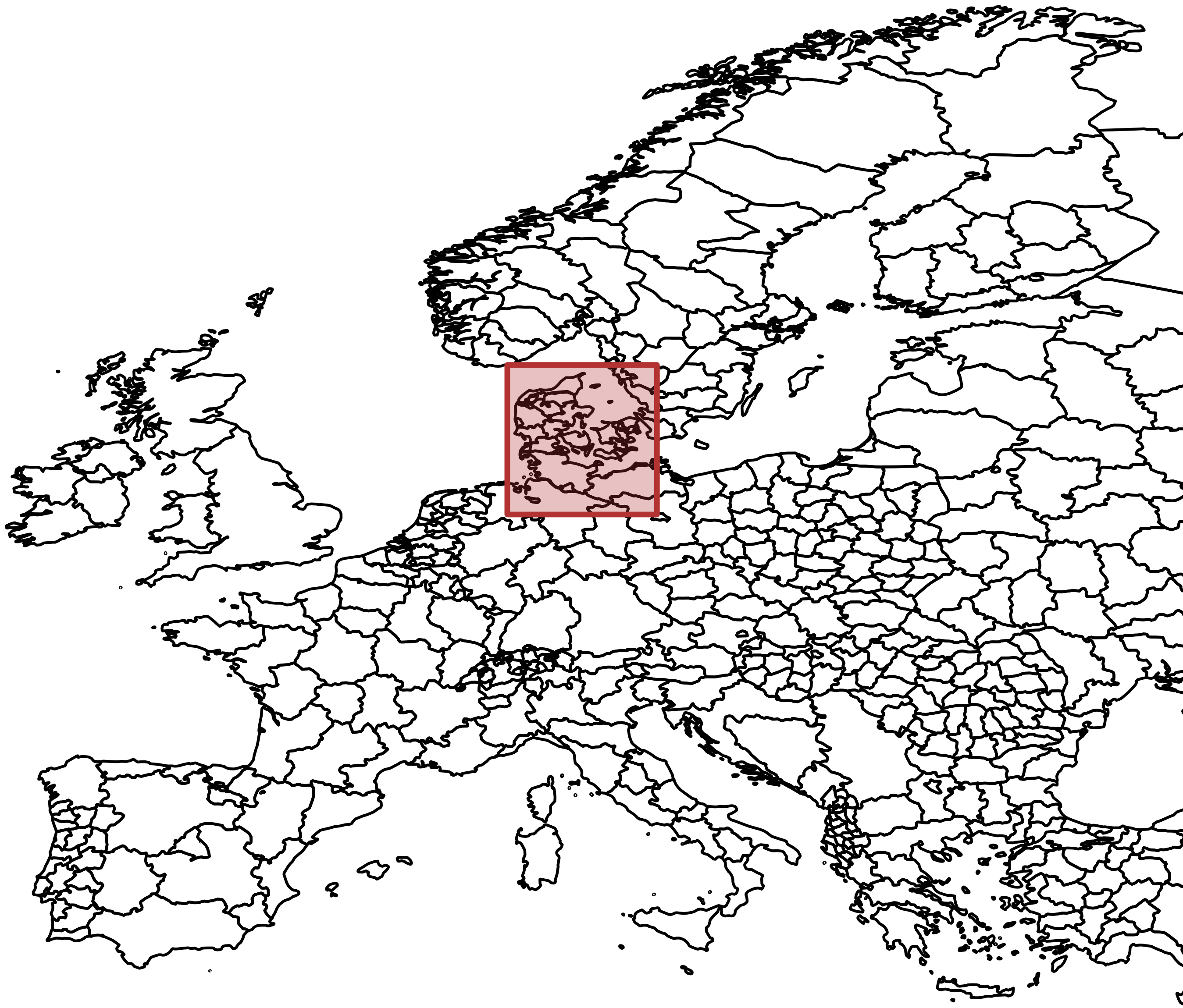
Windowing Queries

Given a set S of n disjoint line segments in the plane.

Store S in a data structure s.t. given a query rectangle R, we can find the segments in S intersecting R efficiently.



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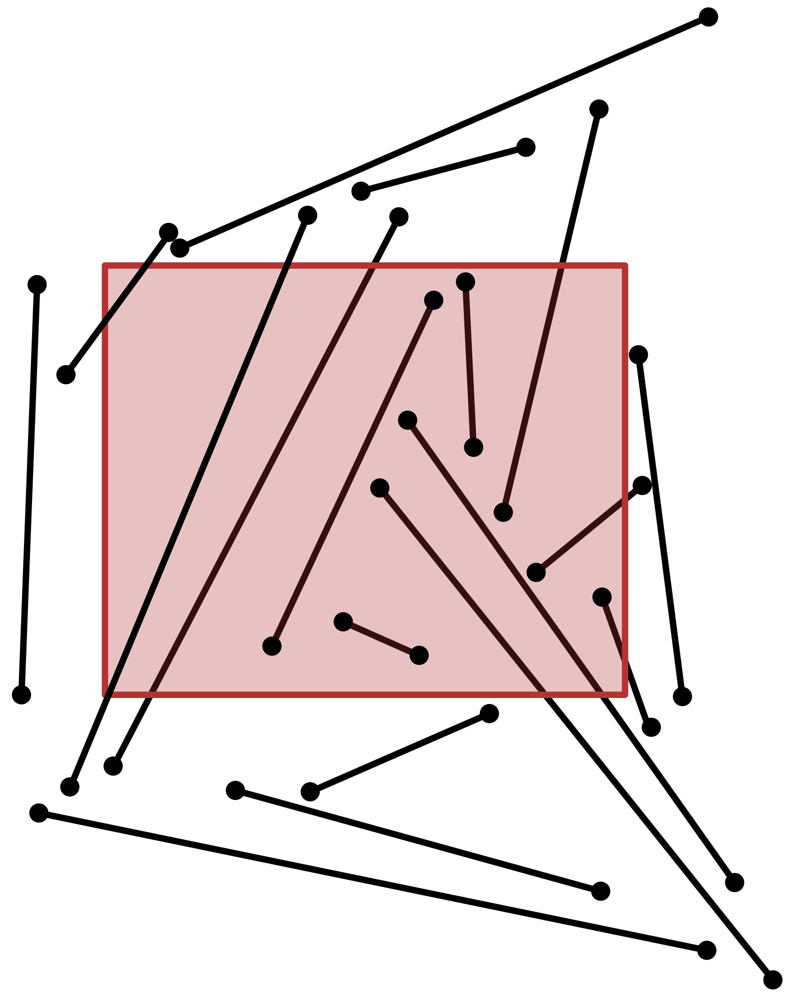

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The segments that intersect R

1) have an endpoint in R, or

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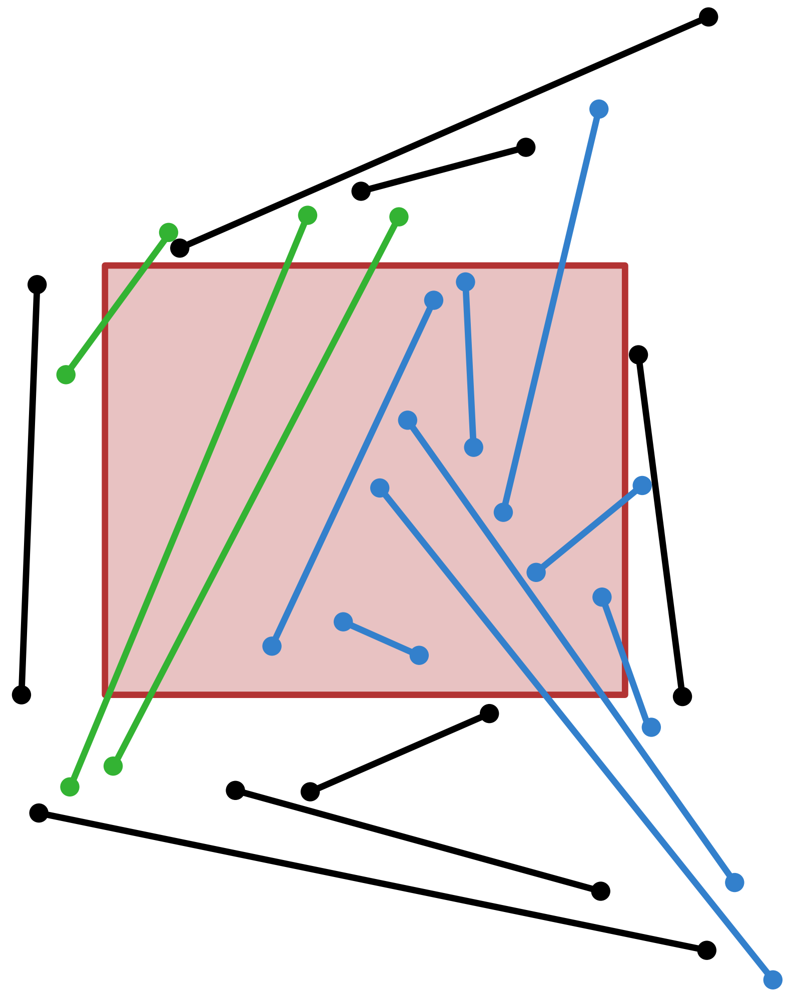

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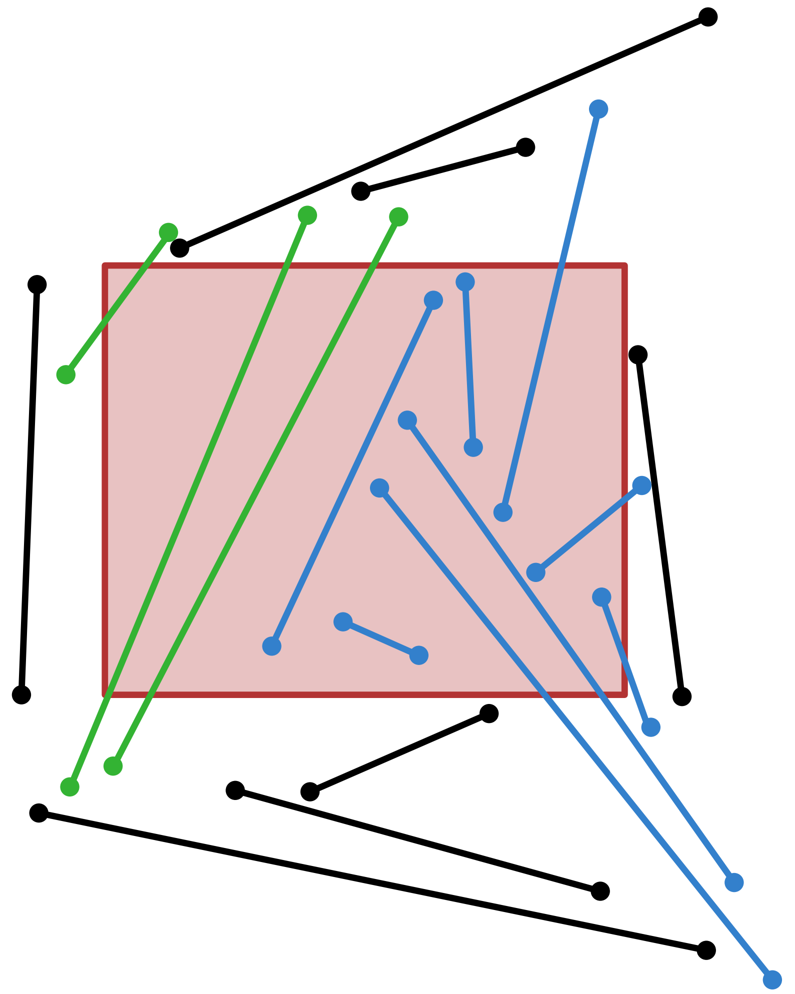

Windowing Queries

Given a set S of n disjoint orthogonal line segments in the plane.

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The segments that intersect R

1) have an endpoint in R, or
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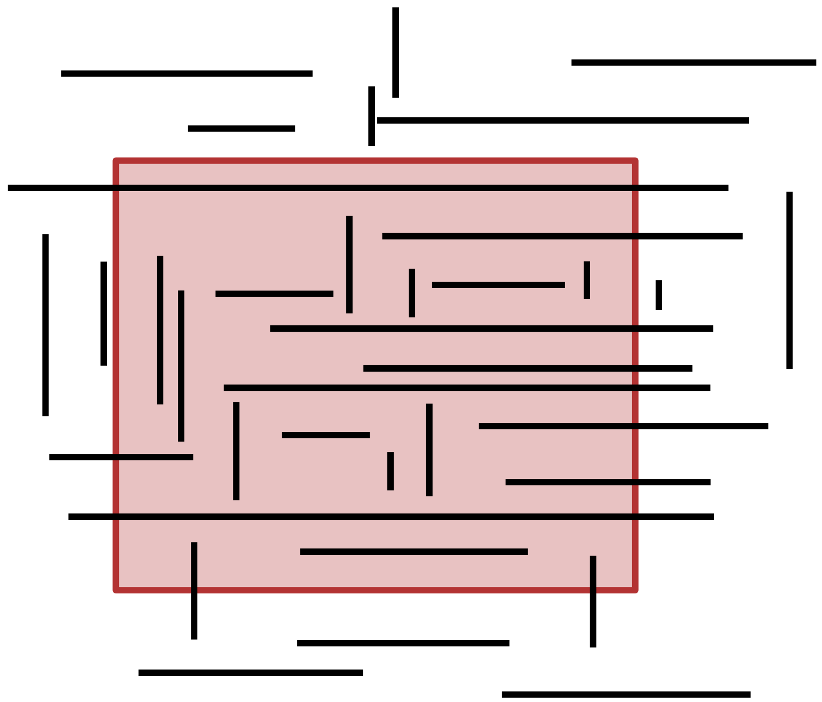

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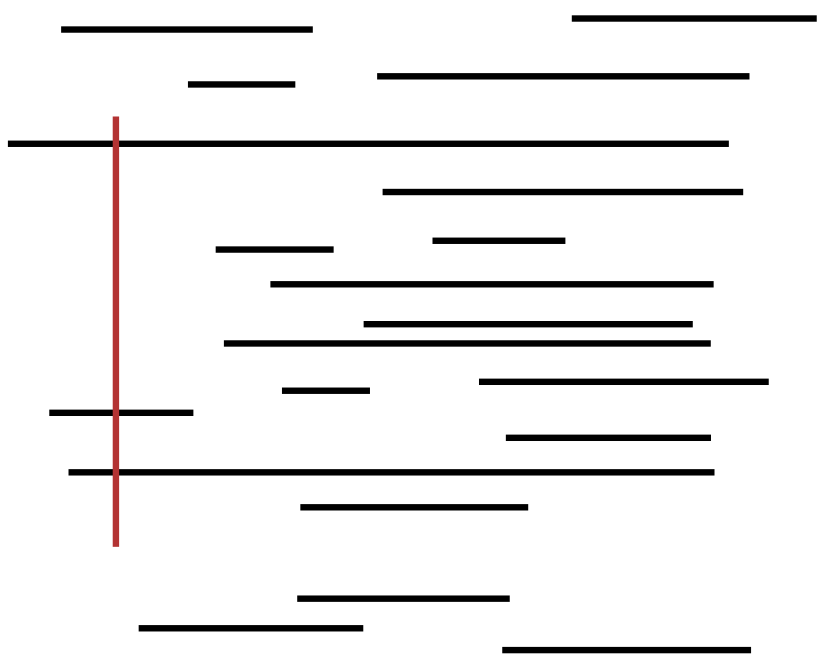
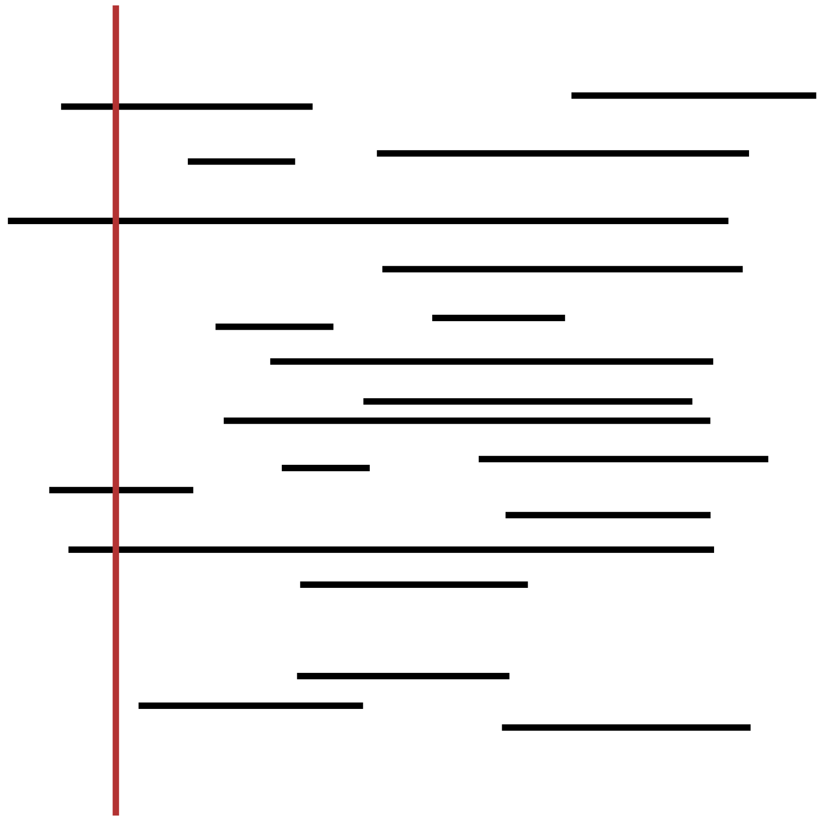


Interval Stabbing Queries

Given a set S of n intervals in $\mathbb{R}^1$

Store S in a data structure s.t. given a query value q, we can find the intervals in S intersecting q efficiently.



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We store S in an interval tree T

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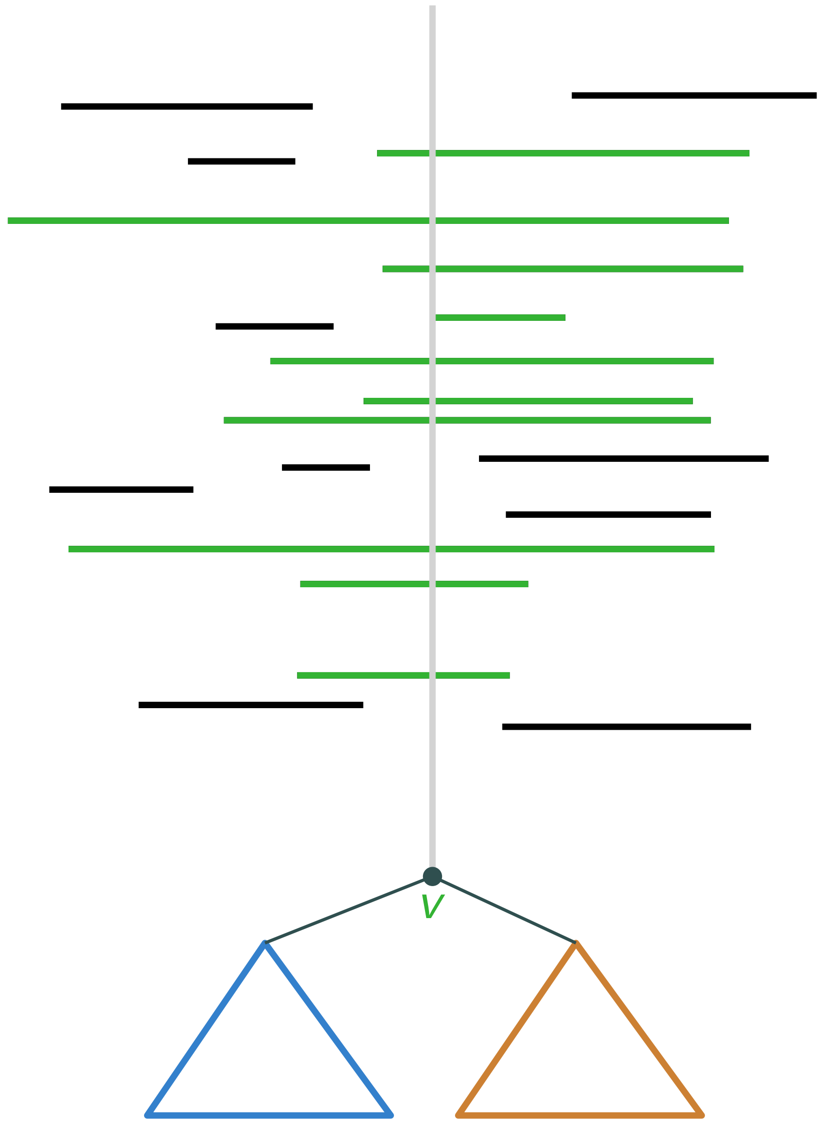
T is a balanced BST on the endpoints

The root of the tree (the median endpoint) v stores the intervals $I(v)$ that contain v



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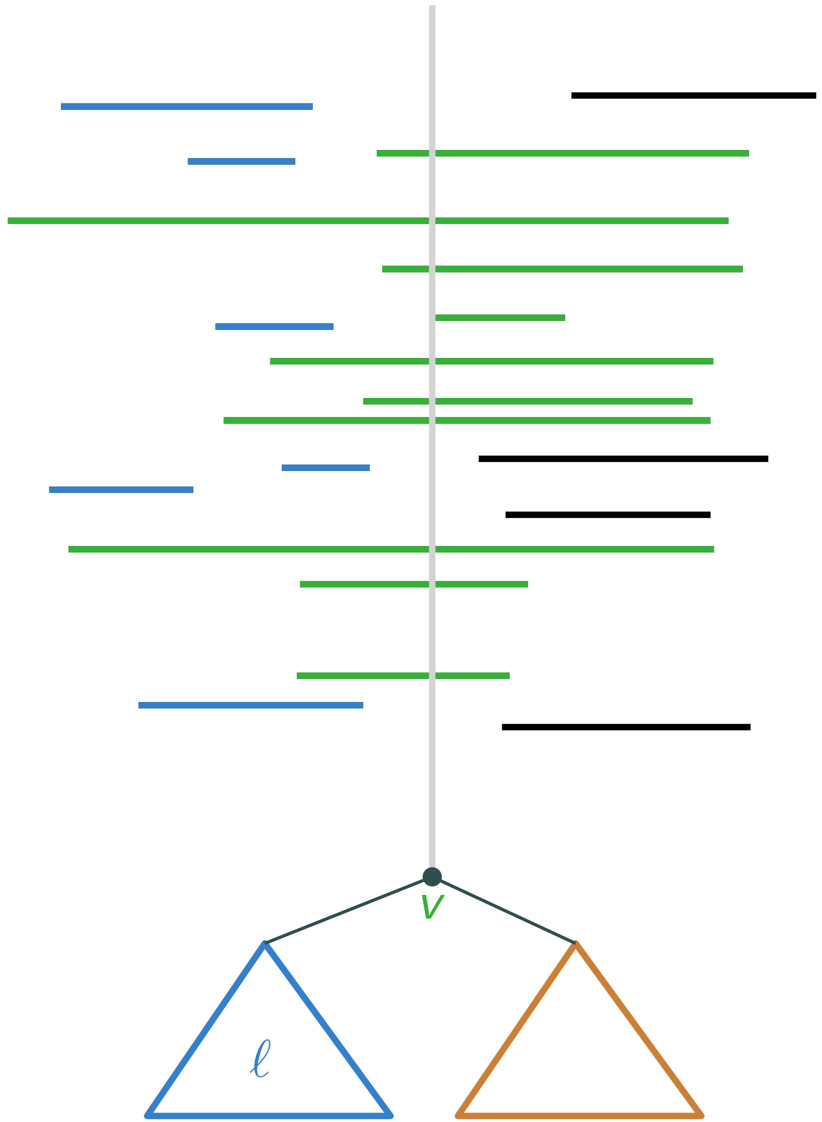
The root of the tree (the median endpoint) v stores the intervals $I(v)$ that contain v

The left subtree ℓ of v stores the intervals that lie completely left of v.



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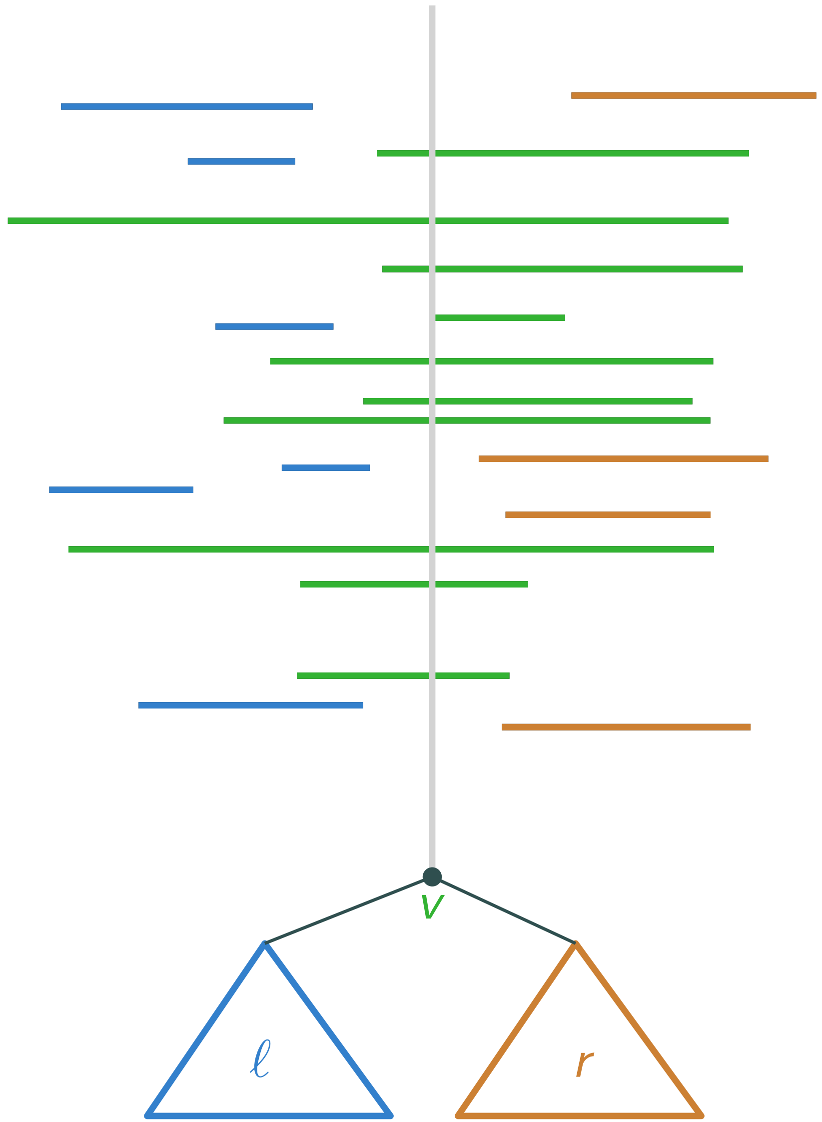
The right subtree r of v stores the intervals that lie completely right of v.



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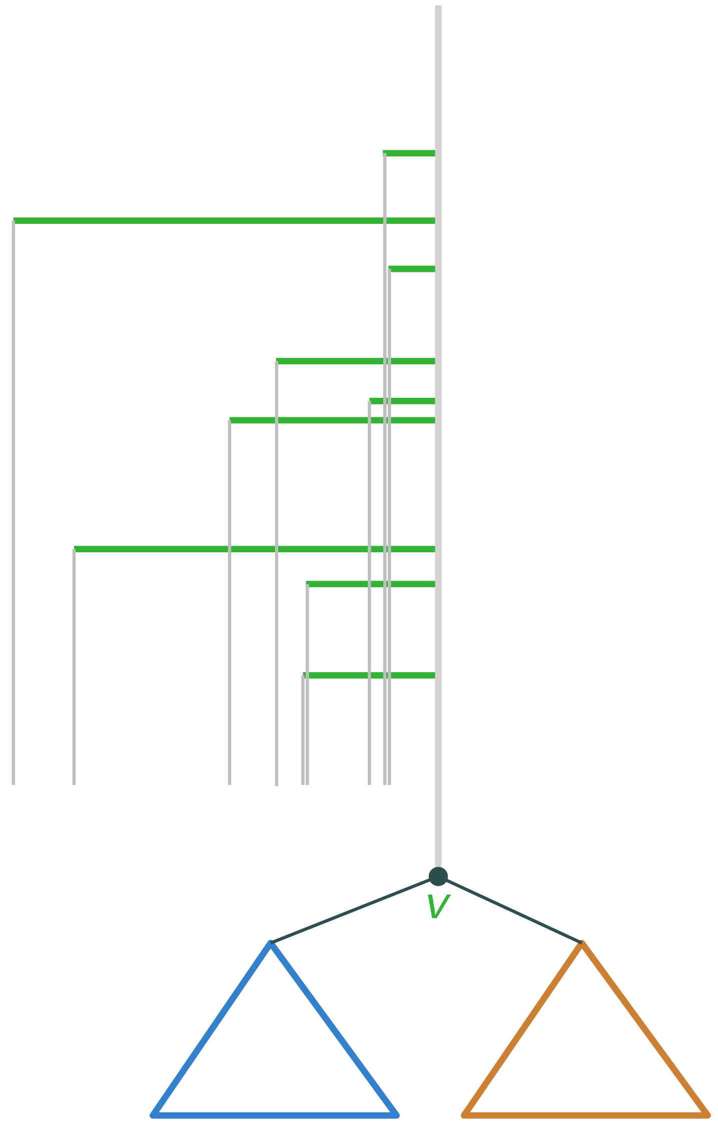
The root of the tree (the median endpoint) v stores the intervals $I(v)$ that contain v

store these intervals twice:

1) sorted on increasing left endpoint

2) sorted on decreasing right endpoint



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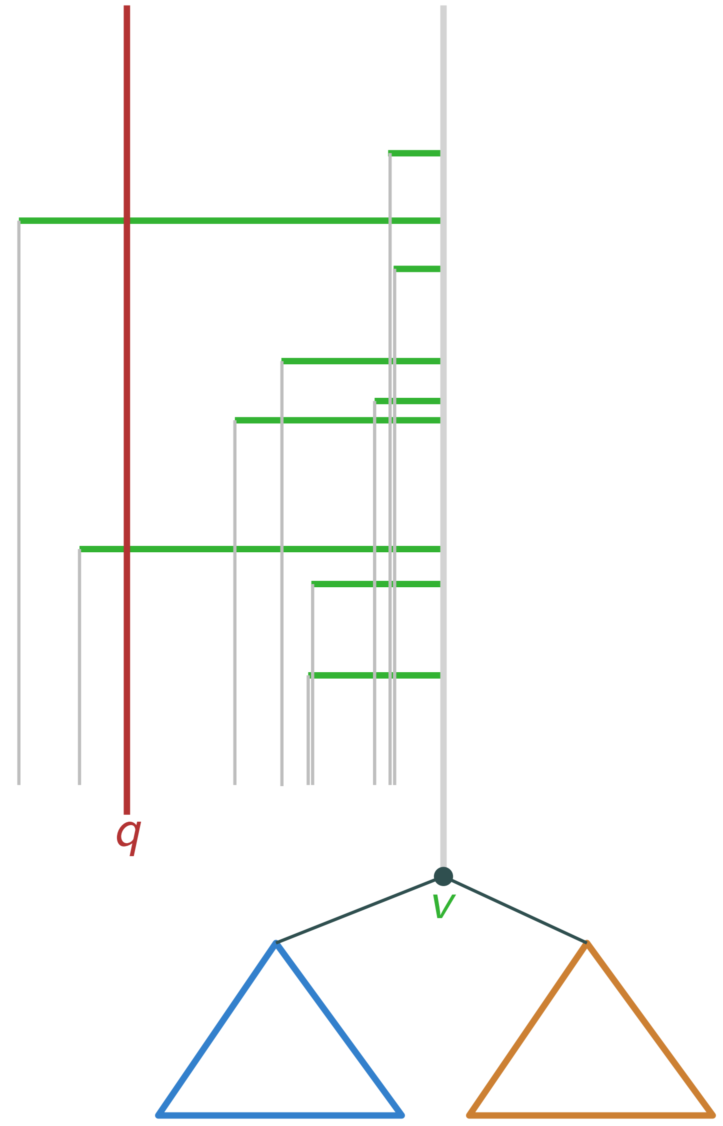

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Query(q, T)
if q left of v **then**
  report intervals from $I(v)$ using the list of left-end points,
  stop at the first interval right of q.
  Query(q, ℓ)
else if q right of v
  ...

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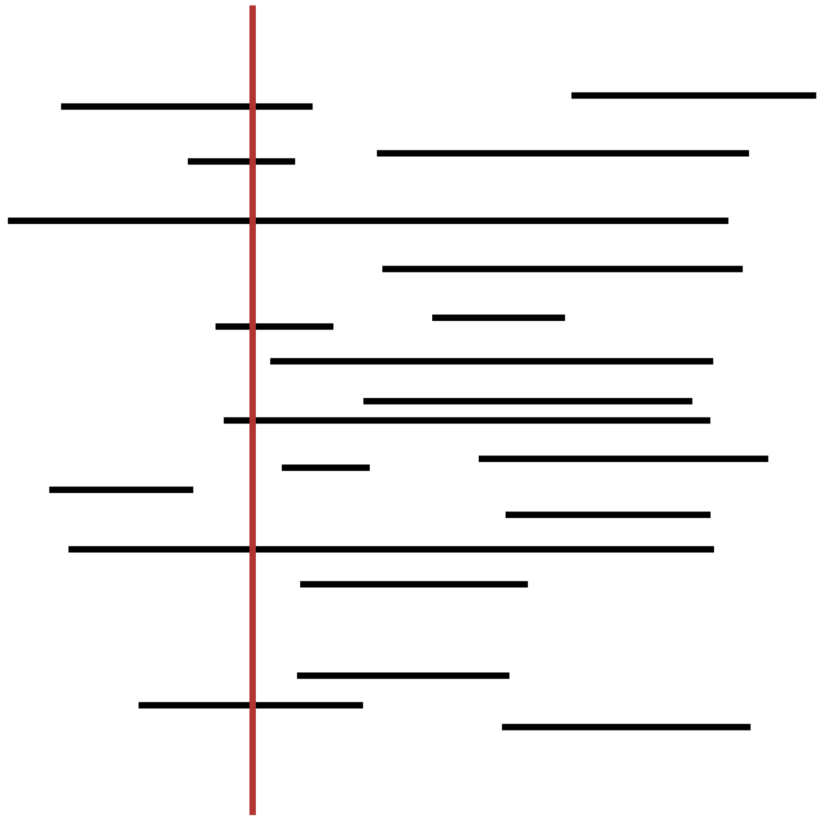

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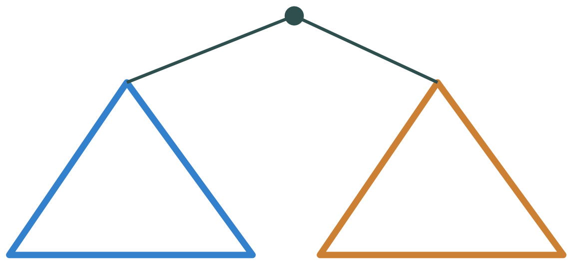
Space usage:

Query time:

Preprocessing time:



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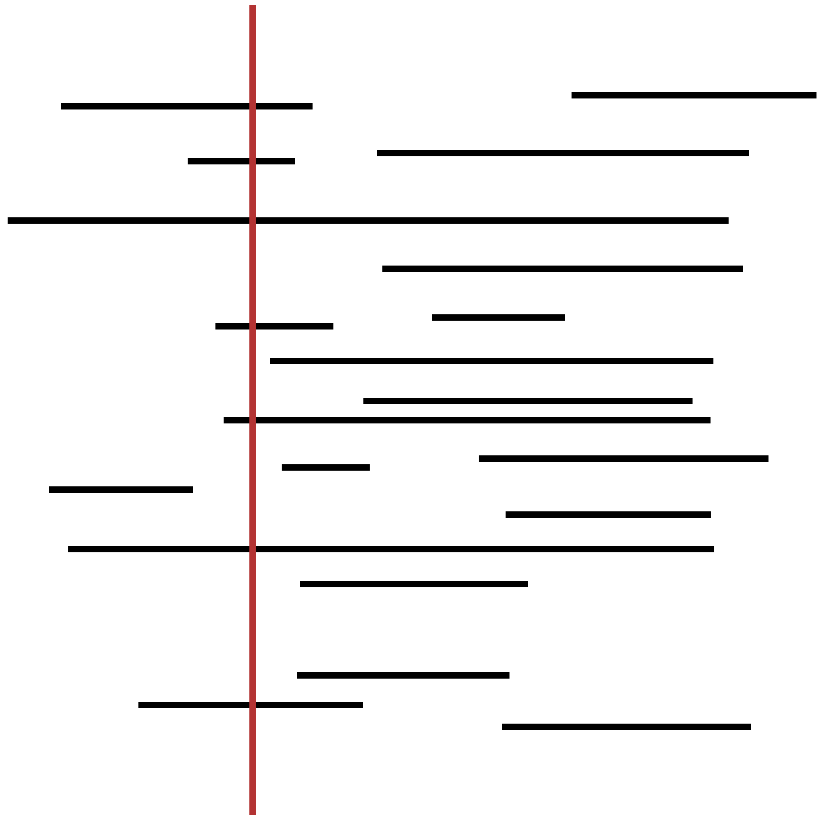

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Space usage: $O(n)$

Query time:

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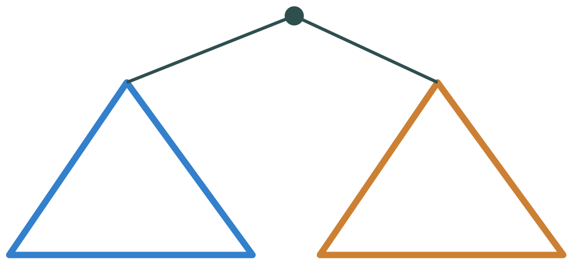

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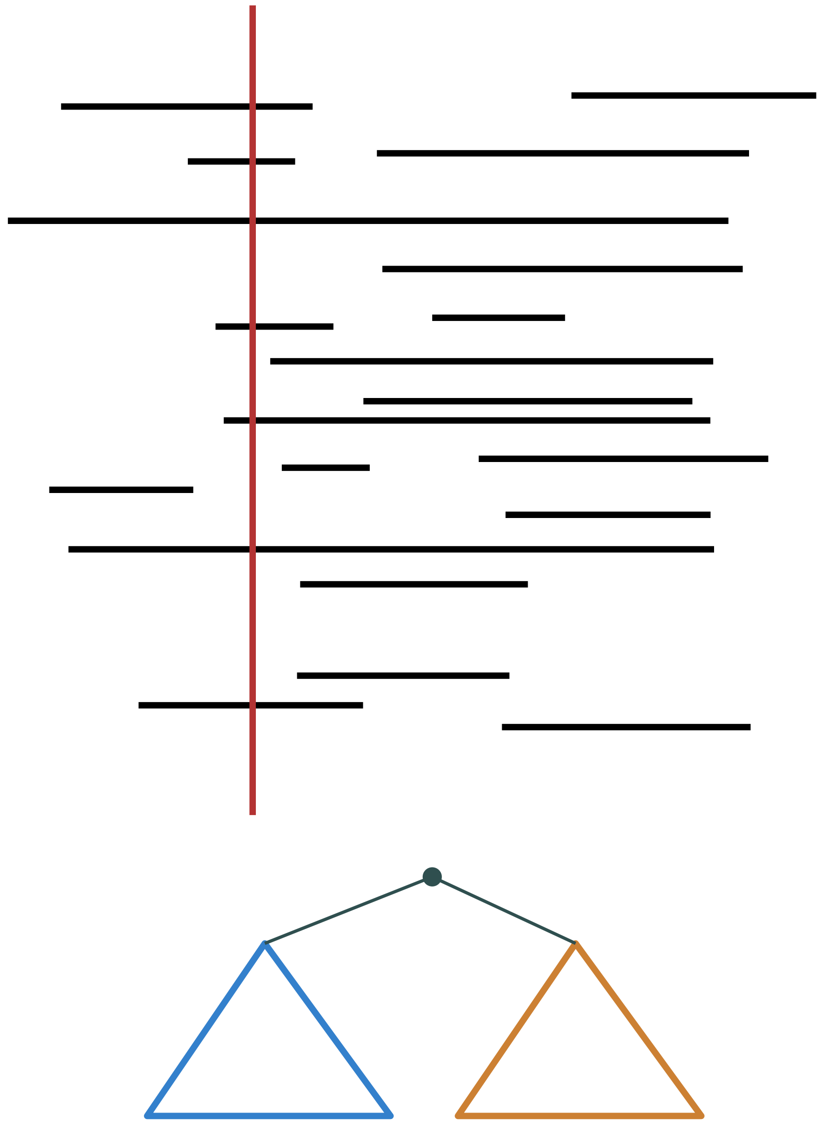
Query time: $O(\log n + k)$

$k =$ #intervals reported

Preprocessing time:



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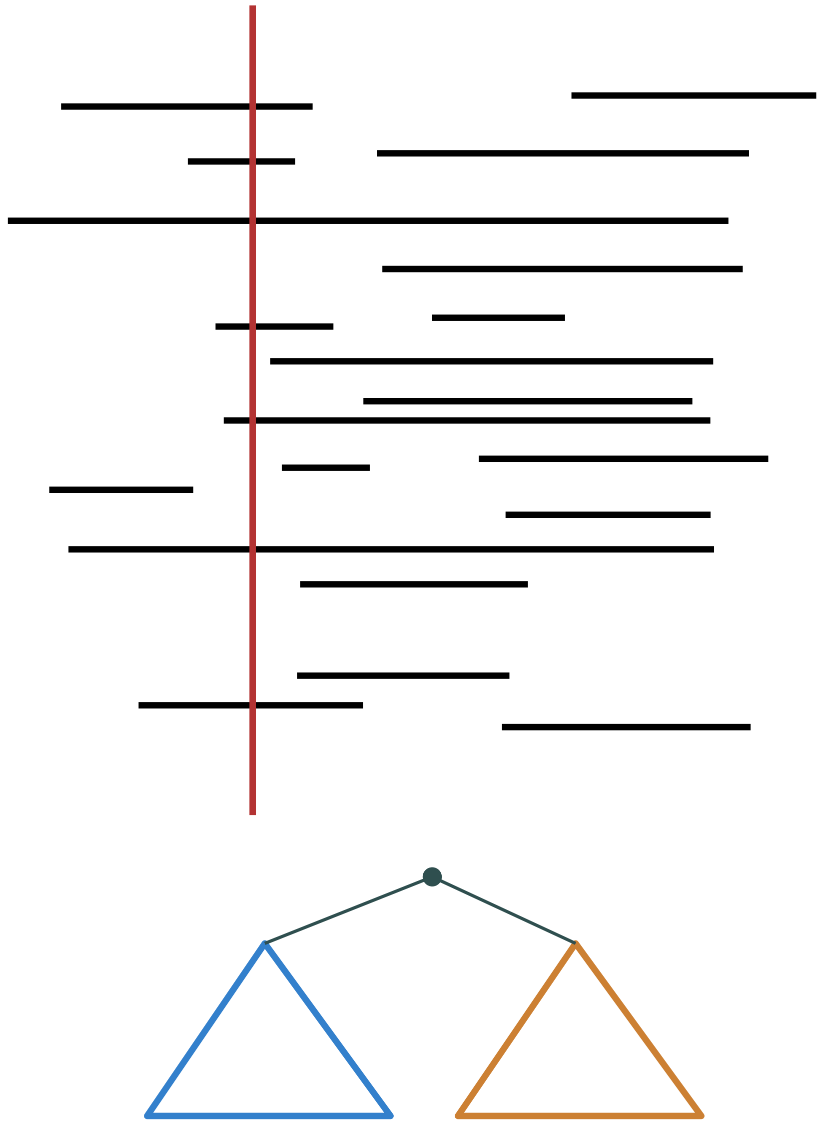

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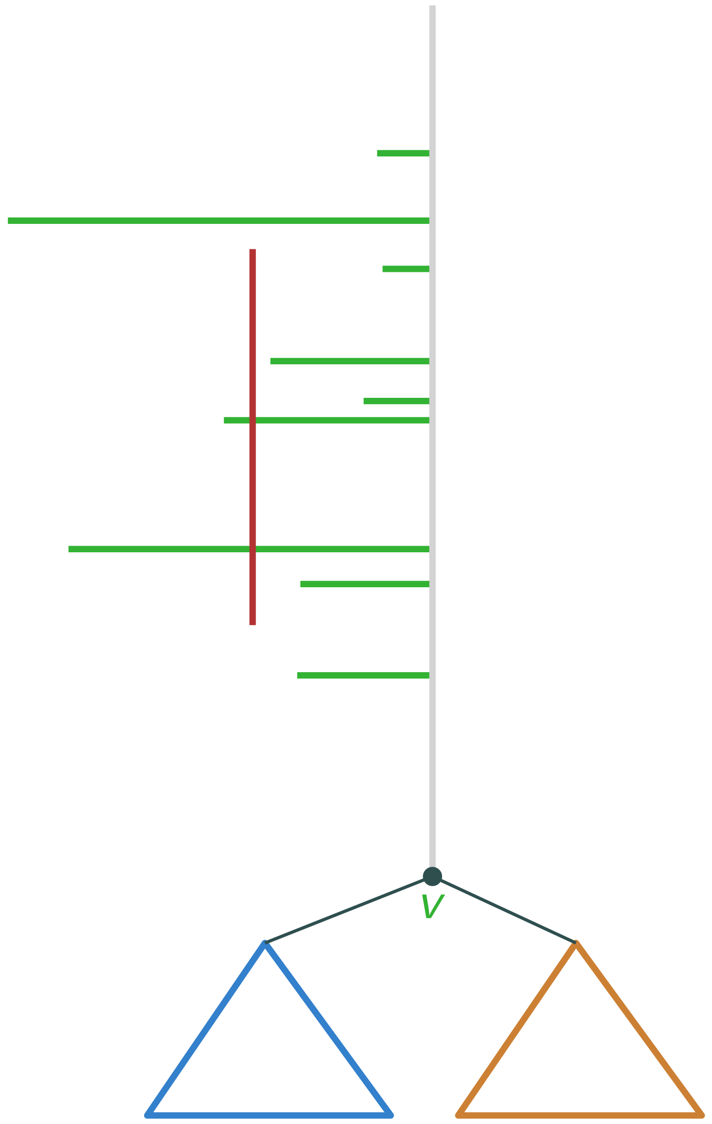

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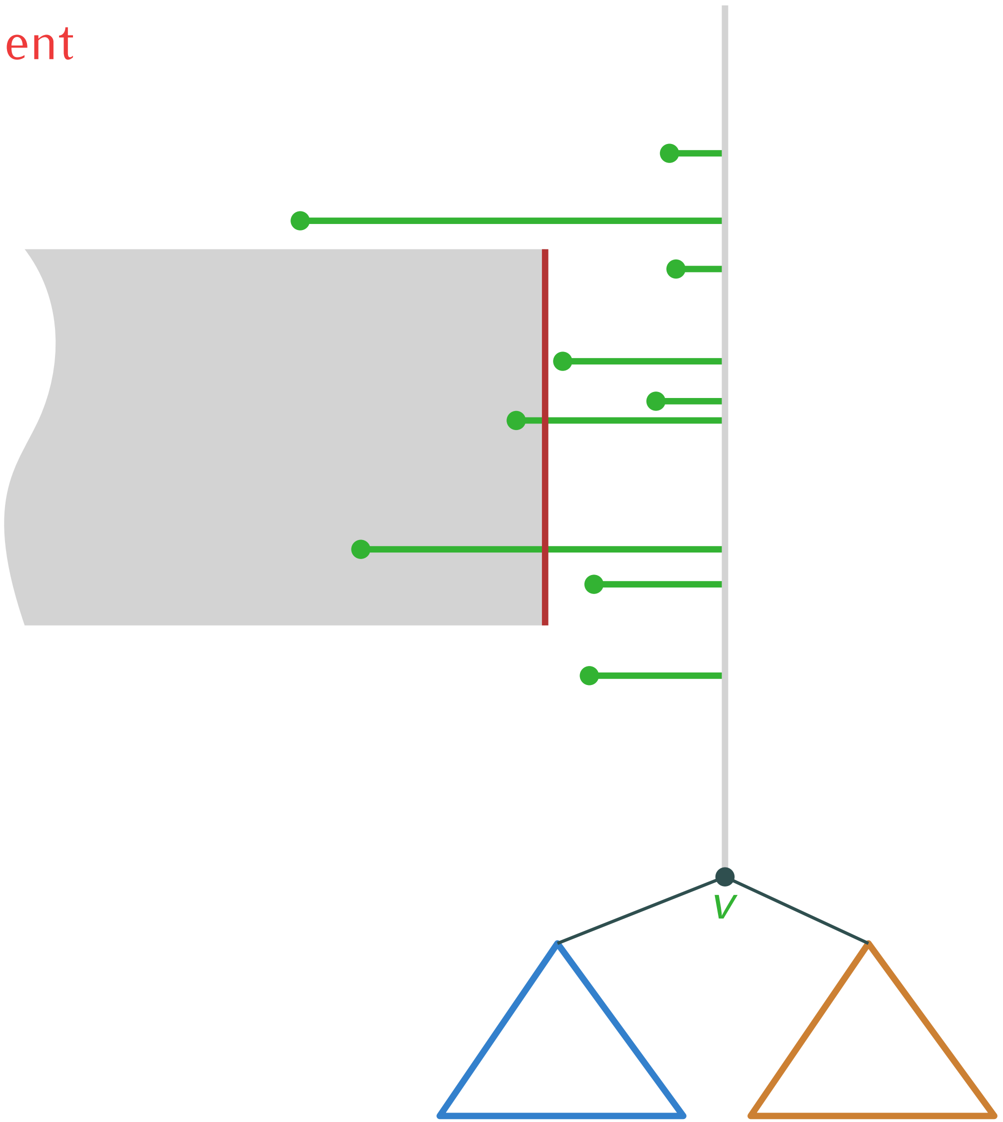

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store these intervals twice:

1) a range tree on their left endpoints

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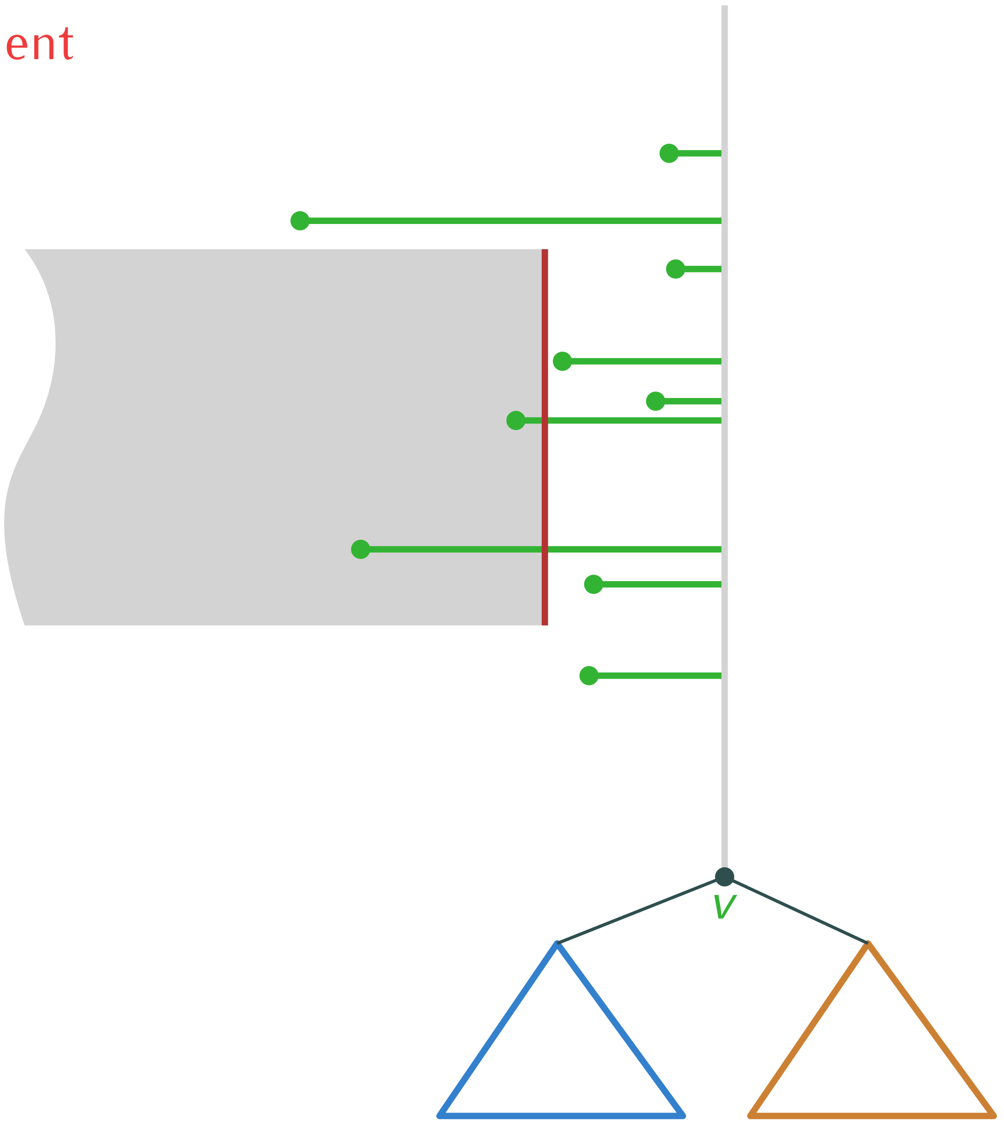

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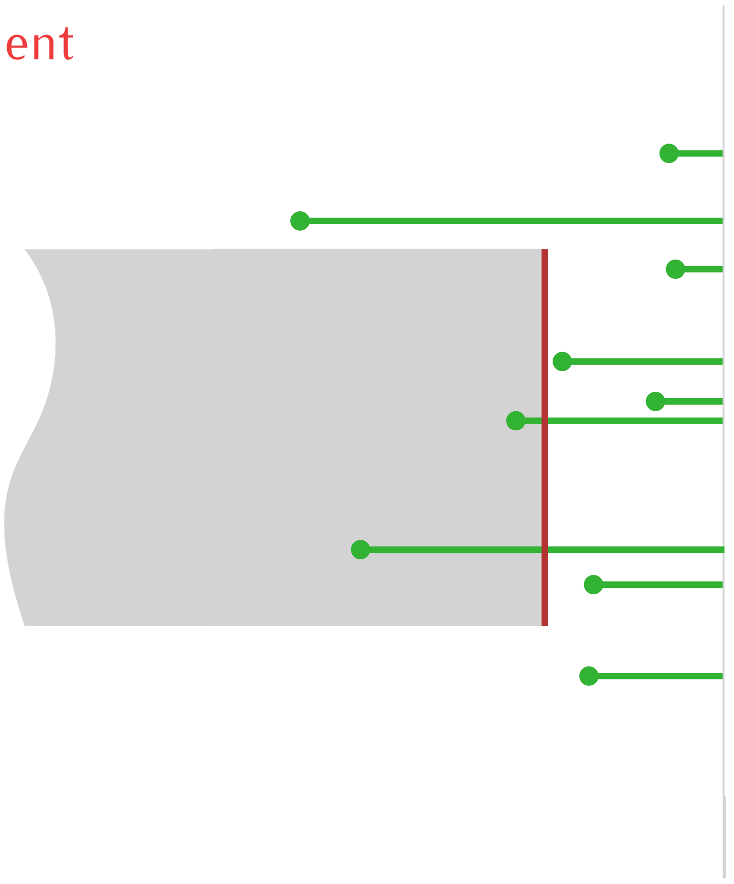

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Space usage:

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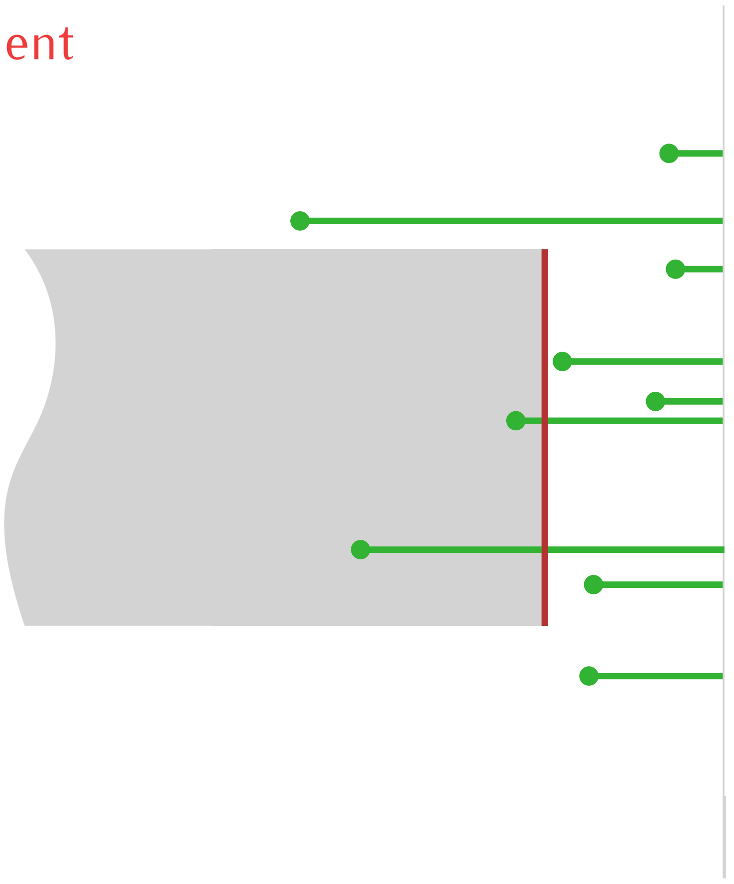

We store S in an interval tree T

Space usage: $O(n \log n)$

Query time:

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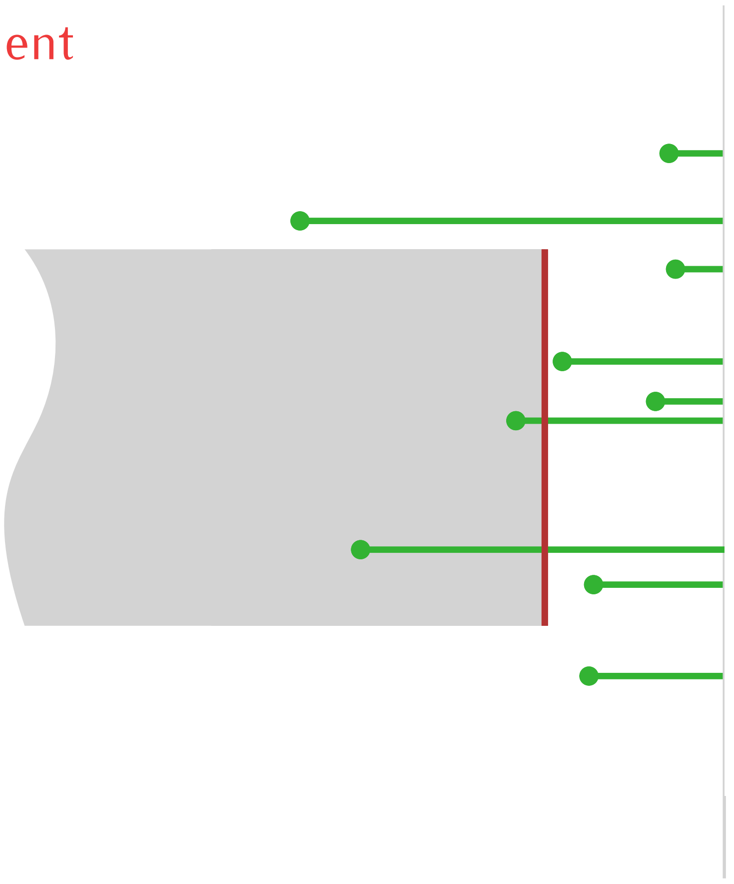

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Space usage: $O(n \log n)$

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$k = \#$intervals reported

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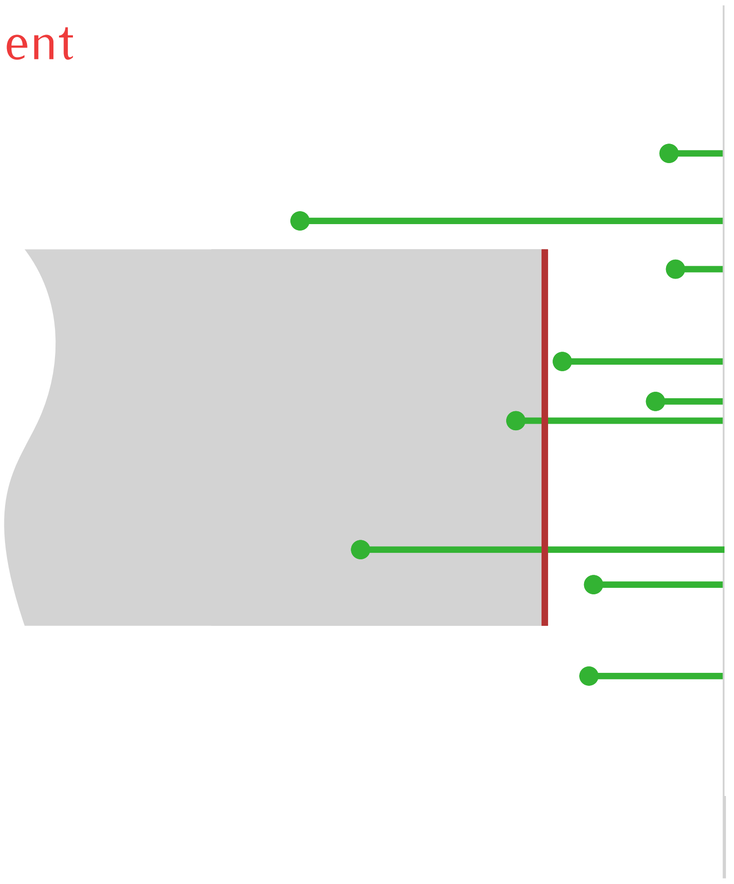

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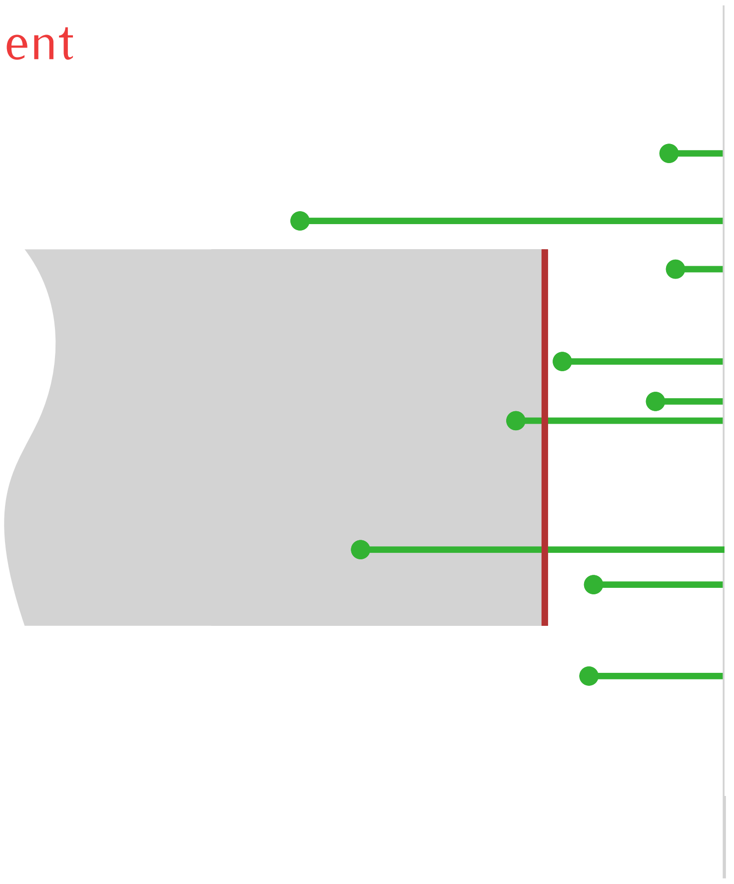

We store S in an interval tree T

Space usage: $O(n)$ using priority search trees

Query time: $O(\log^2 n + k)$

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Preprocessing time: $O(n \log n)$

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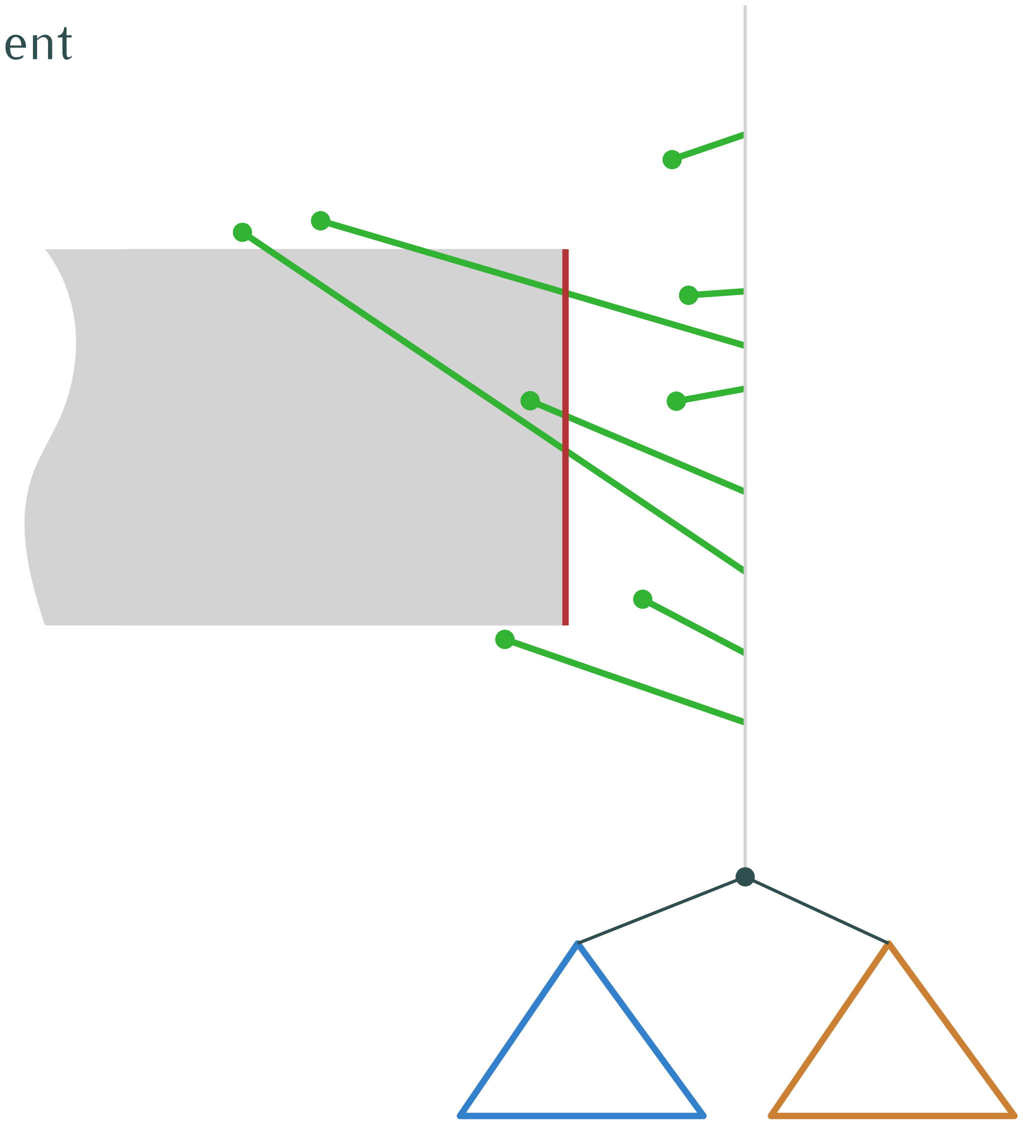
Our solution using an interval tree + range tree (or priority search tree) no longer works



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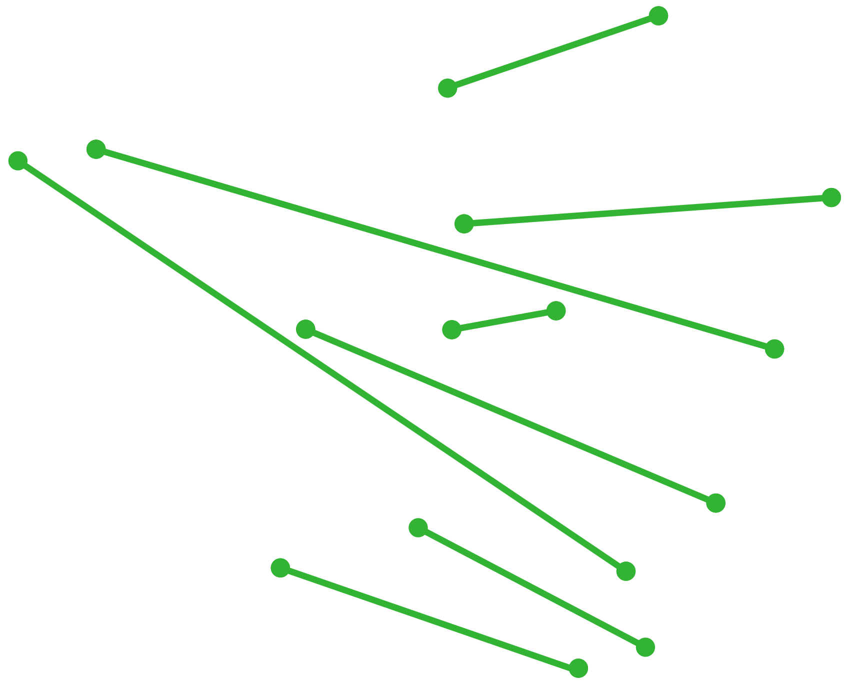

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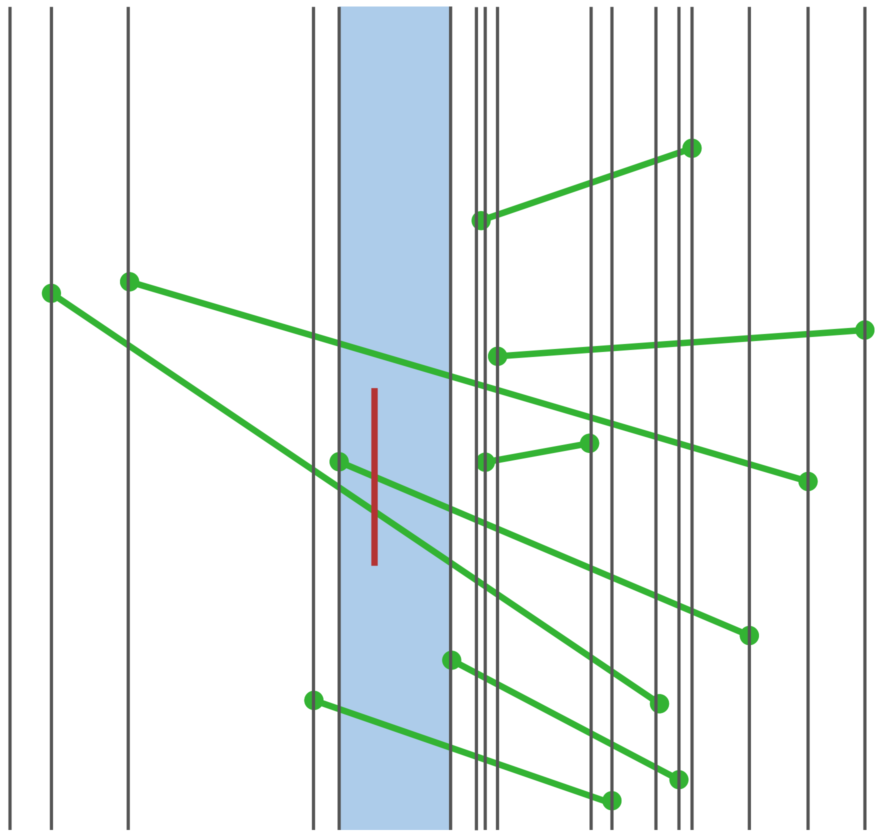
Split into elementary intervals in which a vertical line intersects the same segments.



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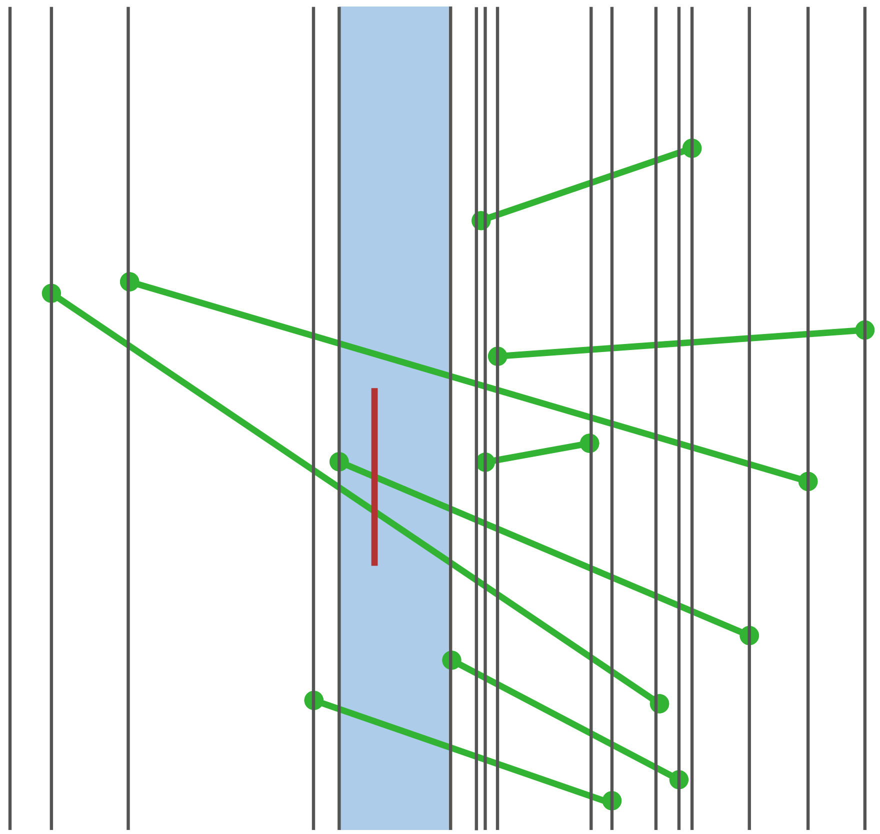
Storing all segments segments in all elementary intervals uses $\Theta(n^2)$ space



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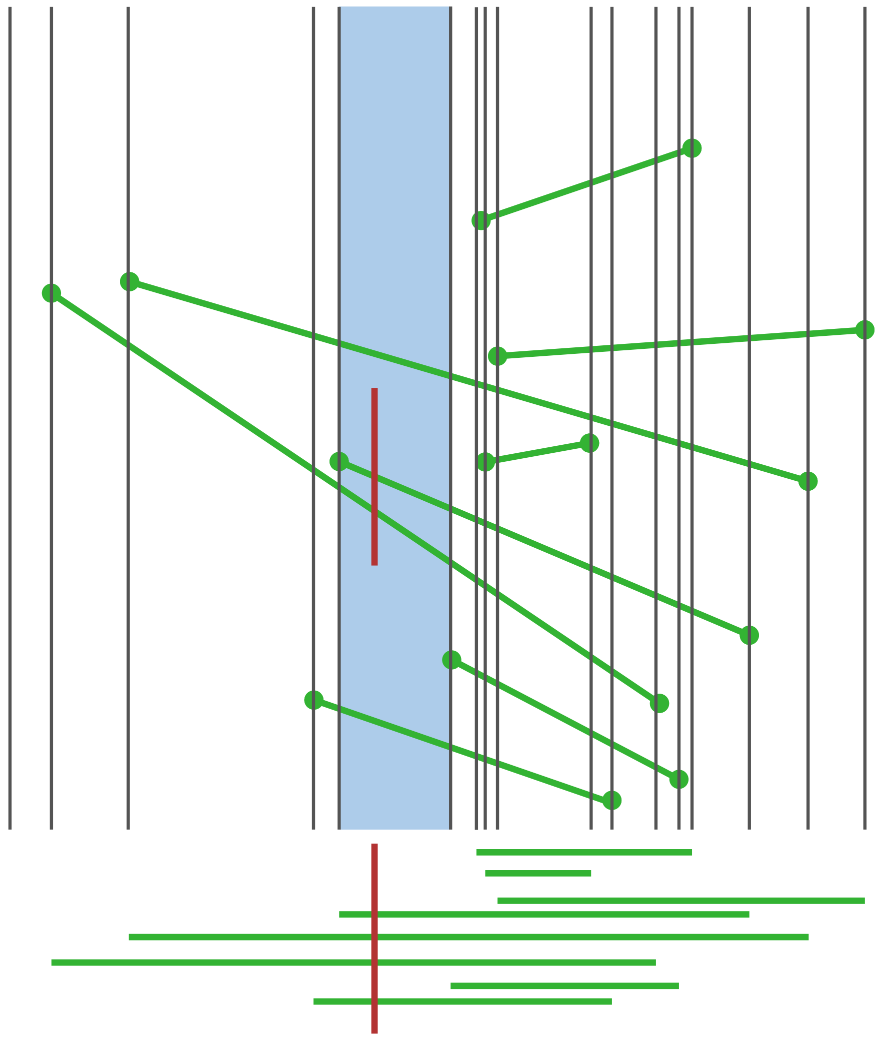
Project the segments onto the x-axis, yielding intervals. We build a different data structure for interval stabbing.



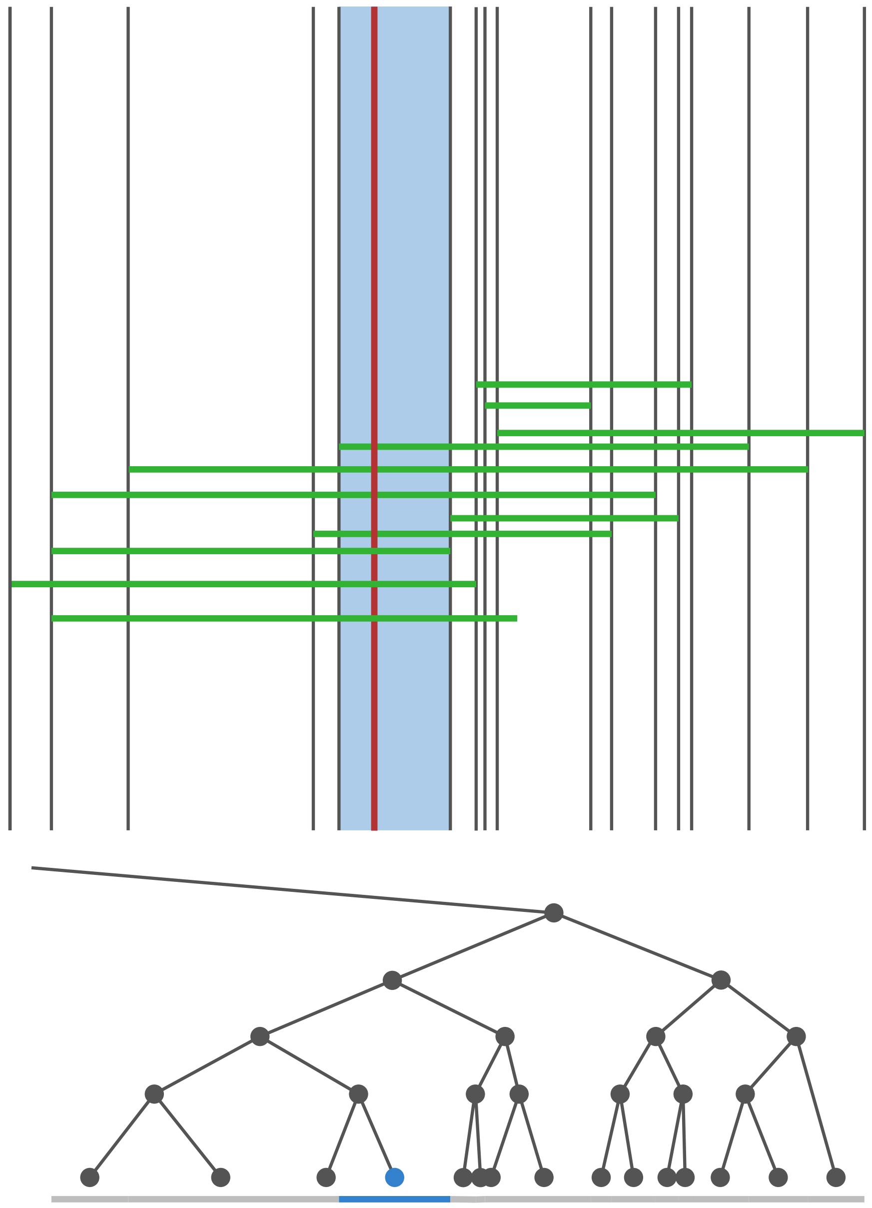
Interval Stabbing Queries

Given a set S of n intervals in $\mathbb{R}^1$

Store S in a data structure s.t. given a query point q, we can find the intervals in S intersecting q efficiently.

Split into elementary intervals in which a vertical line intersects the same segments.

Store the elementary intervals as leaves in a balanced BST T.



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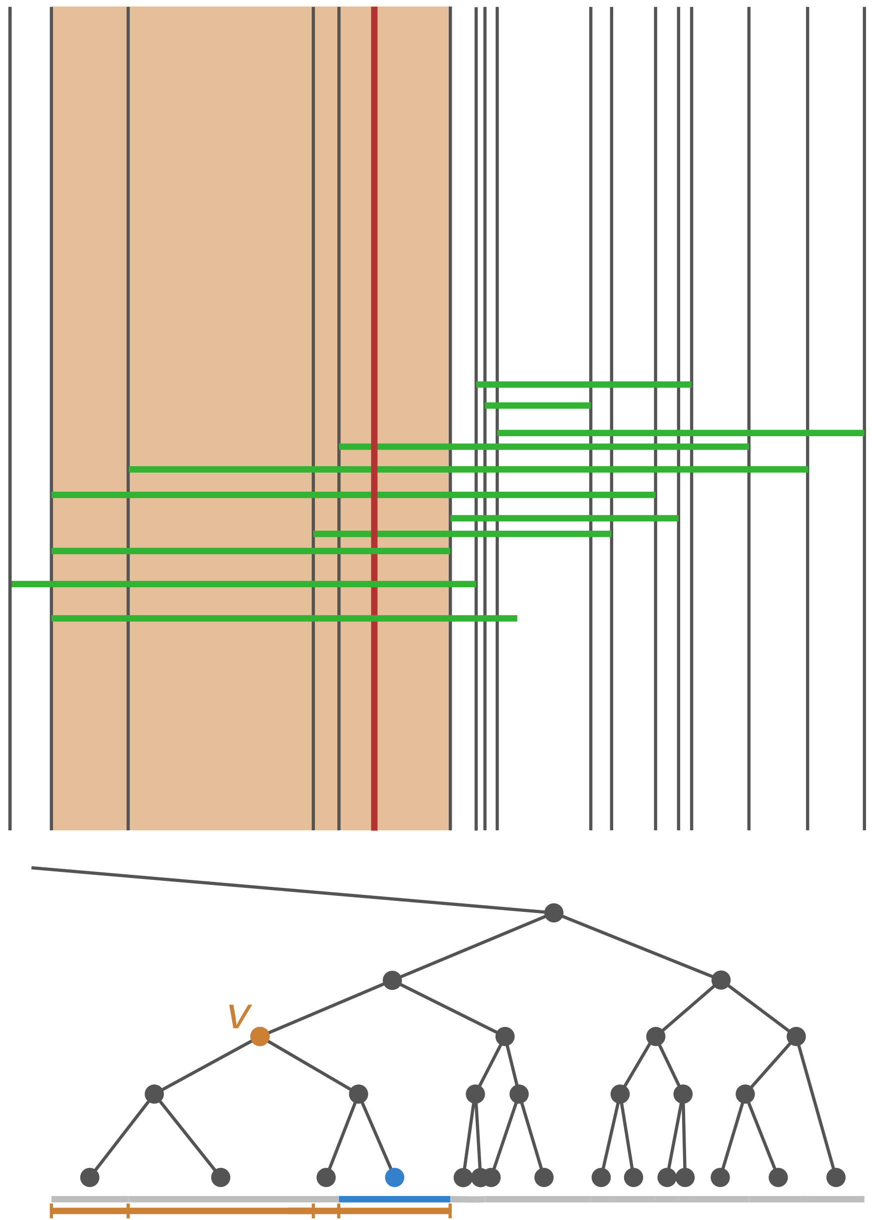
Every node v

- corresponds to an interval I_v, which is the union of the elementary intervals stored in its subtree.



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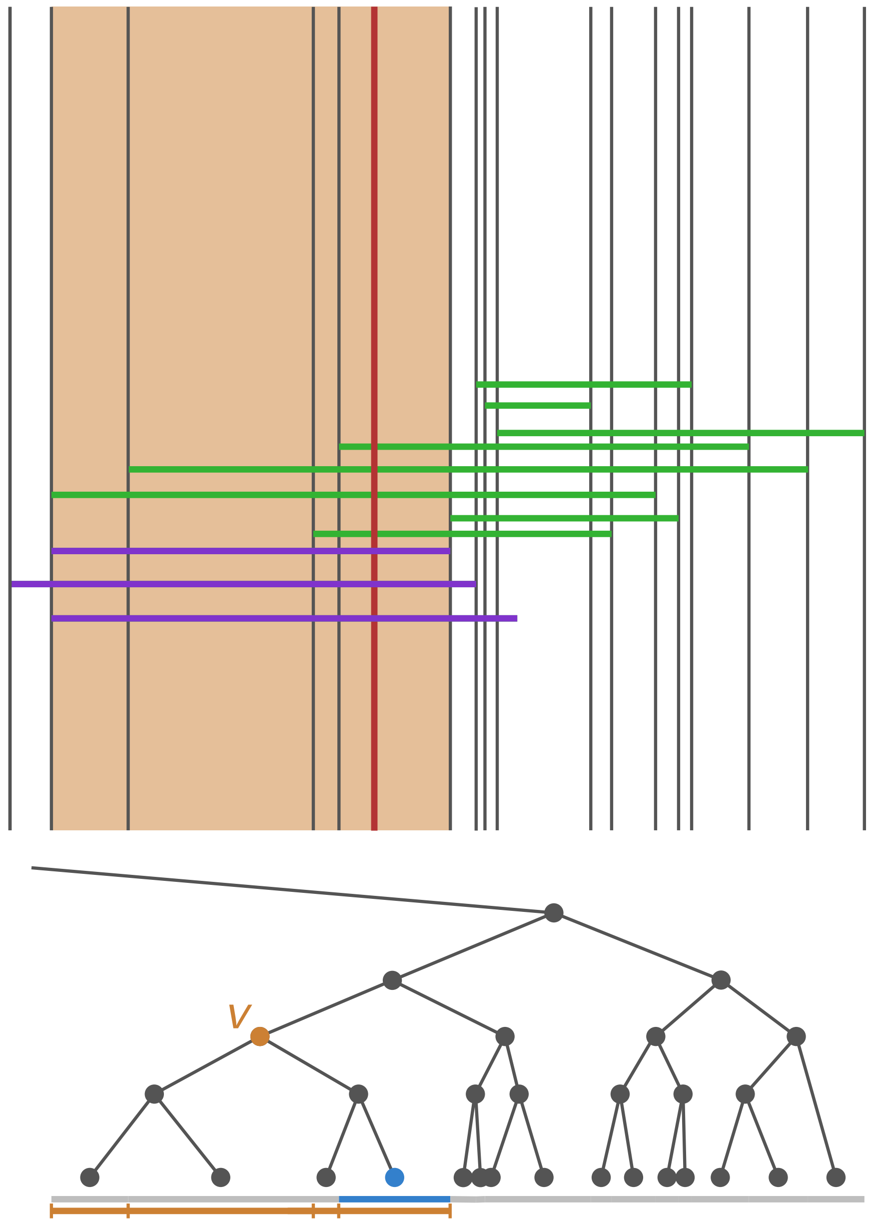

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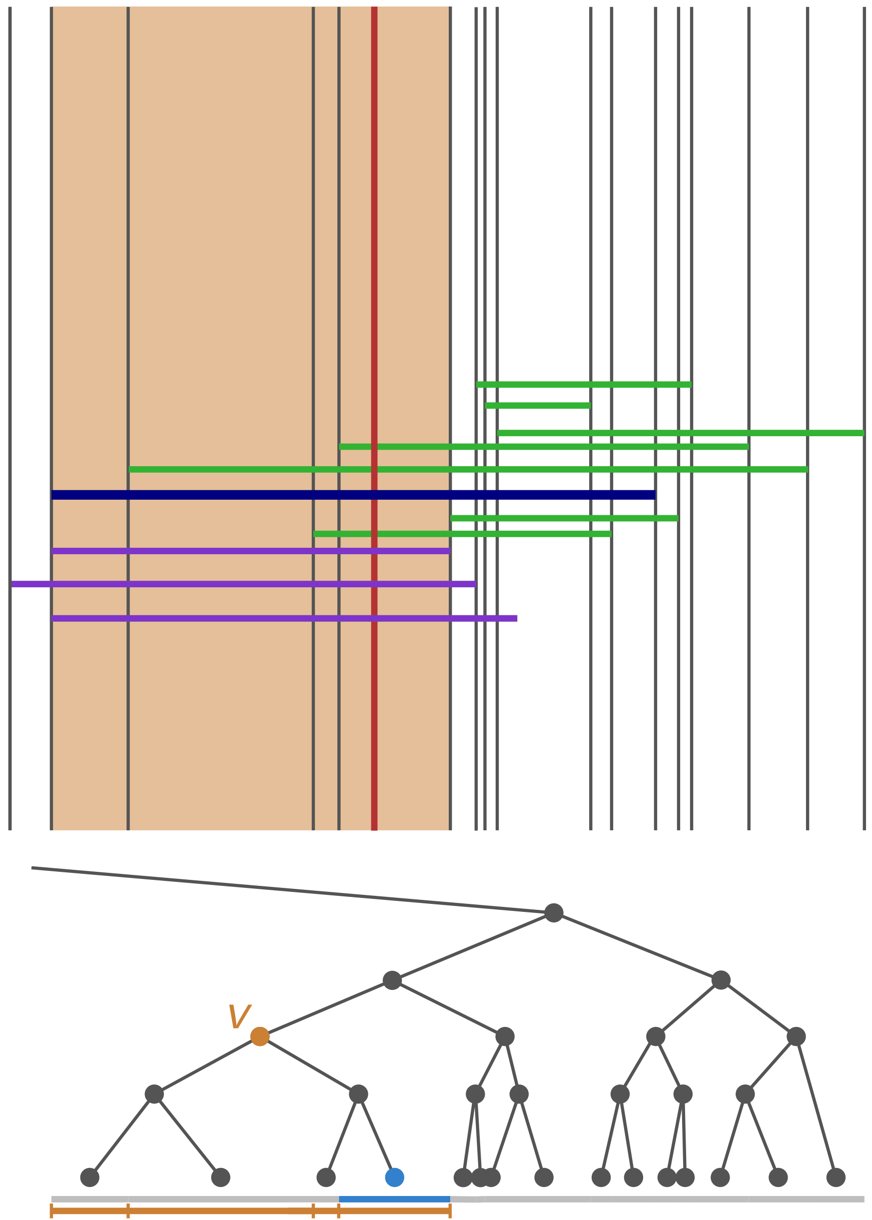

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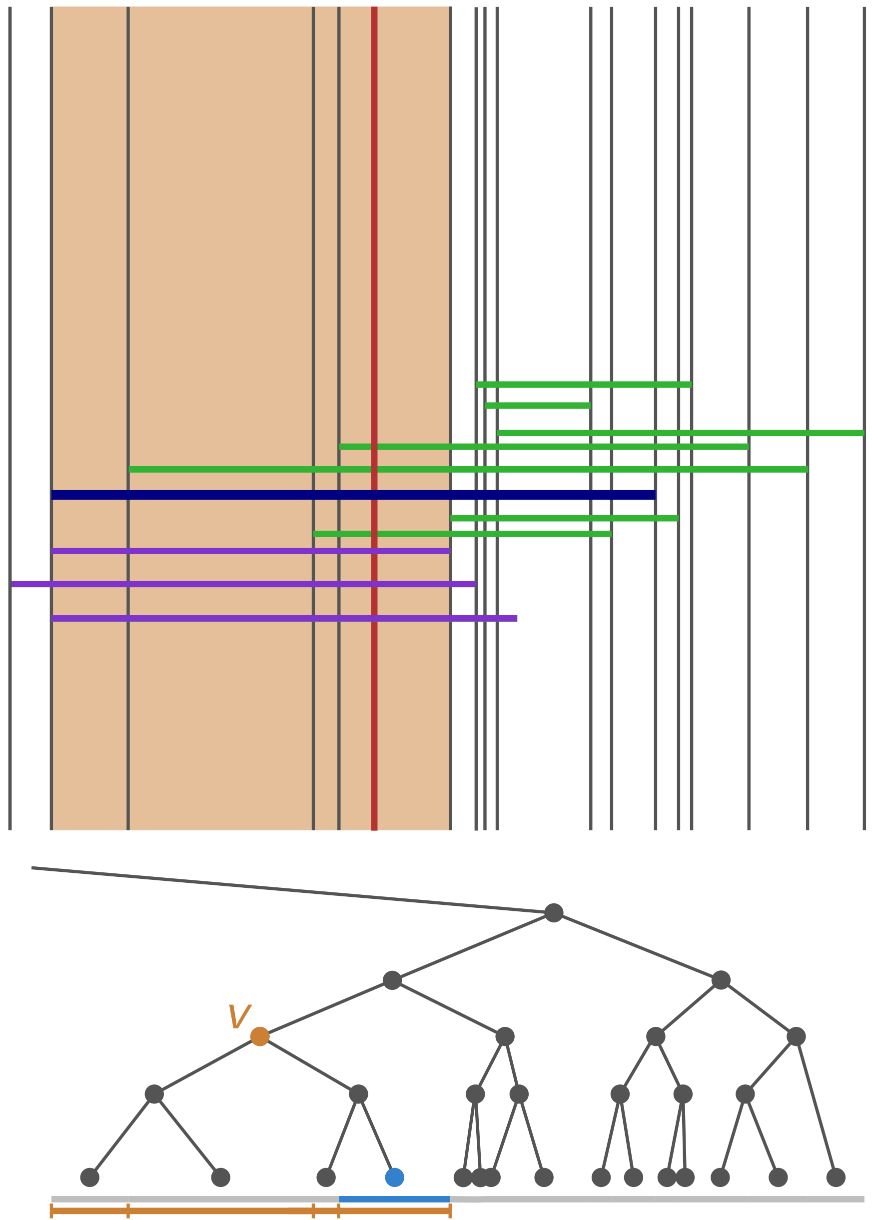

T is a segment tree

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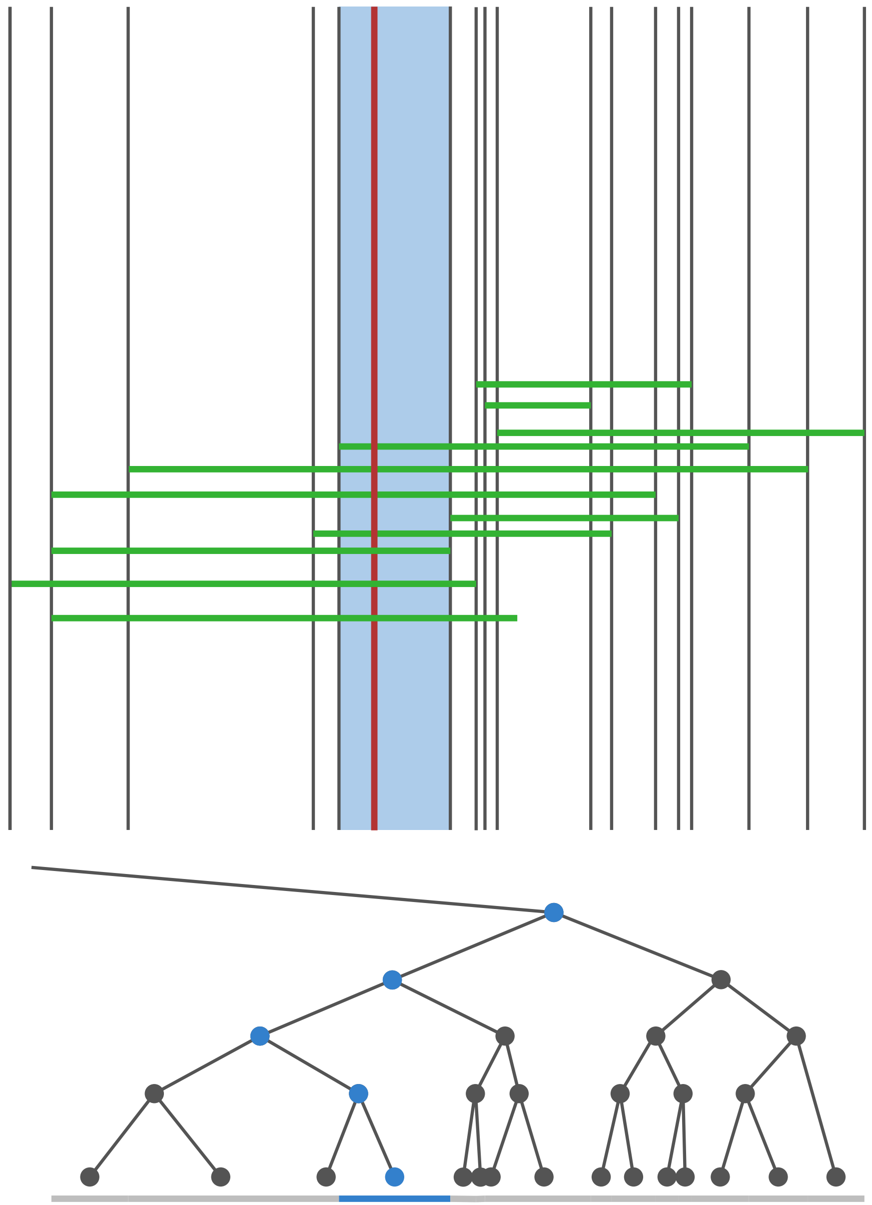
Query: find all nodes v s.t. $q \in I_v$, and for each such node report all intervals in $S(v)$.



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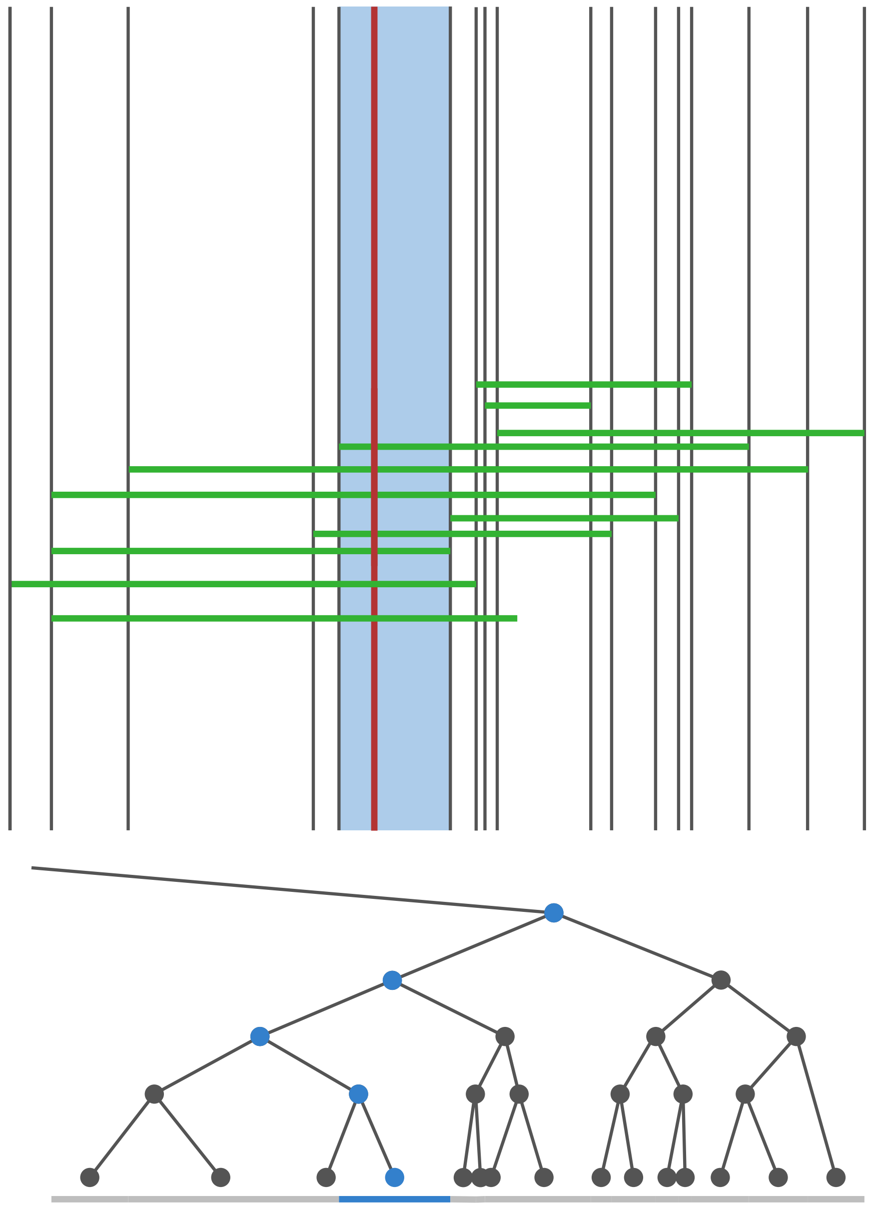
Query time: $O(\log n + k)$, where k is the output size.



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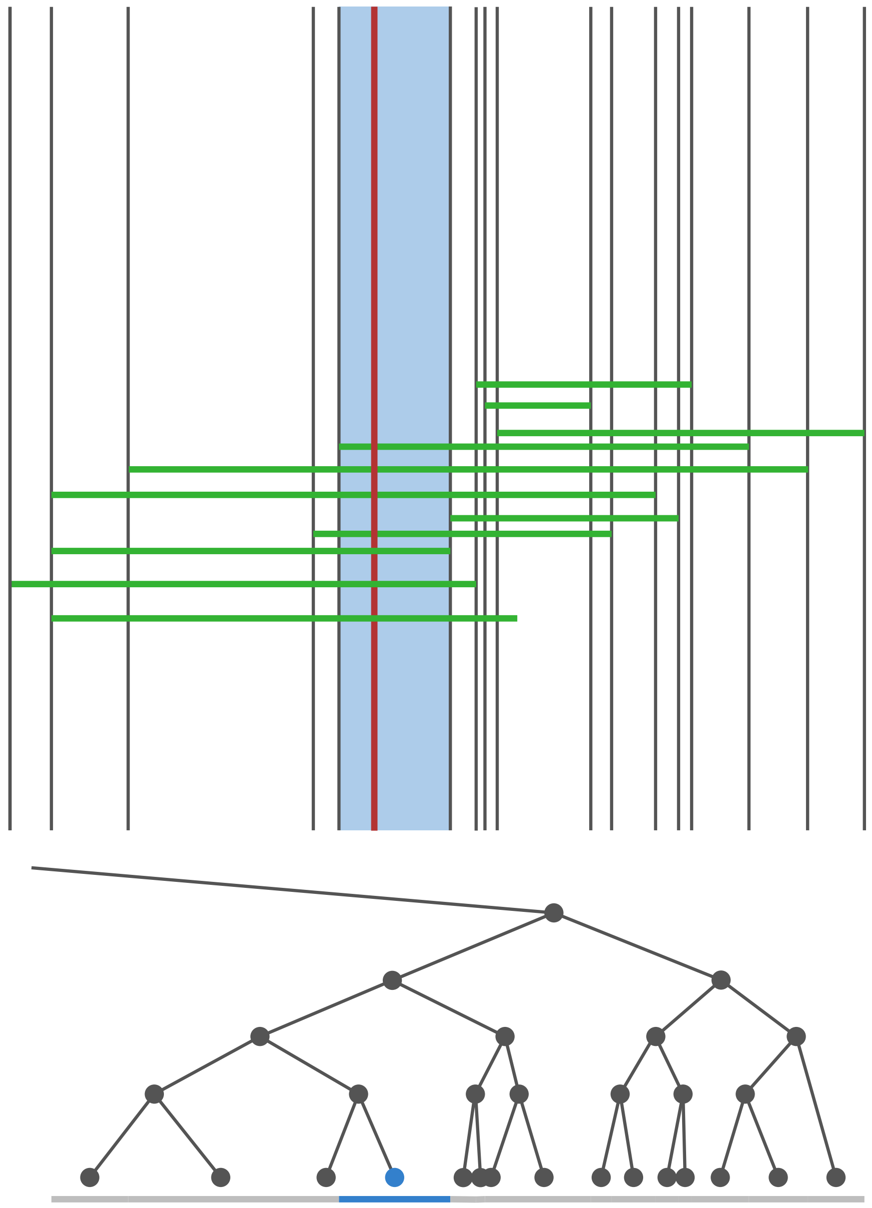
Question: How much storage do we use?



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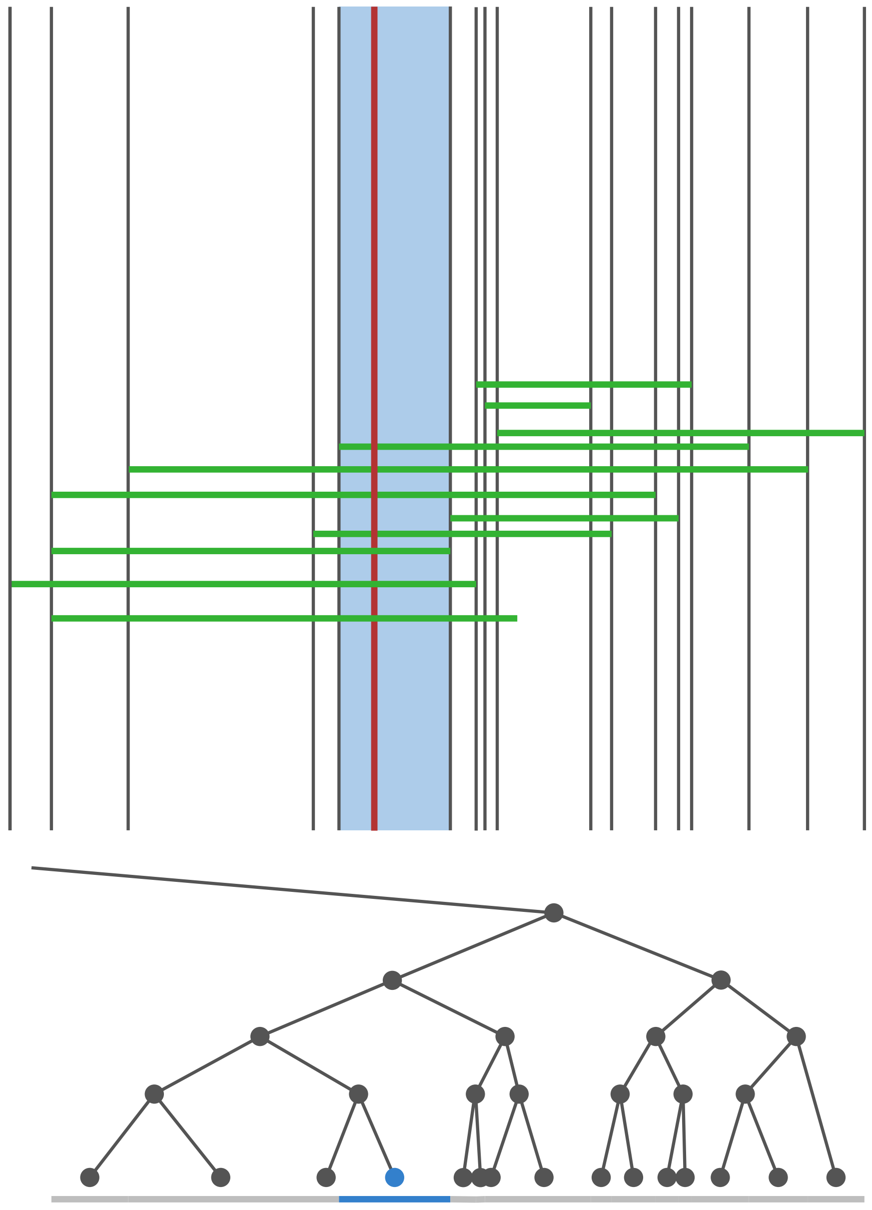
Claim: Every interval is stored $O(\log n)$ times; at most twice per level.

$\implies$ space usage is $O(n \log n)$.



Interval Stabbing Queries

Given a set S of n intervals in $\mathbb{R}^1$

Store S in a data structure s.t. given a query point q, we can find the intervals in S intersecting q efficiently.

Split into elementary intervals in which a vertical line intersects the same segments.

Question: How much storage do we use?

Claim: Every interval is stored $O(\log n)$ times; at most twice per level.

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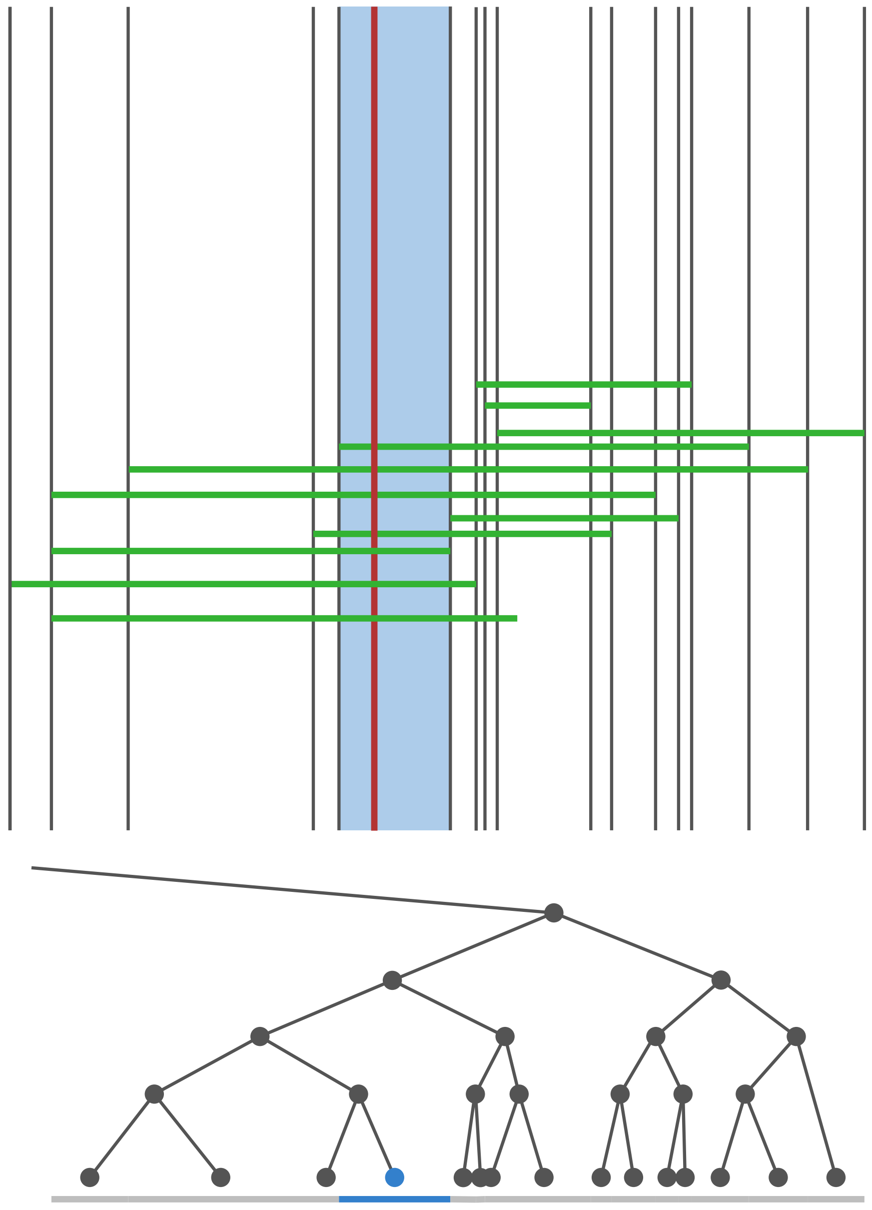

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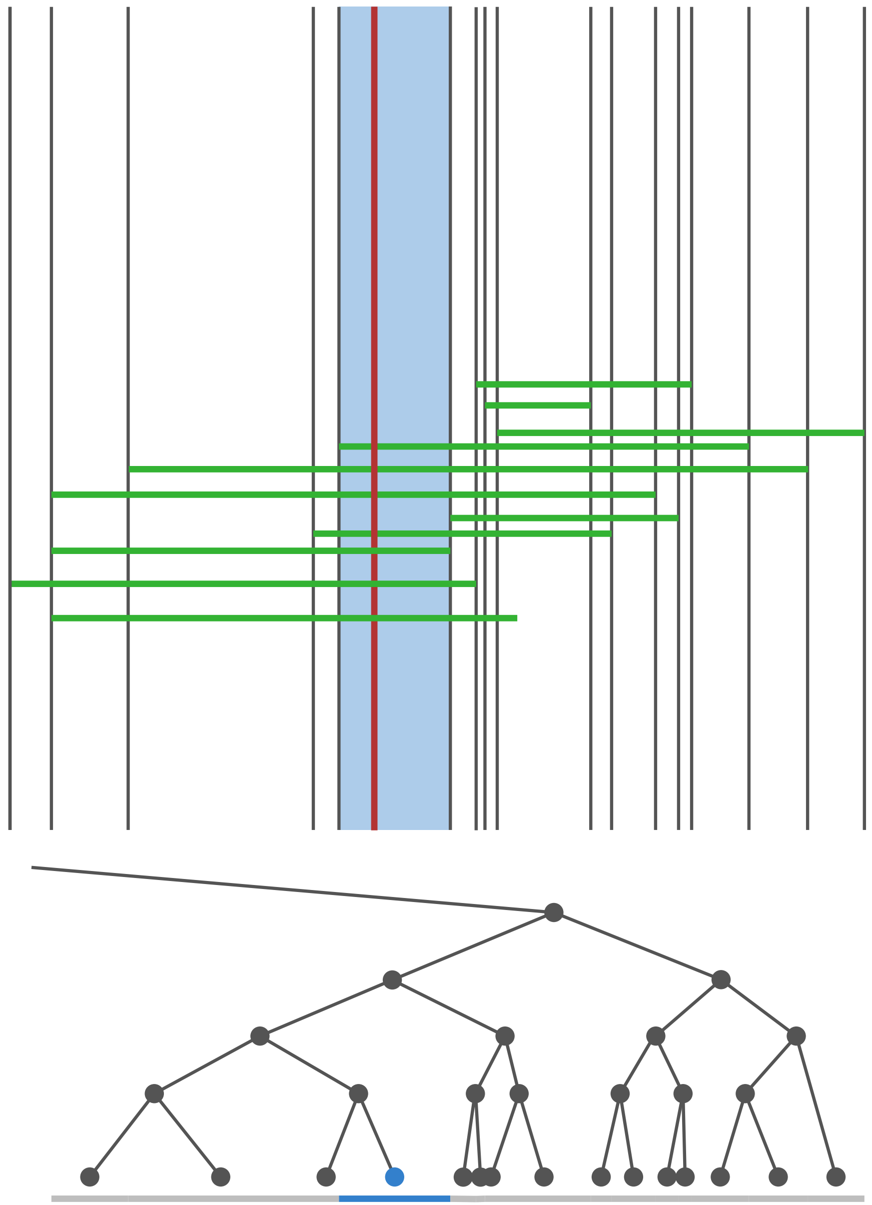

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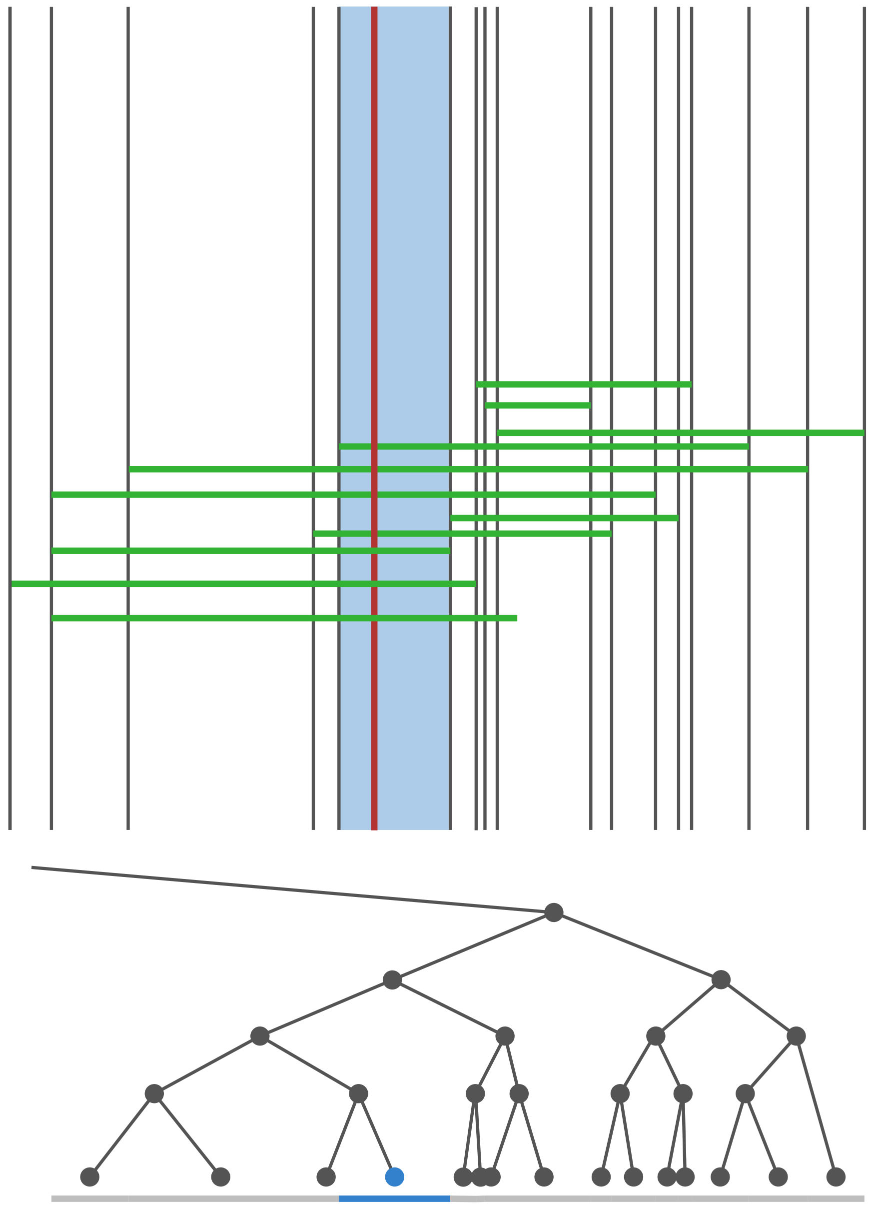
To insert s we visit at most 4 nodes per level



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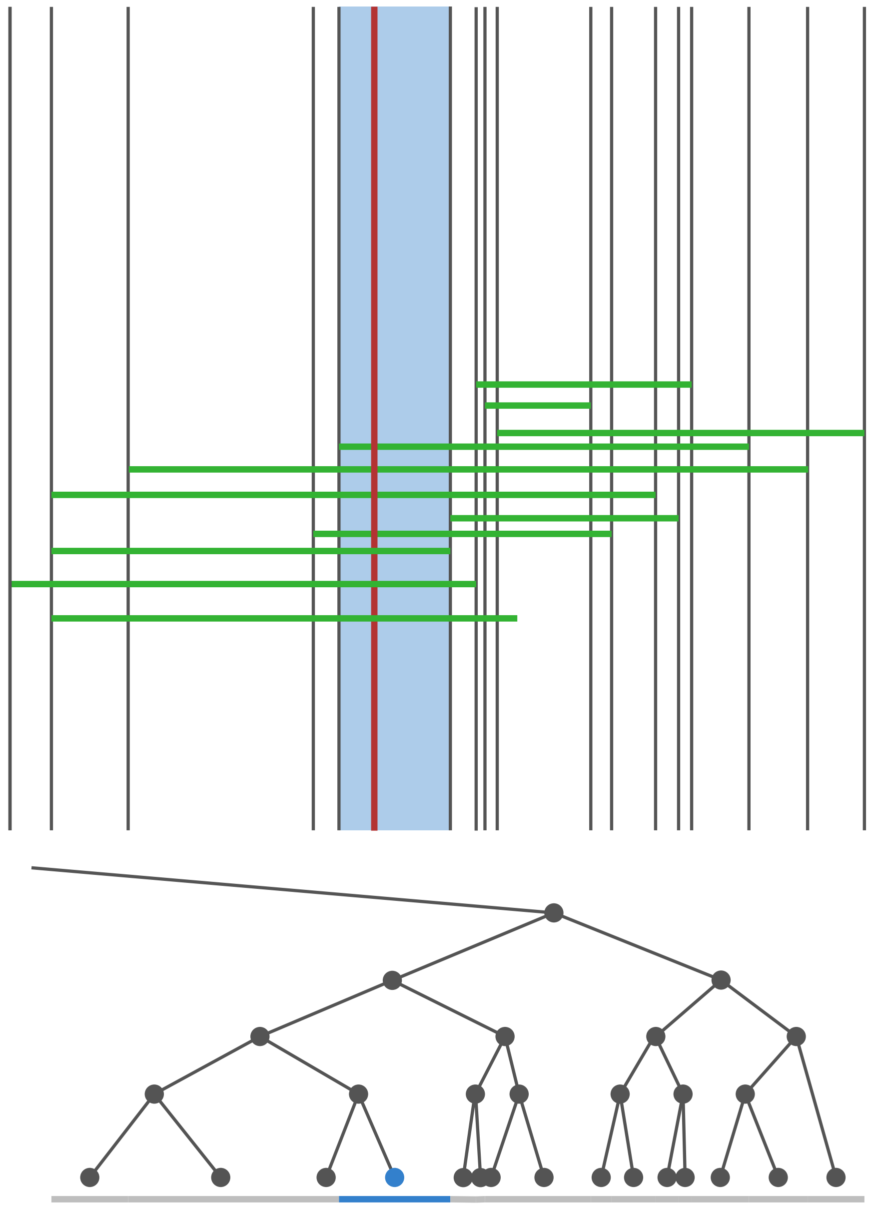
We store S in an segment tree T

Space usage: $O(n \log n)$

Query time: $O(\log n + k)$

$k = $ #intervals reported

Preprocessing time: $O(n \log n)$



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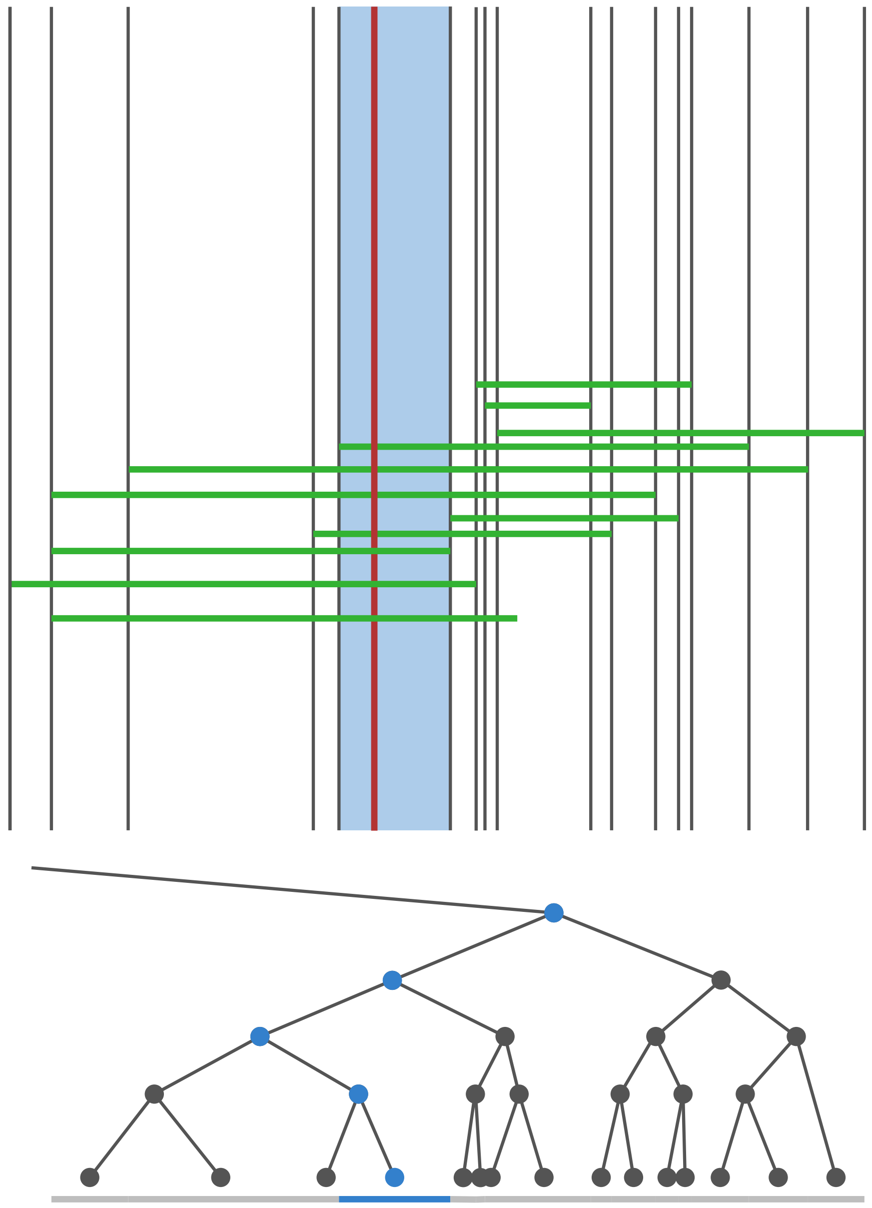

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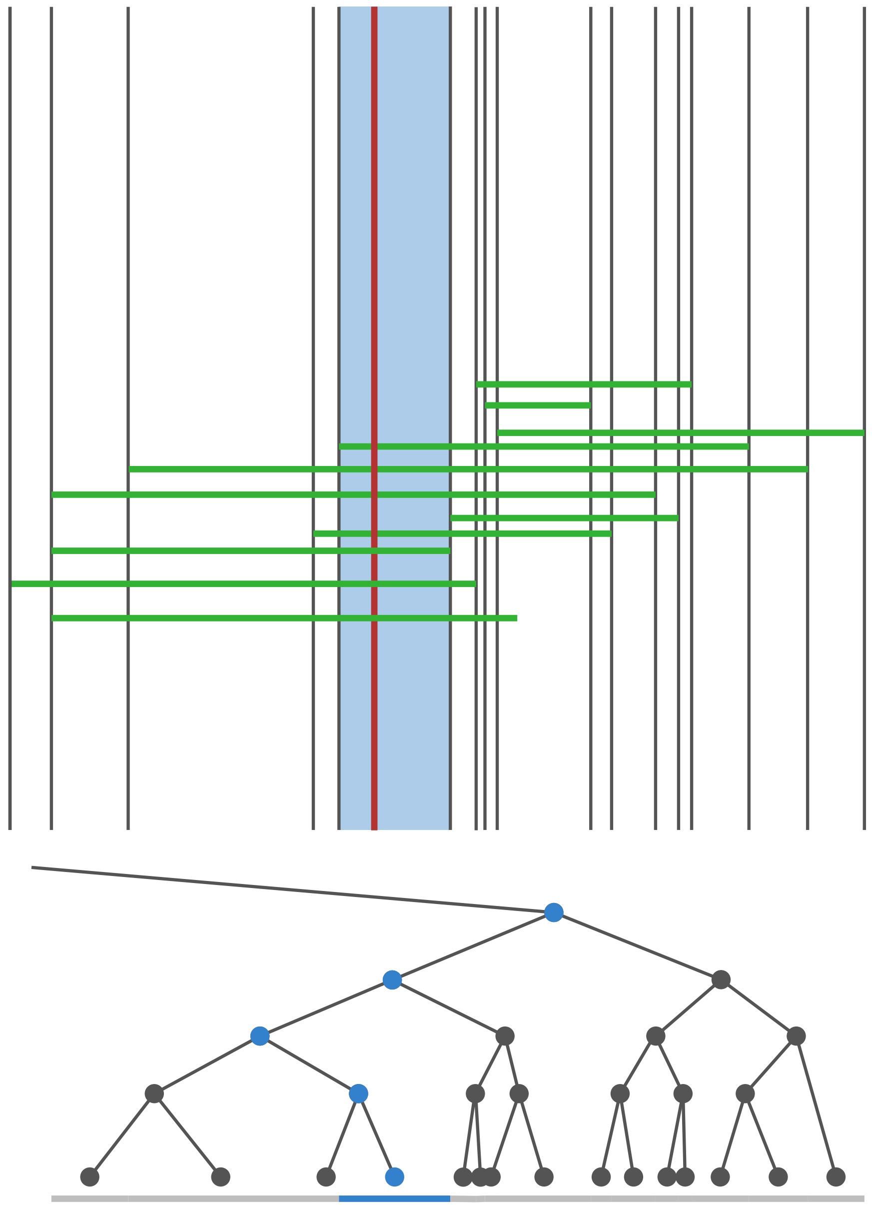

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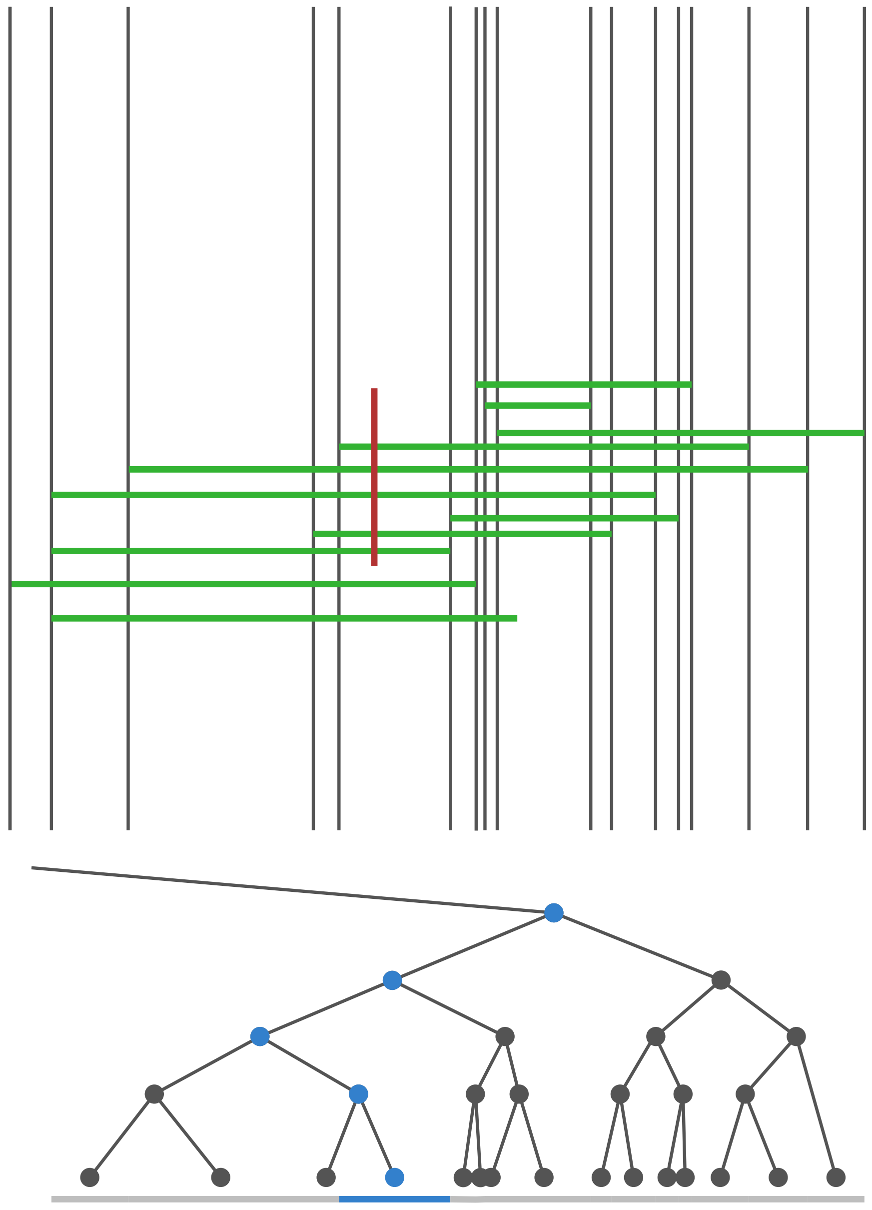
Given a set S of n horizontal line segments in the plane.

Store S in a data structure s.t. given a vertical query segment q, we can find the segments in S intersecting q efficiently.



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Store $S(v)$ in a balanced BST.

$\Longrightarrow$
We can report all segments intersected by q in $O(\log^2 n + k)$ time.

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Given a set S of n disjoint line segments in the plane.

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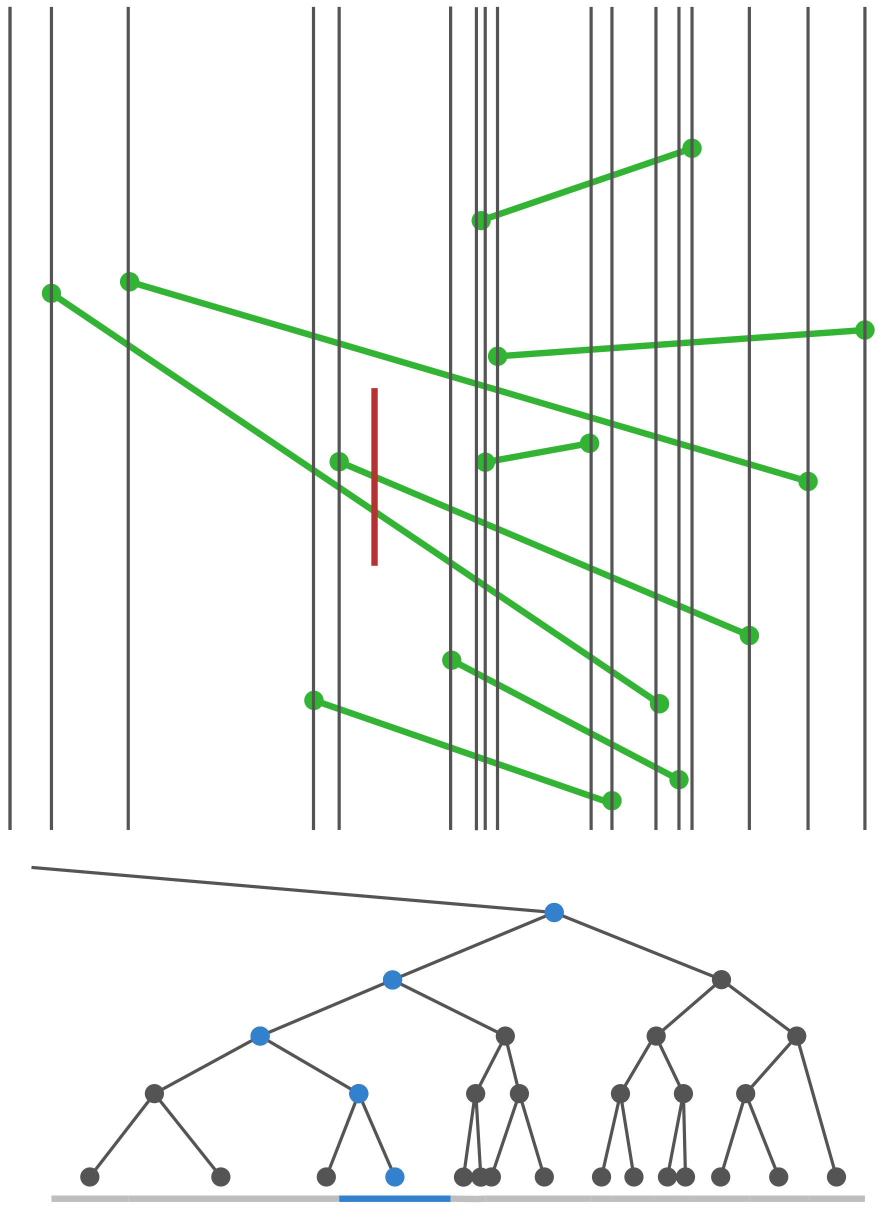

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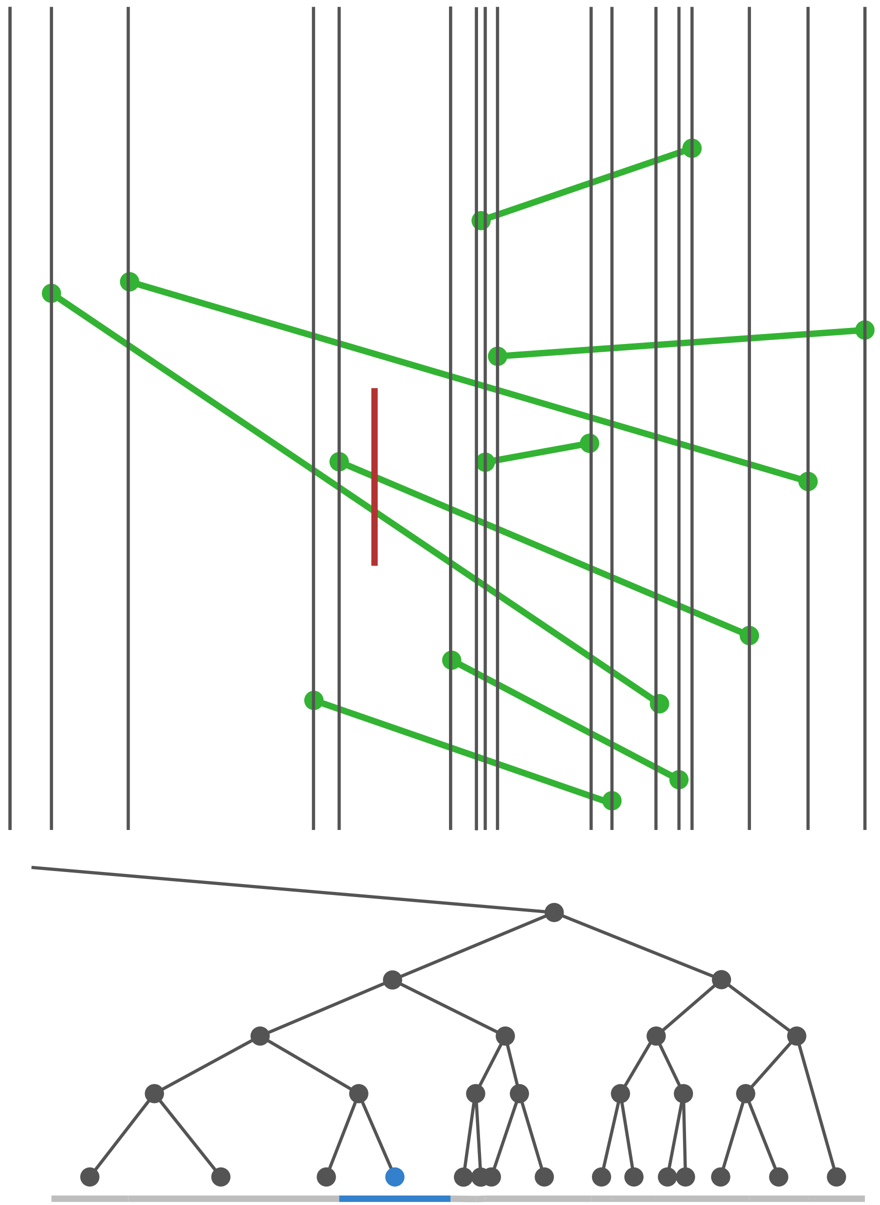


We store S in an segment tree T

Space usage: $O(n \log n)$

Query time: $O(\log^2 n + k)$

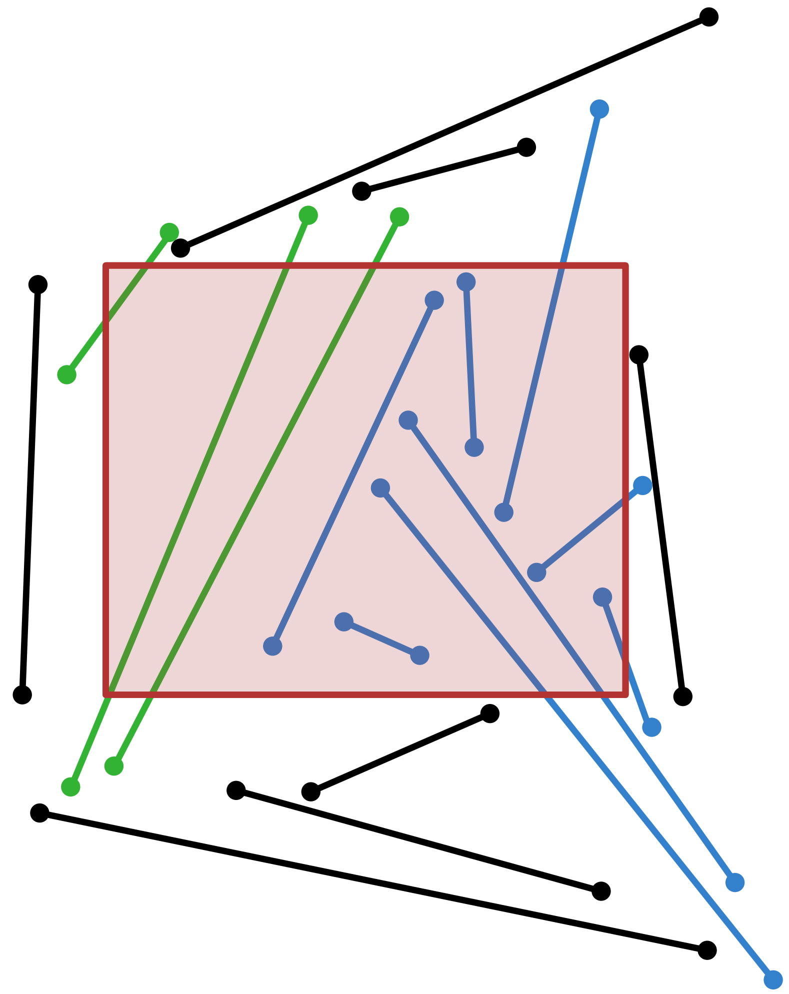
$k =$ #intervals reported

Preprocessing time: $O(n \log n)$

Windowing Queries

Given a set S of n disjoint line segments in the plane.

Store S in a data structure s.t. given a query rectangle R, we can find the segments in S intersecting R efficiently.



The segments that intersect R

1) have an endpoint in R, or

   find them using a range query
   with R on the set of end points
   $\implies O(\log^2 n + k)$ query, $O(n \log n)$ space.

2) intersect the boundary of R.

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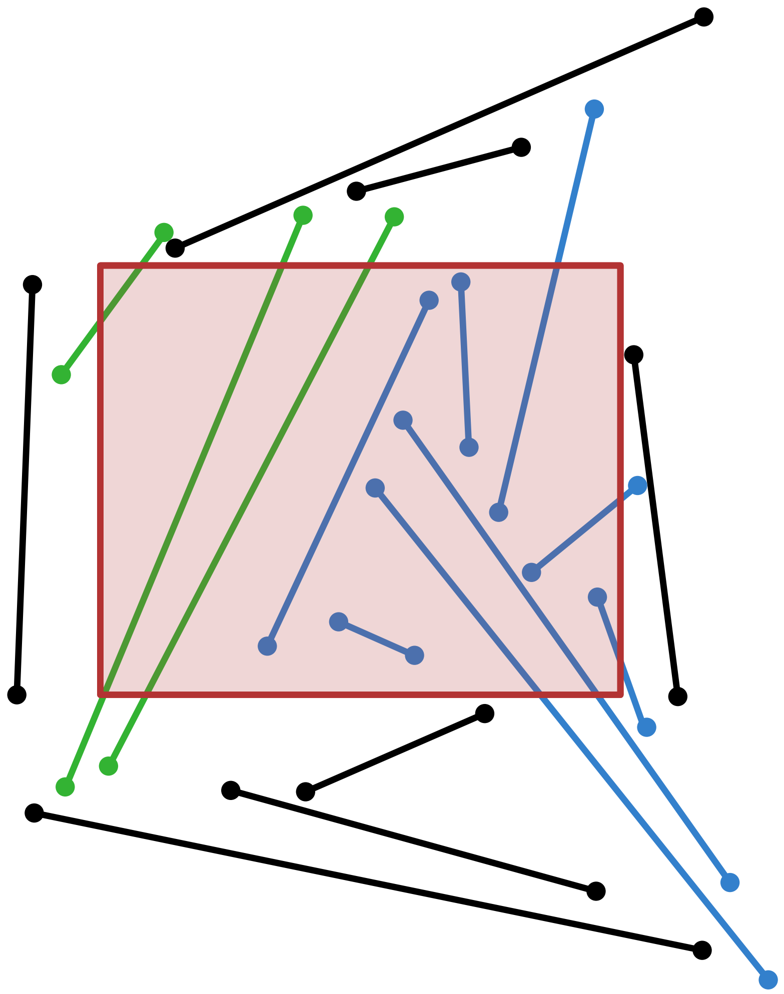

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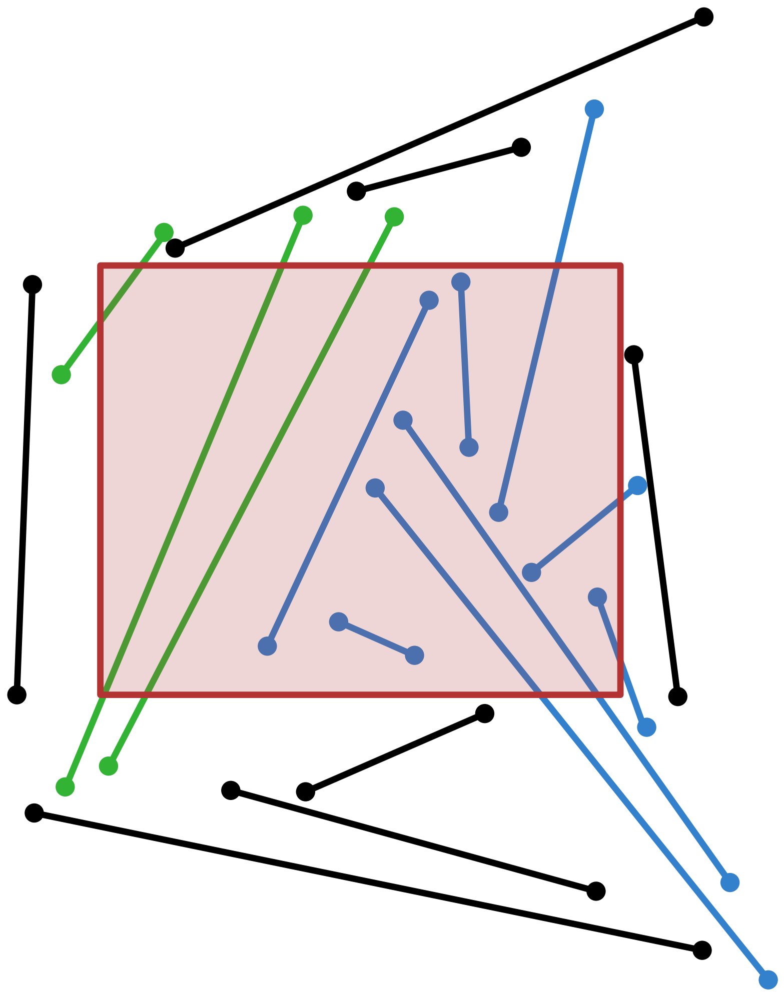

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