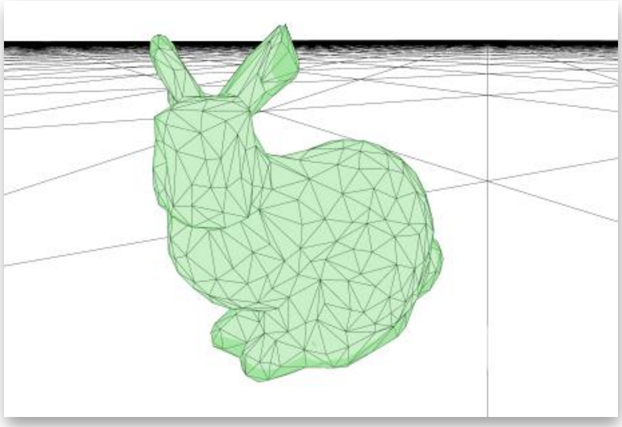
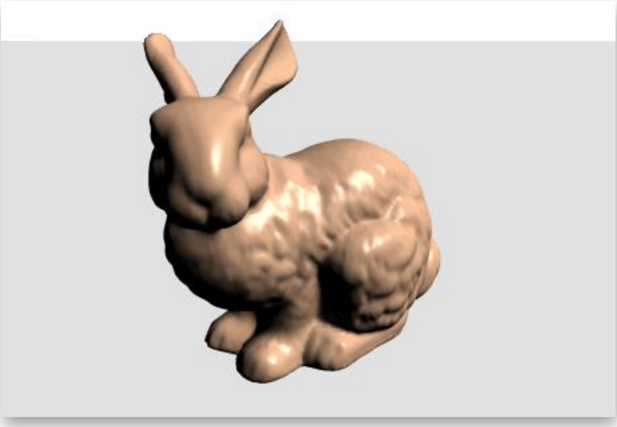
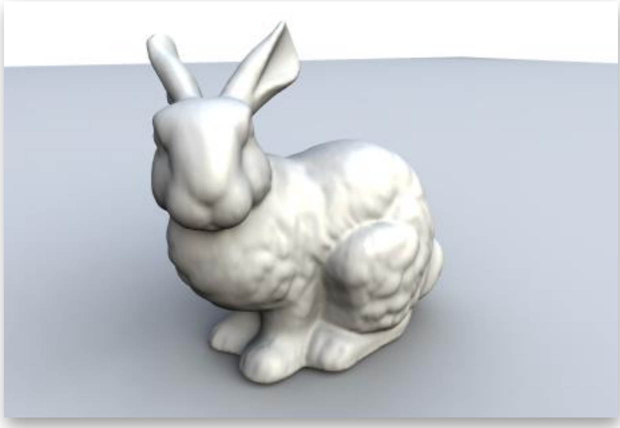
Graphics 2014







Advanced Rasterization

[Faculty of Science]
Information and Computing Sciences

Universiteit Utrecht

Announcements

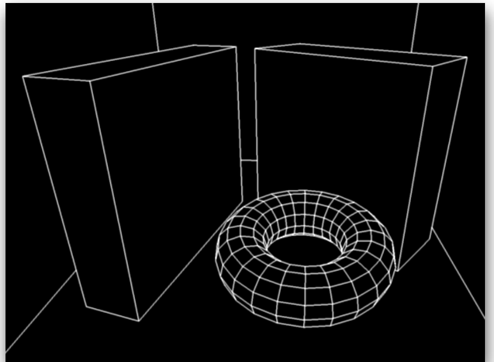
Extra tutorials next week

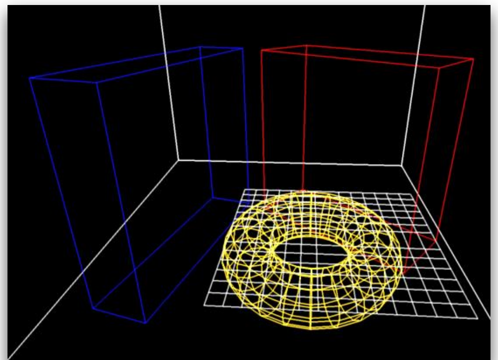
- **The usual times:** Thu 15:15h – 17:00h
- **Rooms:** BBL 023, BBL 079, BBL 083 (not BBL 165)

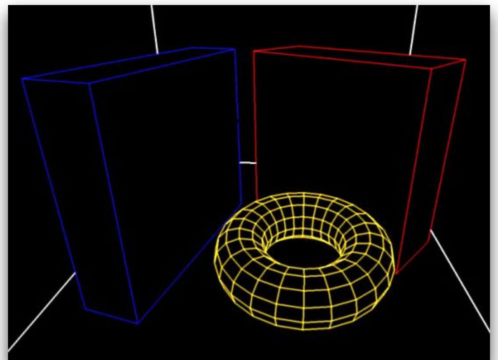
Questions + Answers

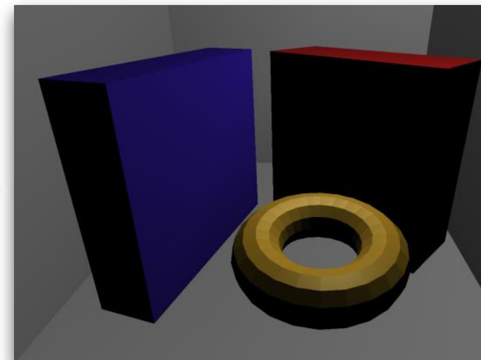
- Please mail me your questions
  - Will be passed on to tutors
  - Best: mail before end of this week
- Preparation for final exam

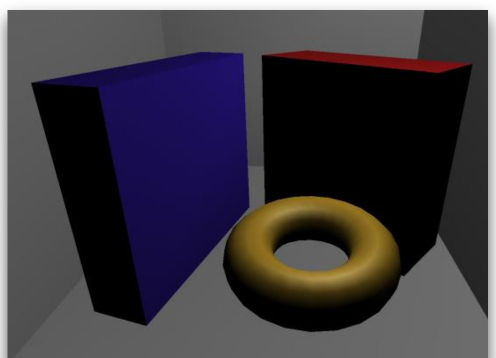
3D Rendering Steps

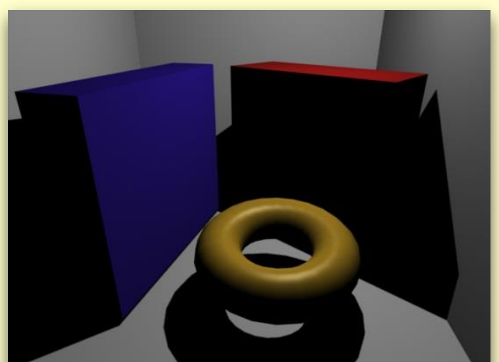

Geometric Model

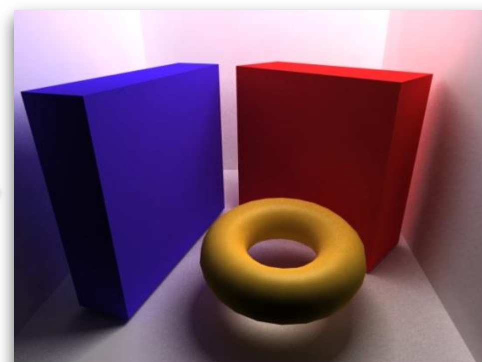

Perspective


Visibility


Local Illumination


Smooth Shading


Advanced Rasterization


Global Illumination

Topics

Supplementary Details

- Projective geometry
- Rasterization and clipping
- Transformations & Normals

Texture Mapping

- Basic idea
- Perspective correction

Topics

Advanced Texture Mapping

- Aliasing, Filtering & Mipmapping (short)
- 2D and 3D Textures
- Shadow maps
- Displacement maps
- Bump mapping / normal maps
- Environment Maps
- Image-based Lighting

Topics

Modern Rasterization Pipeline

- Vertex and Pixel Shaders
  - Extensions
- Render targets
  - Color buffers
  - Float buffers
  - Stencil buffer
- Textures
  - 2D & 3D textures
  - Cube maps

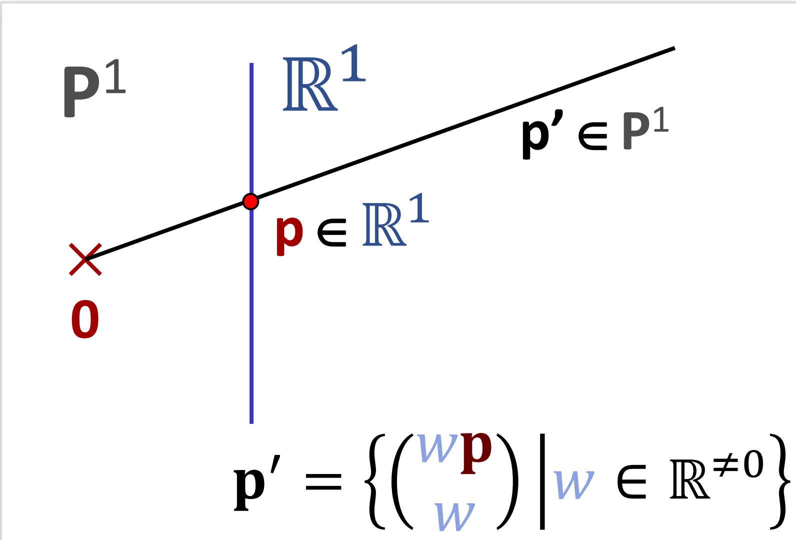
Addendum

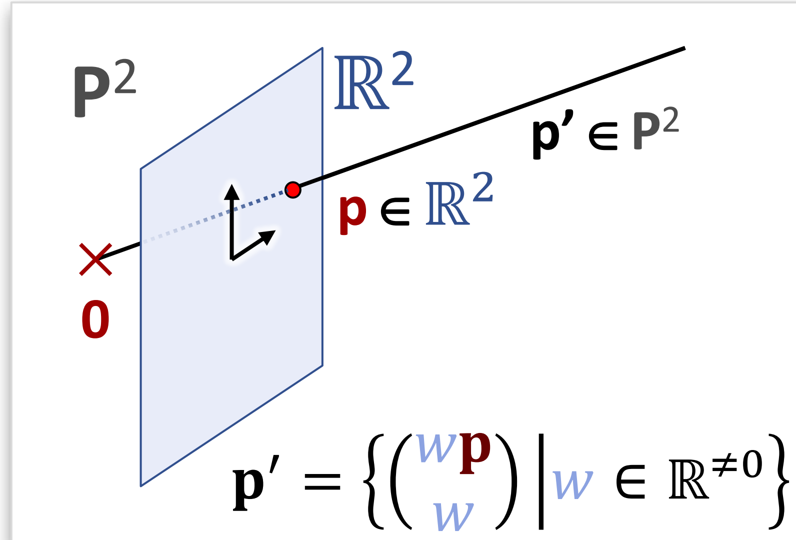
Projective Geometry

ADV

advanced topics
main ideas

Constructing Projective Spaces

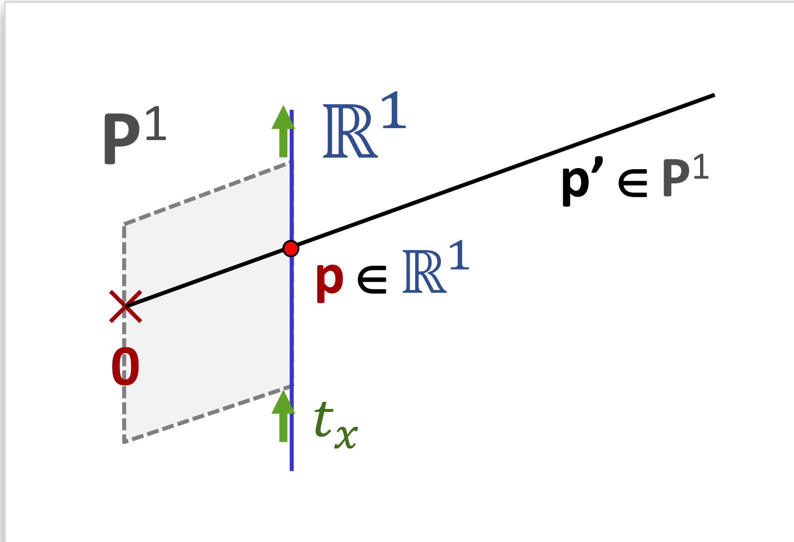




Projective Space P^d:

- Euclidian ("affine") space $\mathbb{R}^d$ embedded in $\mathbb{R}^{d+1}$
- At $w = 1$
- Identify all points on lines through the origin
  - *Representing* the same Euclidian point

Constructing Projective Spaces



Translations:

- Sheering of the projective space

$$\begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

- Translation of the embedded affine space

Normalization

Conversion between

- Cartesian coordinates (Euclidian space)
- Homogeneous coordinates (projective space)

$$\mathbf{x} \rightarrow \begin{pmatrix} w\mathbf{x} \\ w \end{pmatrix}$$

Cartesian coordinates
(Euclidian space)

homogenous coordinates
(projective space)

$$\frac{1}{w}\mathbf{x} \leftarrow \begin{pmatrix} \mathbf{x} \\ w \end{pmatrix}$$

normalization*)

*) overloaded name
do not confuse with $\mathbf{x}/\|\mathbf{x}\|$

Mathematical Language

Form equivalence classes

- $(\mathbf{x} \equiv \mathbf{y}) \Leftrightarrow \exists \lambda \in \mathbb{R}: (\mathbf{x} = \lambda \mathbf{y})$
- Think of overloading `operator=()`

Even more formally (math students)

- Consider group of uniform scalings
- $G = \left\{ \begin{pmatrix} \lambda & & 0 \\ & \ddots & \\ 0 & & \lambda \end{pmatrix} \middle| \lambda \neq 0 \right\}$
- Symmetry group of the representation:
$$\mathrm{P}^d = \mathbb{R}^d \bmod G$$
  - Ignore "irrelevant information"

Properties

Projective Maps

- Linear maps in the higher dimensional space
- Scale at any time:

$$\mathbf{y} = \mathbf{M} \cdot \mathbf{x} \equiv \frac{\mathbf{M} \cdot \mathbf{x}}{\mathbf{x}.w} \equiv \frac{\mathbf{M} \cdot \mathbf{x}}{\mathbf{y}.w} \quad (\text{for } w \neq 0)$$

- Why? Scaling yields the same point!

Properties

Important:

- We have: $\mathbf{x} \equiv \lambda \mathbf{x}$ (for $\lambda \neq 0$)
- But in general: $\mathbf{x} + \mathbf{y} \not\equiv \mathbf{x} + \lambda \mathbf{y}$
  - For correct result:
    Normalize first (same w)

Vectors & Points

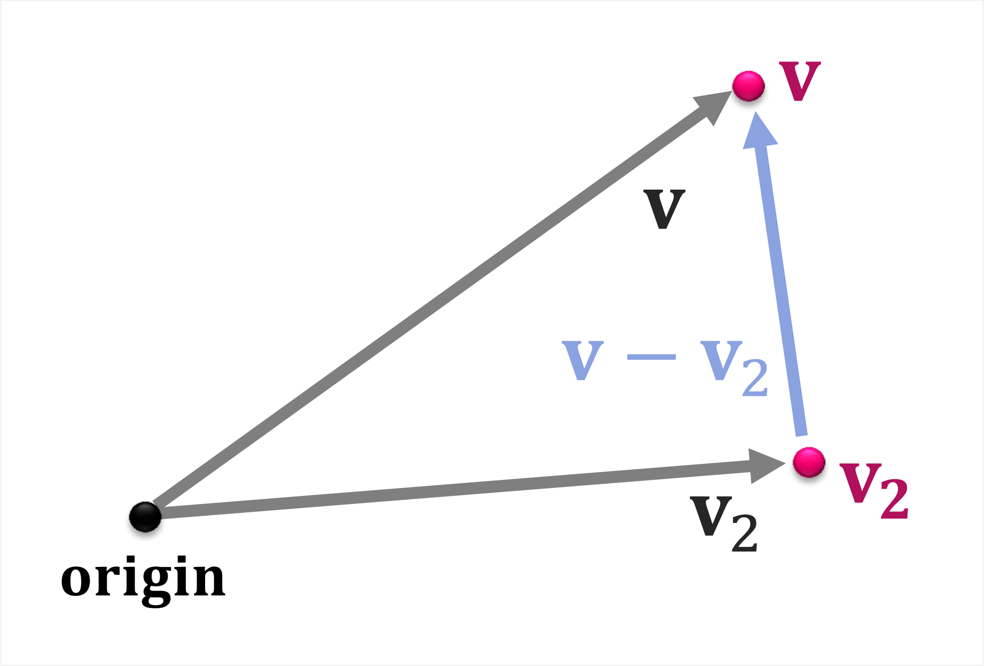
Interpretation

- Points: $\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}, w \neq 0$
- Vectors: $\begin{pmatrix} x \\ y \\ z \\ 0 \end{pmatrix}$ – “pure directions”

Vectors & Points

Rules

- Substracting points yields vectors
  - Normalize first!
- Vectors can be added to
  - Other vectors
  - Points (normalize first!)



Rasterization and Clipping

CORE

core topics

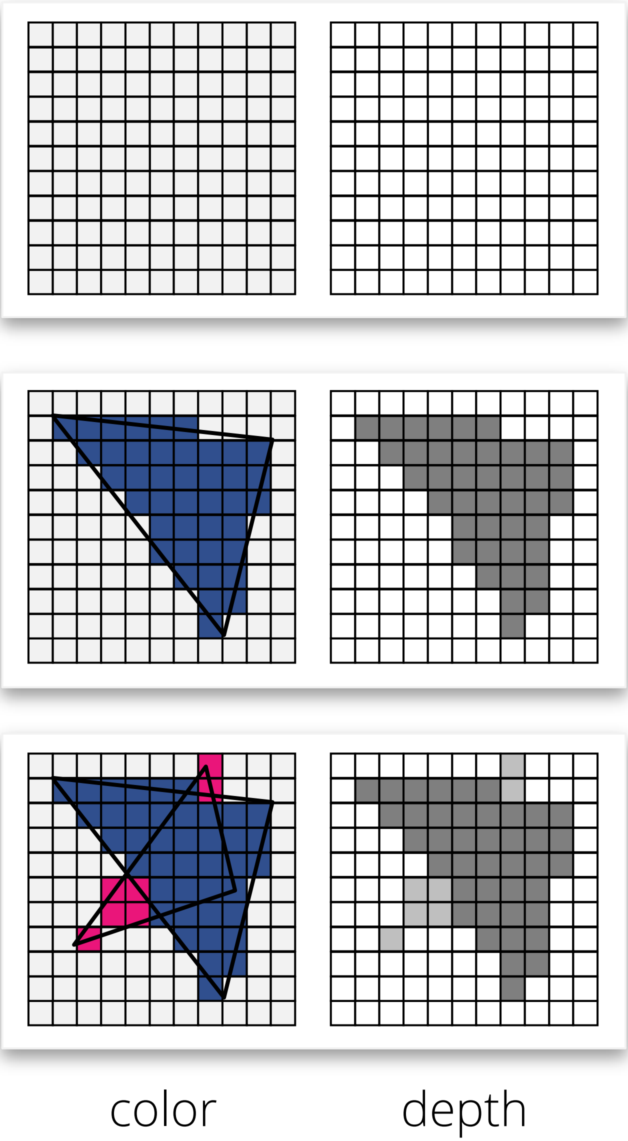
important

Rasterization

How to rasterize Primitives?

Two problems

- Rasterization
- Clipping

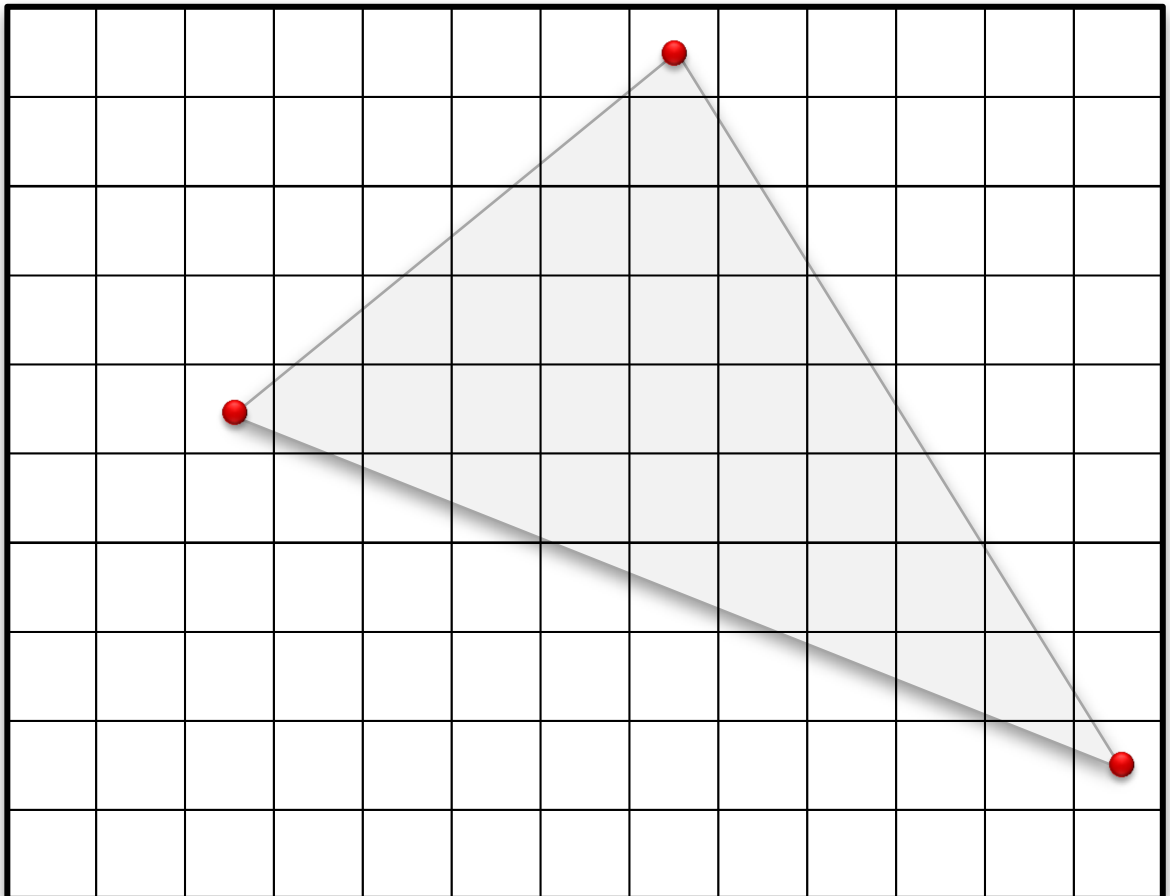


Rasterization

Assumption

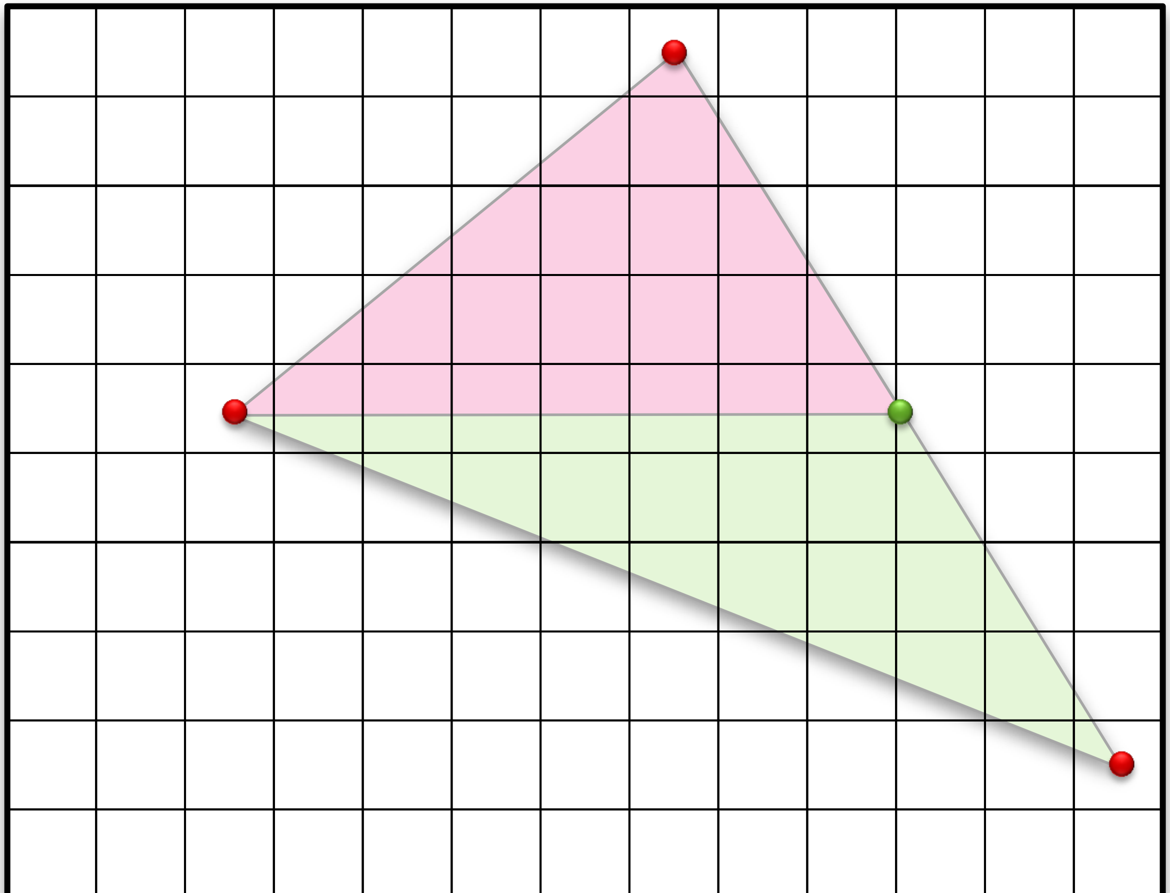
- Triangles only
- Triangle not outside screen
- No clipping required

Triangle Rasterization



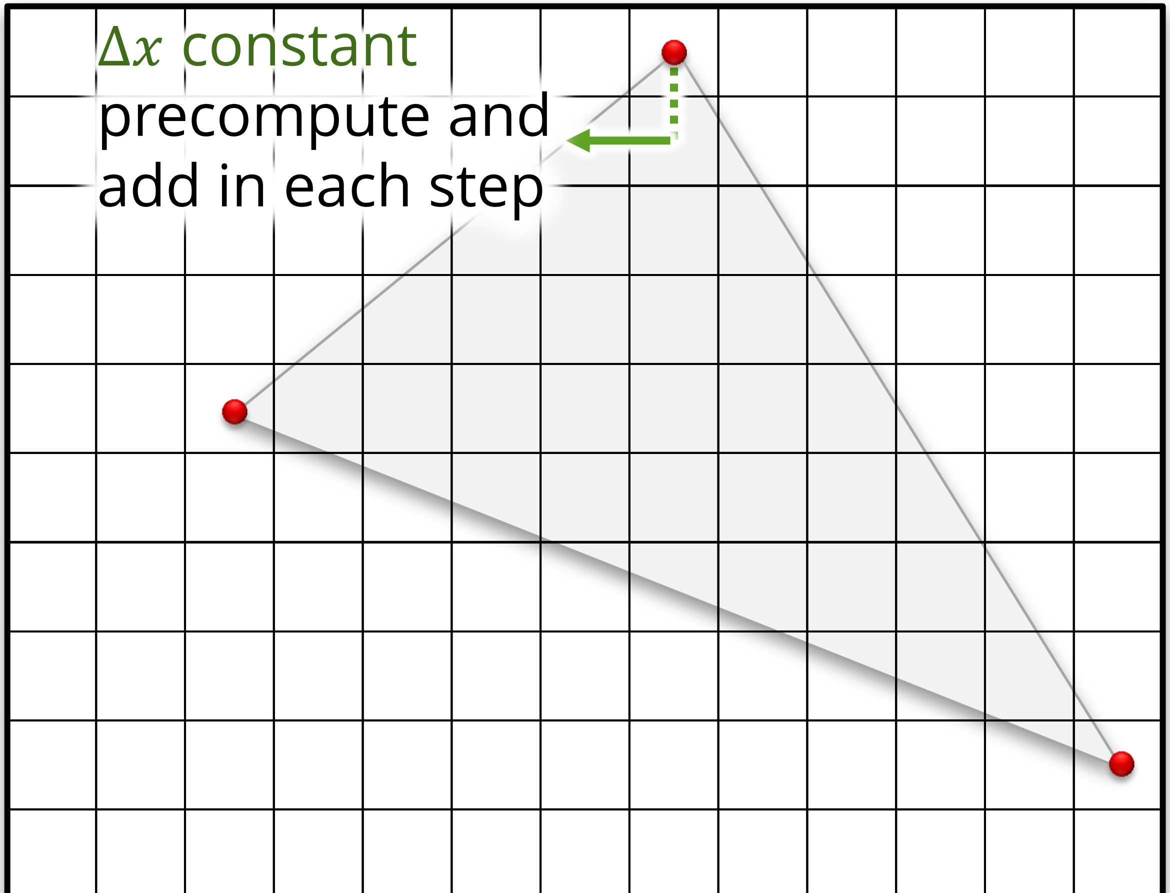
Several Algorithms…

Triangle Rasterization



Example: two slabs (tutorials)

Triangle Rasterization



Incremental rasterization

Incremental Rasterization

Precompute steps in x, y-direction

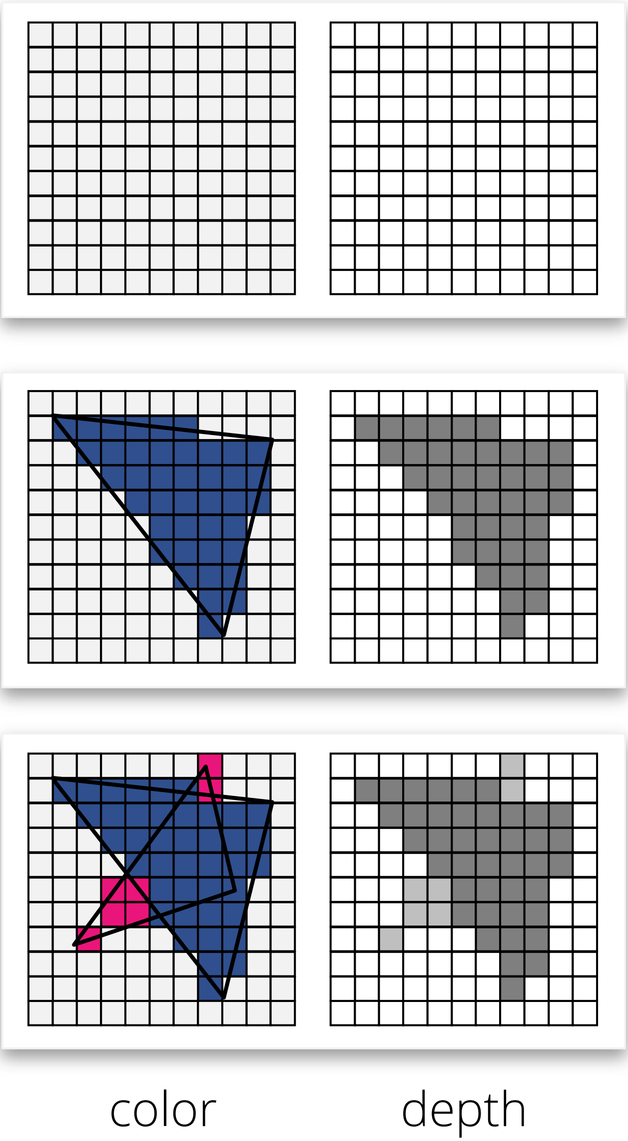
- For boundary lines
- For linear interpolation within triangle
  - Colors
  - Texture coordinates (more later)
- Inner loop
  - Only one addition (“DDA” algorithm)
  - Floating point value
  - Strategies
    - Fixed-point arithmetics
    - Bresenham / midpoint algorithm
      (requires if; problematic on modern CPUs)

Rasterization

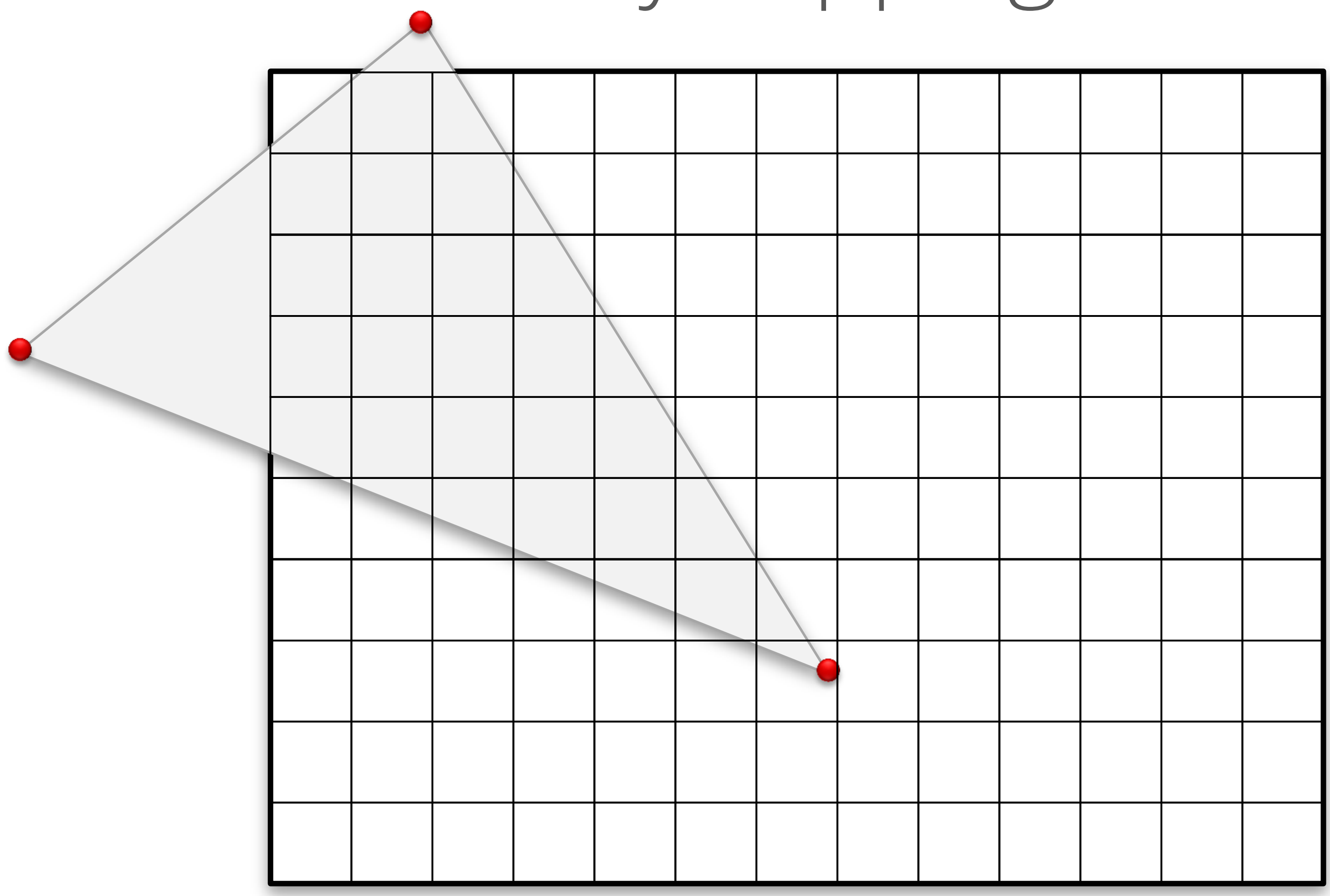
How to rasterize Primitives?

Two problems

- Rasterization
- Clipping



Why Clipping?



Crashes – write to off-screen memory!

Clipping Strategies

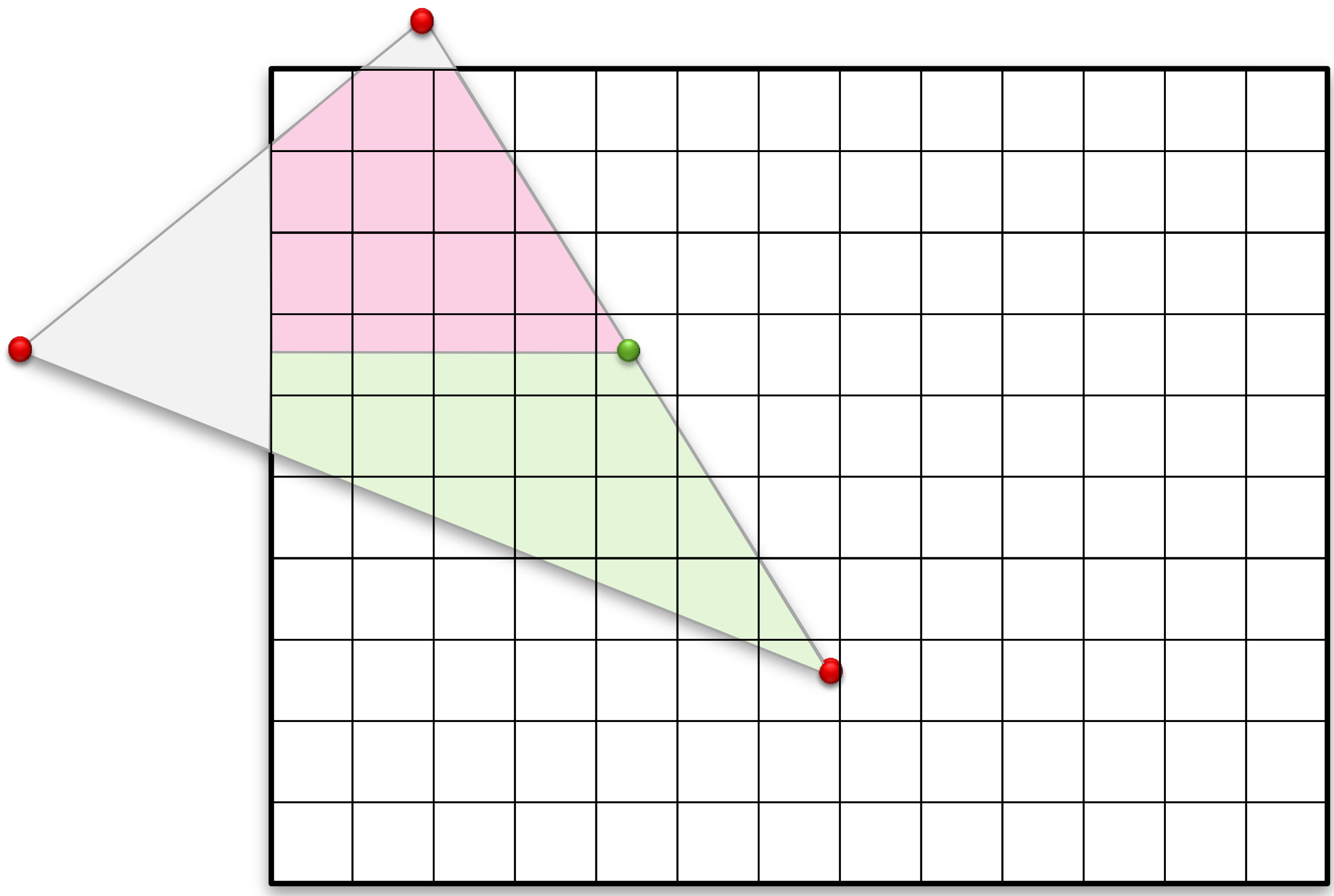
Pixel Rejection

- “if (x,y ∉ screen) continue;”
  - Can be arbitrarily slow (large triangles)
  - Nope. Not a good idea.

Screen space clipping

- Modify rasterizer to jump to visible pixels
  - See tutorial 5
- Efficient
- Still problems with when crossing camera plane ($w = 0$) ⇒ a semi-good idea

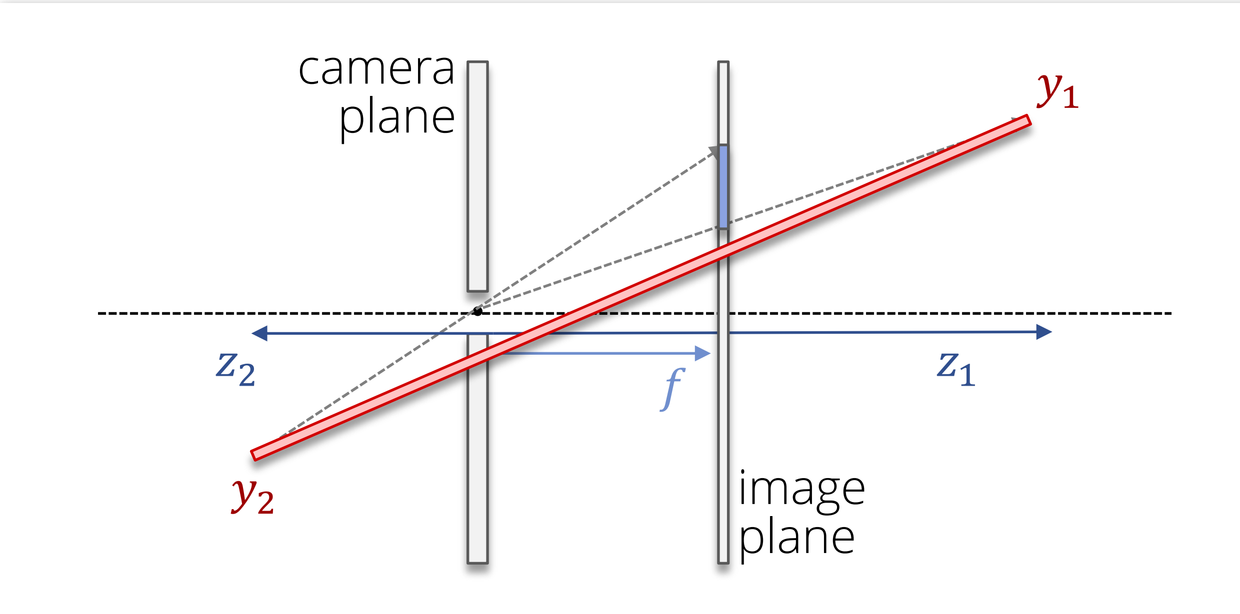
Smart Slab Renderer



Does not crash, optimal complexity

- $O(k)$ for k output fragments

Problem

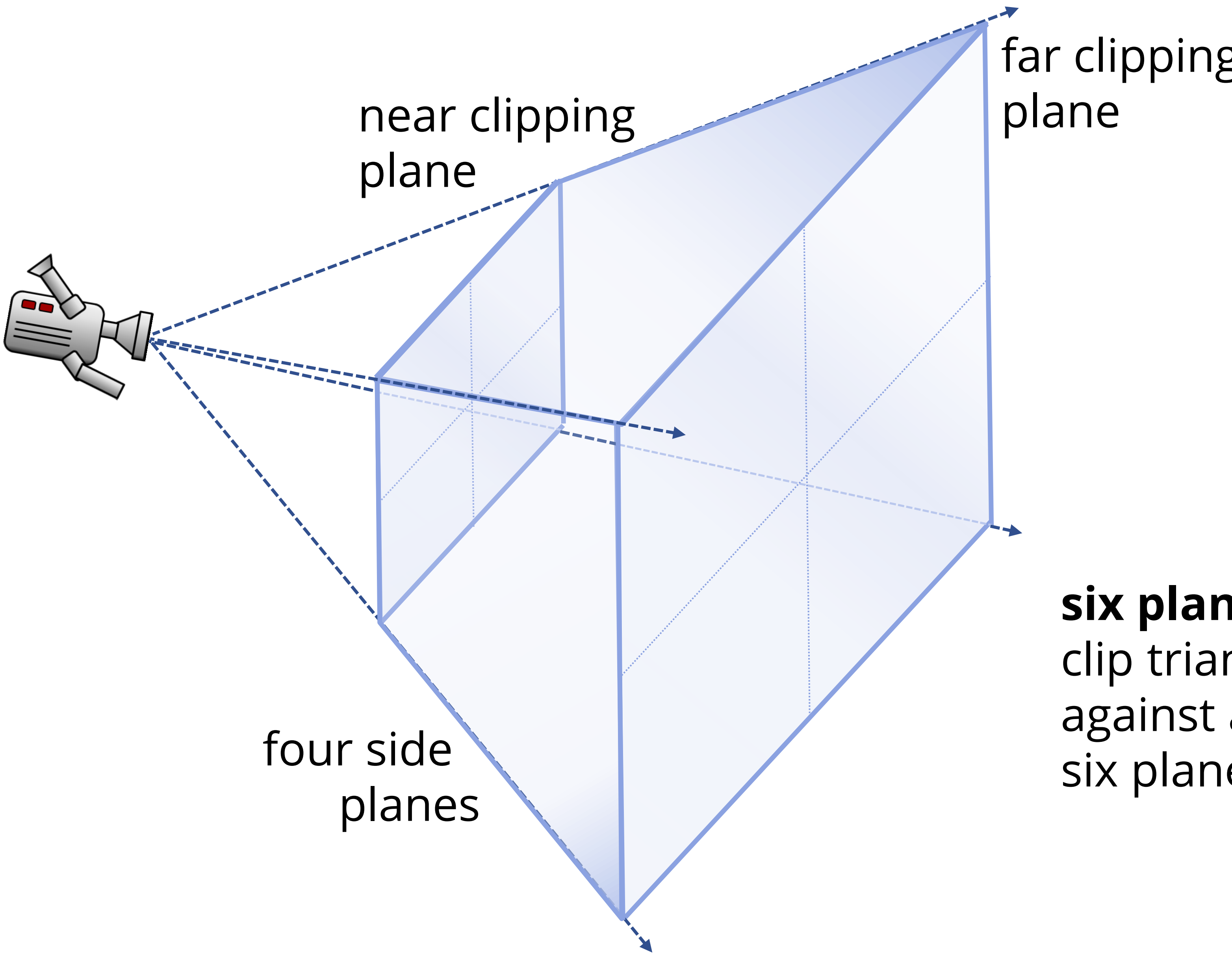


$$y' = f\frac{y}{z}$$

Problem:

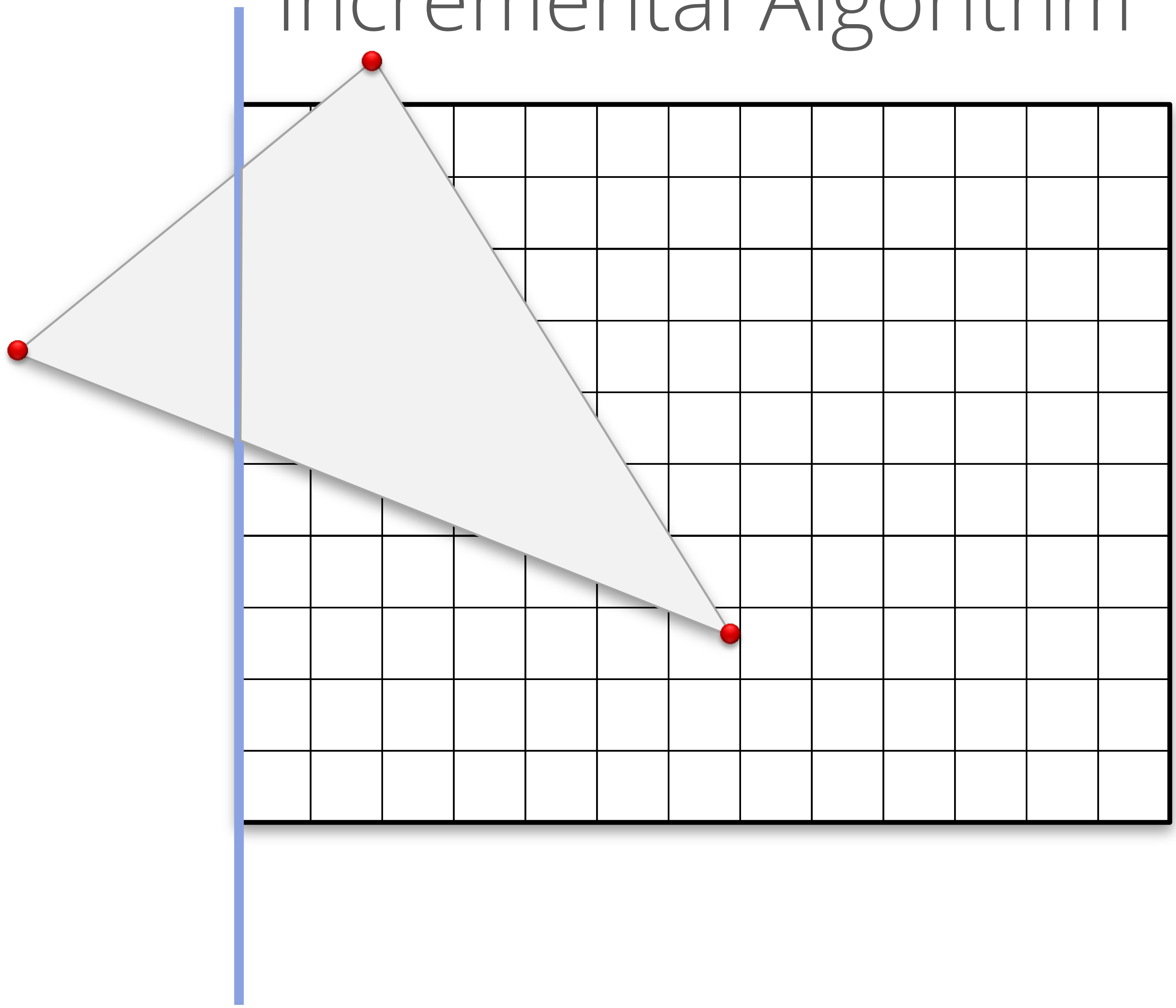
- Triangles crossing camera plane!
  - Wrong results
- Need object space clipping

View Frustum Clipping

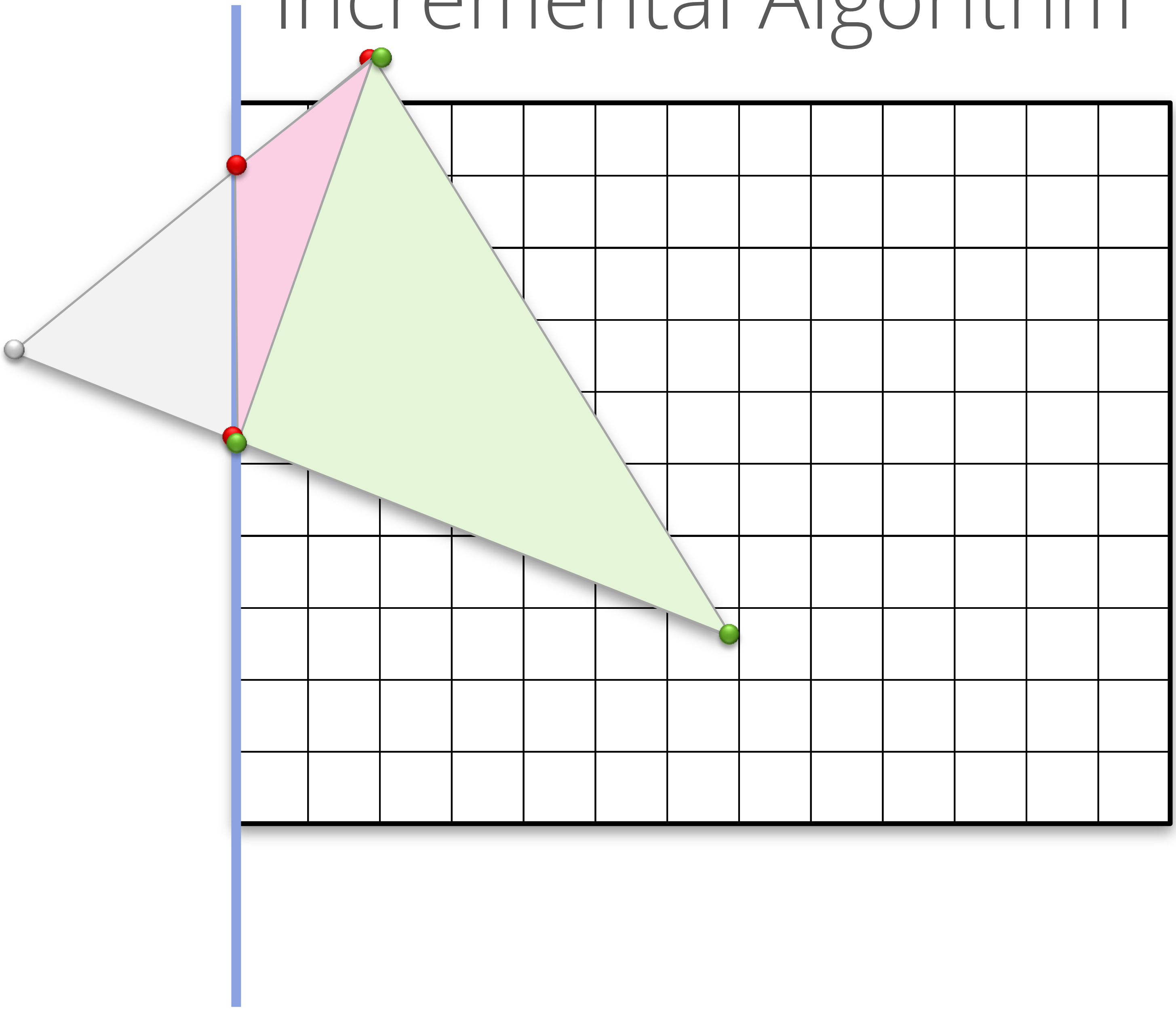


six planes
clip triangles
against all
six planes

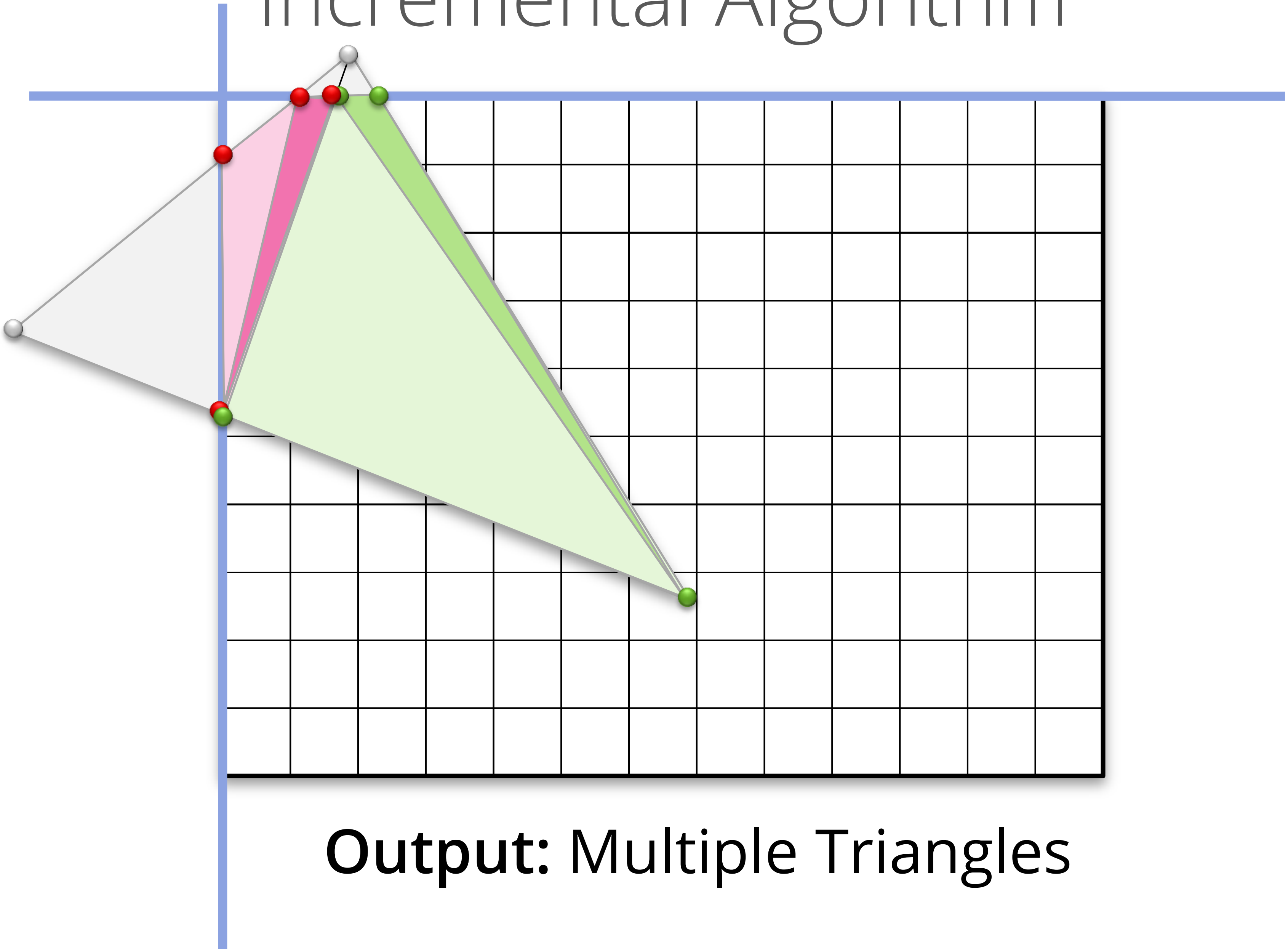
Incremental Algorithm



Incremental Algorithm



Incremental Algorithm



Output: Multiple Triangles

Further Optimization

View Frustum Culling

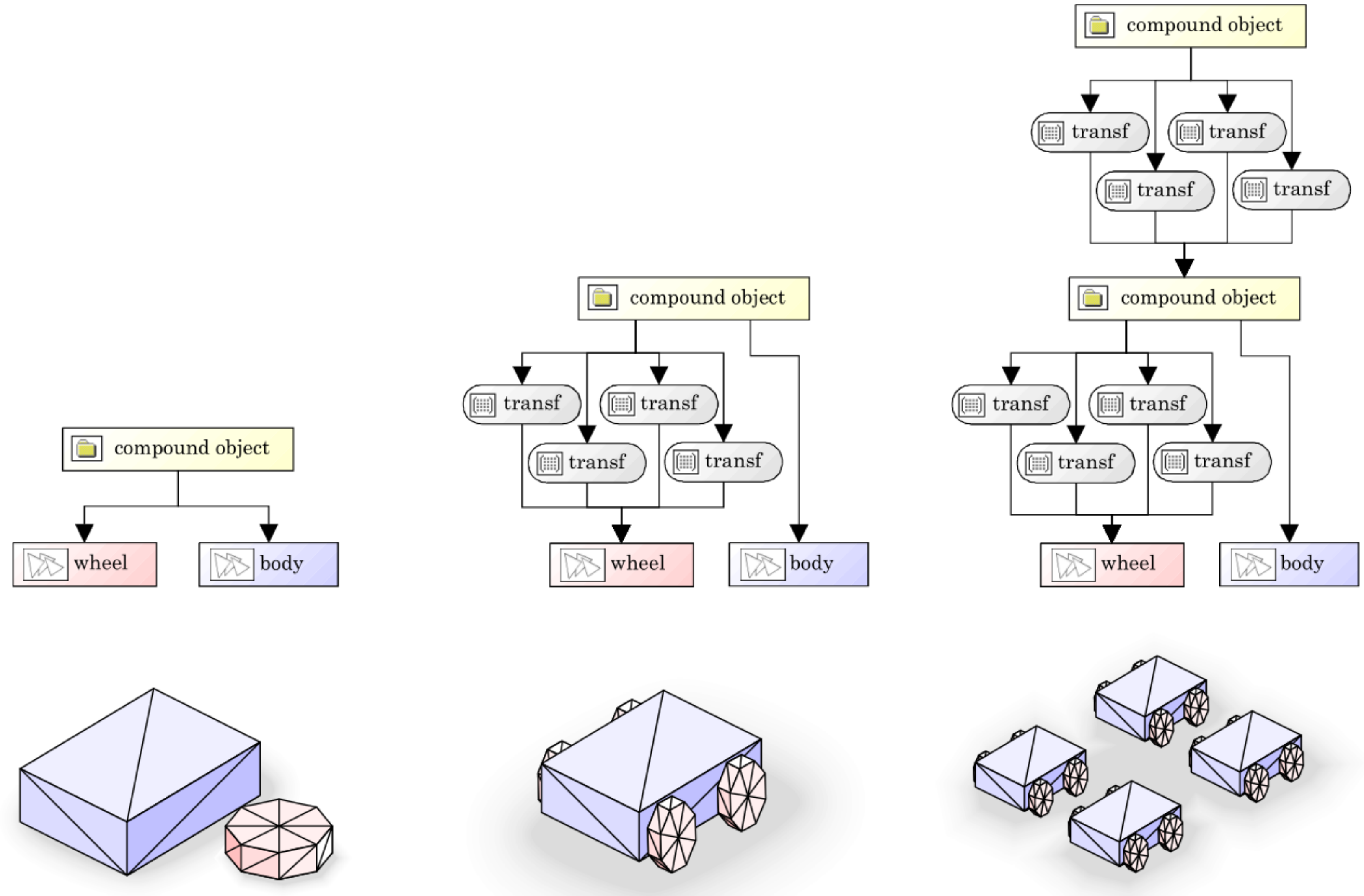
- Complex shapes (whole bunnies)
- Coarse bounding volume (superset)
  - Cube, Sphere
  - Often: Axis-aligned bounding box
- Reject all triangles inside if bounding volume outside view frustrum

Transformations & Normals

CORE

core topics
important

Remark 1: Scene Graphs



(a) two objects **(b) instantiation** **(c) hierarchical instantiation**

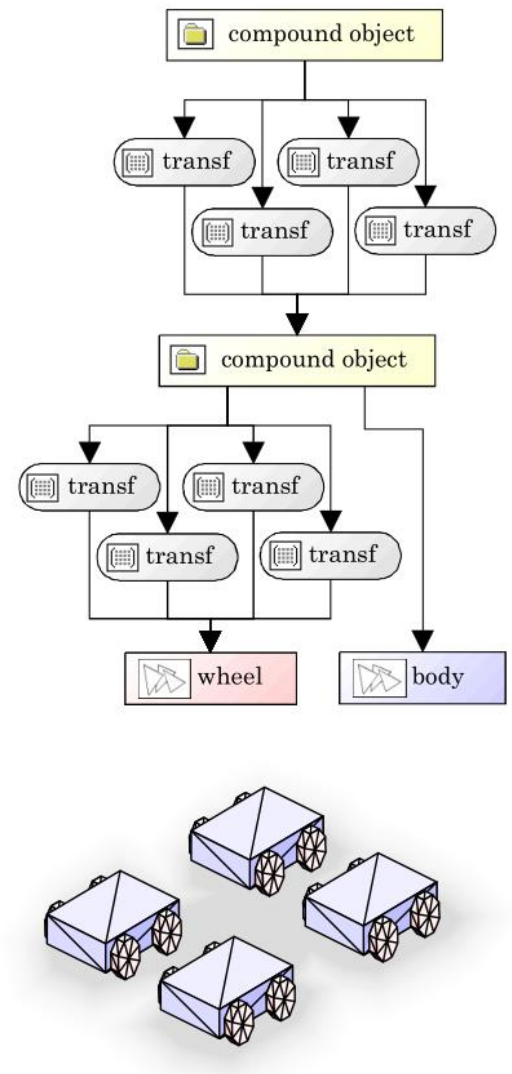
Animation

Hierarchical Animation

- Rotate wheels
- Move car with rotating wheels

“Kinematic Chains”

- Body, upper arm, lower arm, hand, fingers,…
- Relative transformations handled correctly automatically



(c) hierarchical instantiation

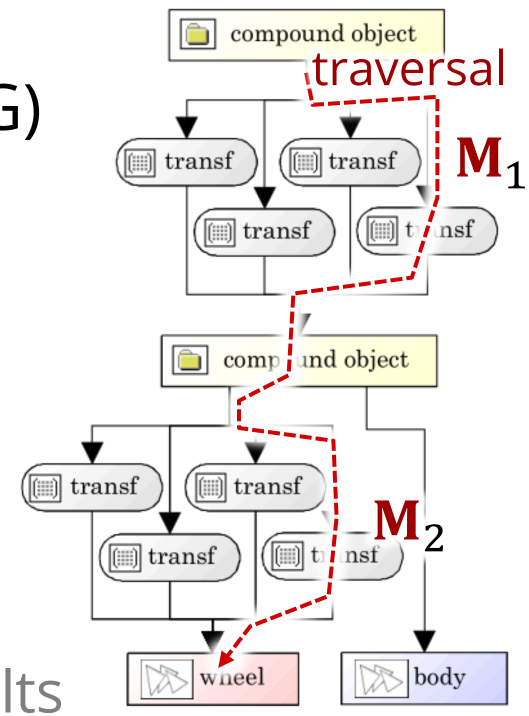
Implementation

Data Structure

- Simplest version: Tree
- Instancing: Directed Acyclic Graph (DAG)

Algorithm

- Depth-first-traversal
- Multiply transformation nodes
- Use associativity to order
  - Matrix stack to store intermediate results



- $$\left(\left((\mathbf{M}_1 \cdot \mathbf{M}_2) \cdots \mathbf{M}_{n-1}\right) \cdot \mathbf{M}_n\right)$$

deepest (applied first)
topmost (visited first)

Remark 2: Transforming Normals

How to transform normals of a surface?

Three cases

- Translations
  - Do not apply to normals!
- Orthogonal transformations
  - Rotations, reflections
  - Transform normals and points the same way
- General linear transformations
  - Points: $\mathbf{p}' = \mathbf{M}\mathbf{p}$
  - Normals: $\mathbf{n}' = \left(\mathbf{M}^{\mathrm{T}}\right)^{-1}\mathbf{n}$

← nothing to worry about

← be careful in this case!

Explanation

Implicit plane equation

$$\langle \mathbf{n}, \mathbf{p} \rangle = \mathbf{n}^{\mathrm{T}} \cdot \mathbf{p} = d$$

- $\mathbf{p}$ is a vector
- $\mathbf{n}$ is a co-vector

Change of coordinates:

- $\mathbf{p} \rightarrow \mathbf{M} \cdot \mathbf{p}$
- $\mathbf{n}^{\mathrm{T}} \rightarrow \left(\mathbf{M}^{-1} \cdot \mathbf{n}\right)^{\mathrm{T}}$

Result: same plane

$$\left(\mathbf{M}^{-1} \cdot \mathbf{n}\right)^{\mathrm{T}} \cdot \mathbf{M} \cdot \mathbf{p} = \mathbf{n}^{\mathrm{T}}\left(\mathbf{M}^{-1} \cdot \mathbf{M}\right) \cdot \mathbf{p} = \mathbf{n}^{\mathrm{T}} \cdot \mathbf{p} = d$$

Texture Mapping

CORE

core topics

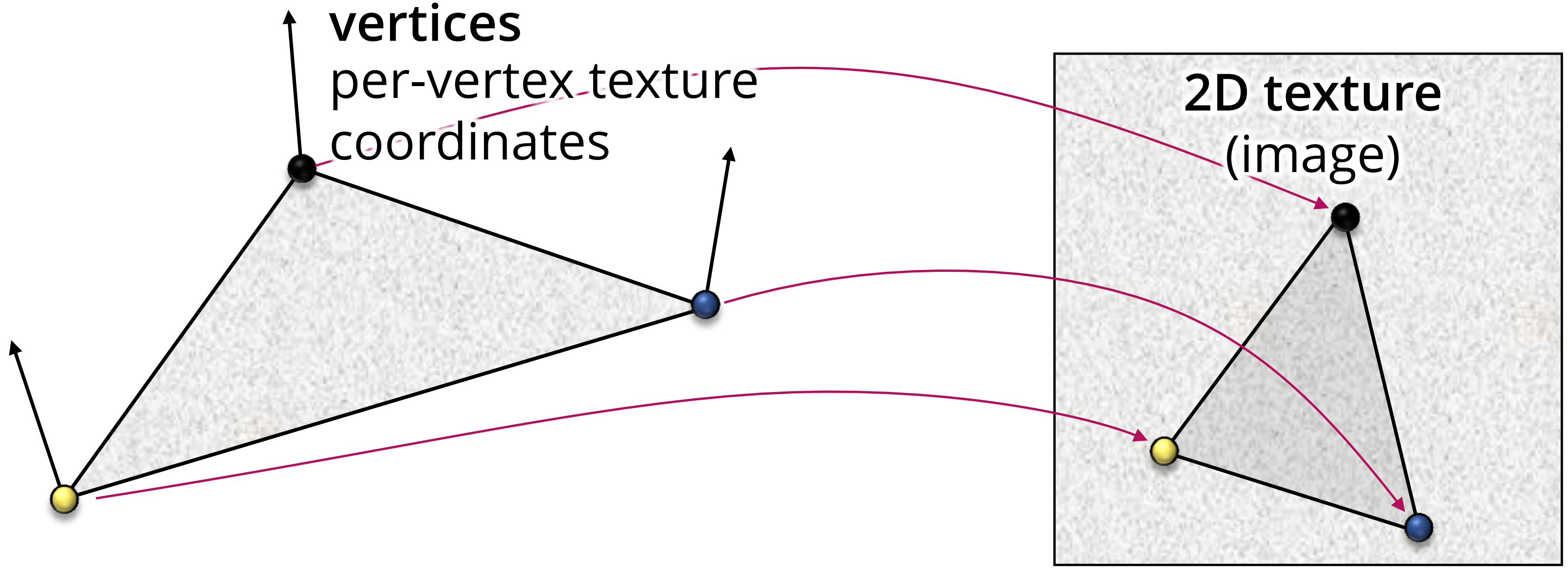
important

Texture Mapping

Idea:

- Map image to triangle
- Additional details
- Hard to model with geometry
- Much cheaper than fine geometric tessellation

Texture Coordinates

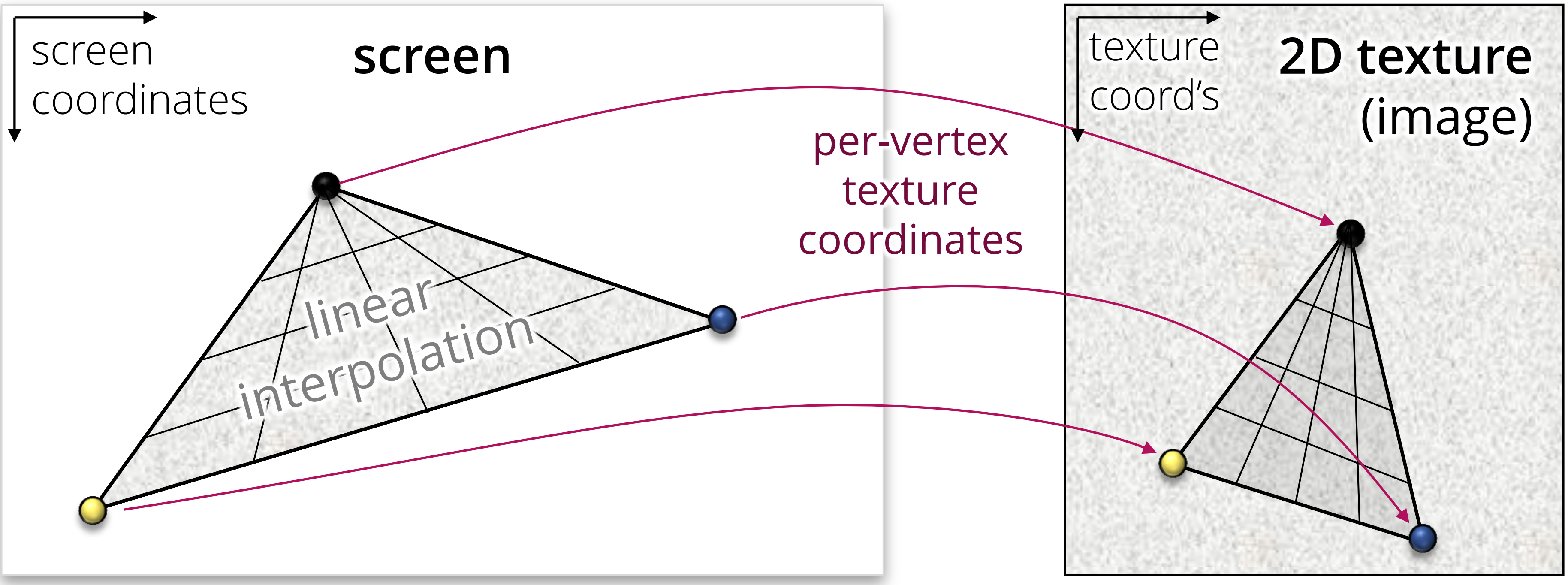


Define Mapping to Image

- Texture coordinates at vertices
- In between: linear interpolation
- Defines an affine map

2D Texture Mapping

Texture Coordinates

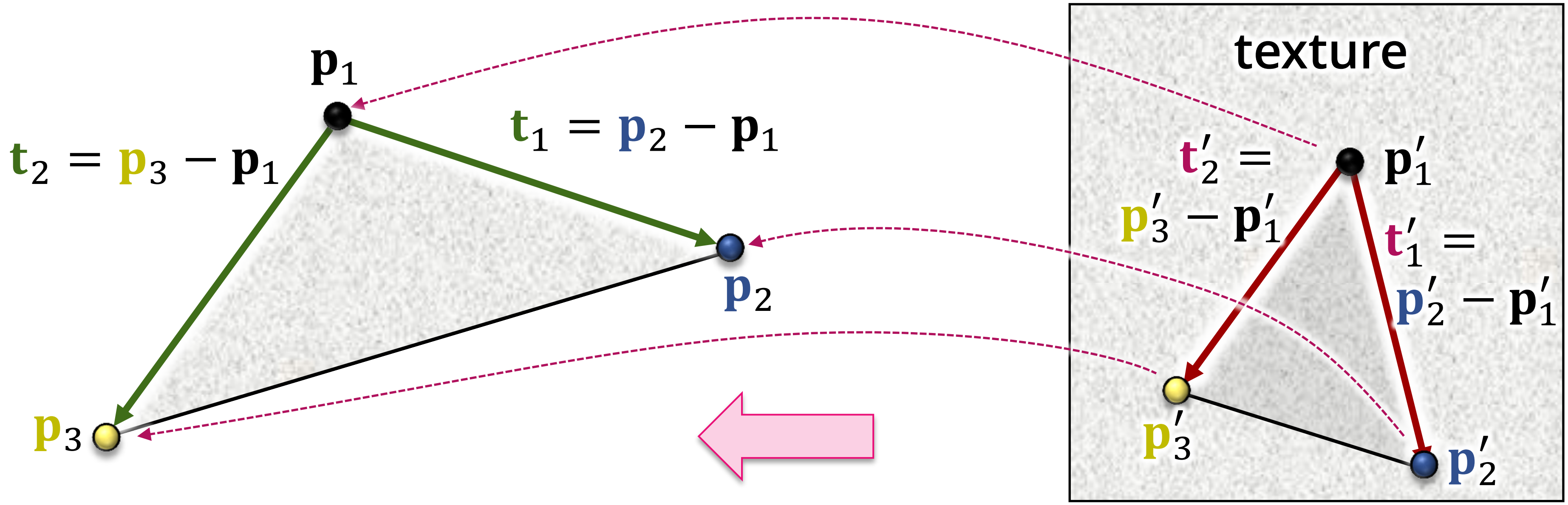


Define Mapping to Image

- Texture coordinates at vertices
- In between: affine ("linear") interpolation
- Defines an affine map

technically, this is an affine map,
but people often call it
"linear interpolation"

Affine Map

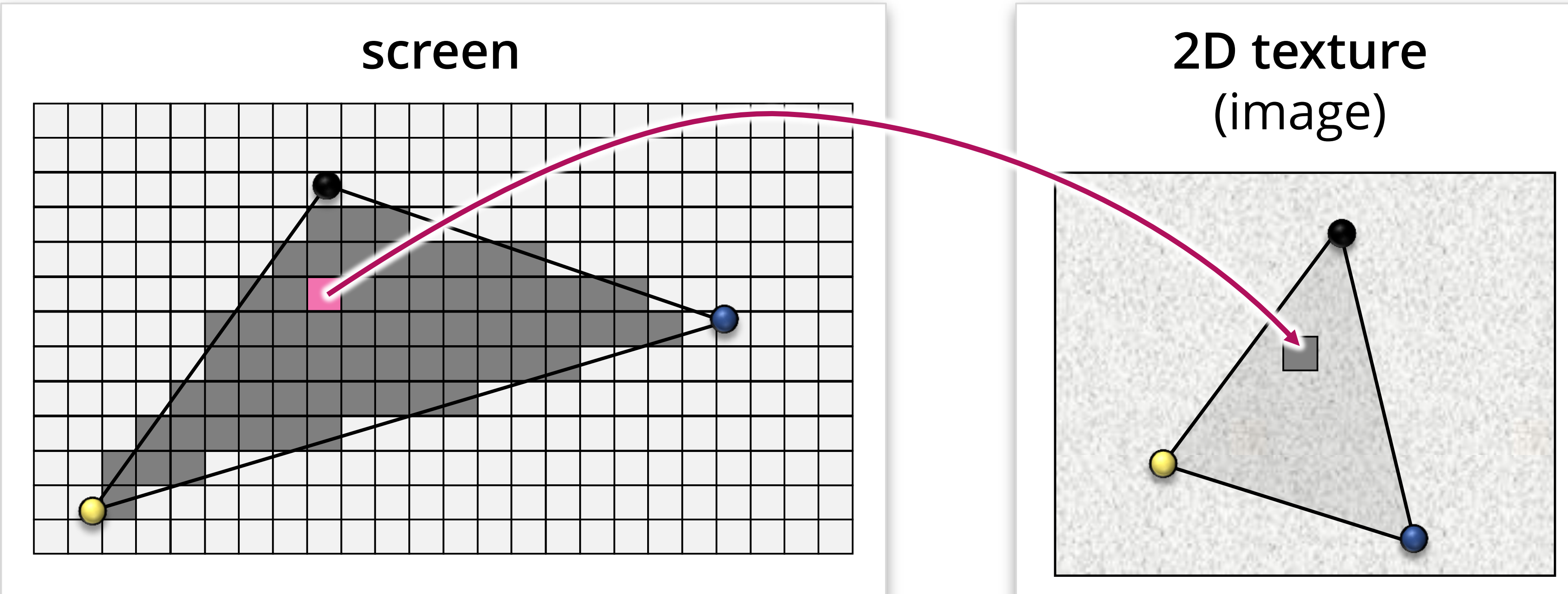


Affine Map

- Map coordinate system $\{\mathbf{p}'_1, (\mathbf{t}'_1, \mathbf{t}'_2)\}$ to $\{\mathbf{p}_1, (\boldsymbol{t}_1, \boldsymbol{t}_2)\}$

$$\mathbf{x} \to \mathbf{p}_1 + \begin{pmatrix} | & | \\ \boldsymbol{t}_1 & \boldsymbol{t}_2 \\ | & | \end{pmatrix} \cdot \begin{pmatrix} | & | \\ \mathbf{t}'_1 & \mathbf{t}'_2 \\ | & | \end{pmatrix}^{-1} (\mathbf{x} - \mathbf{p}'_1)$$

Rasterization



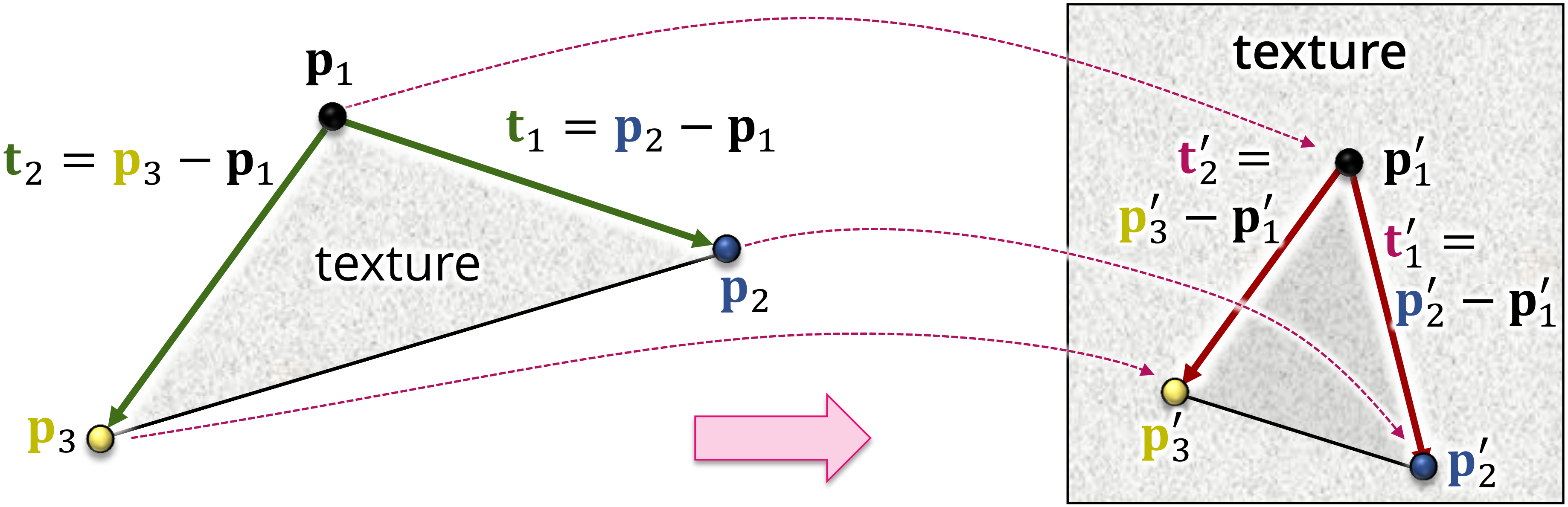
Rasterization

- Project vertices
  - Keep texture coordinates as specified
- Create fragments
  - Lookup texture color

Texture Lookup

lookup color
for each fragment

Rasterization: Inverse Map



Affine Map

- Map coordinate system $\{\mathbf{p}_1, (\mathbf{t}'_1, \mathbf{t}'_2)\}$ to $\{\mathbf{p}_2, (t_1, t_2)\}$

$$\mathbf{x} \rightarrow \mathbf{p}'_1 + \begin{pmatrix} | & | \\ \mathbf{t}'_1 & \mathbf{t}'_2 \\ | & | \end{pmatrix} \cdot \begin{pmatrix} | & | \\ t_1 & t_2 \\ | & | \end{pmatrix}^{-1} (\mathbf{x} - \mathbf{p}_1)$$

Texture Mapping

Texture space to screen space:

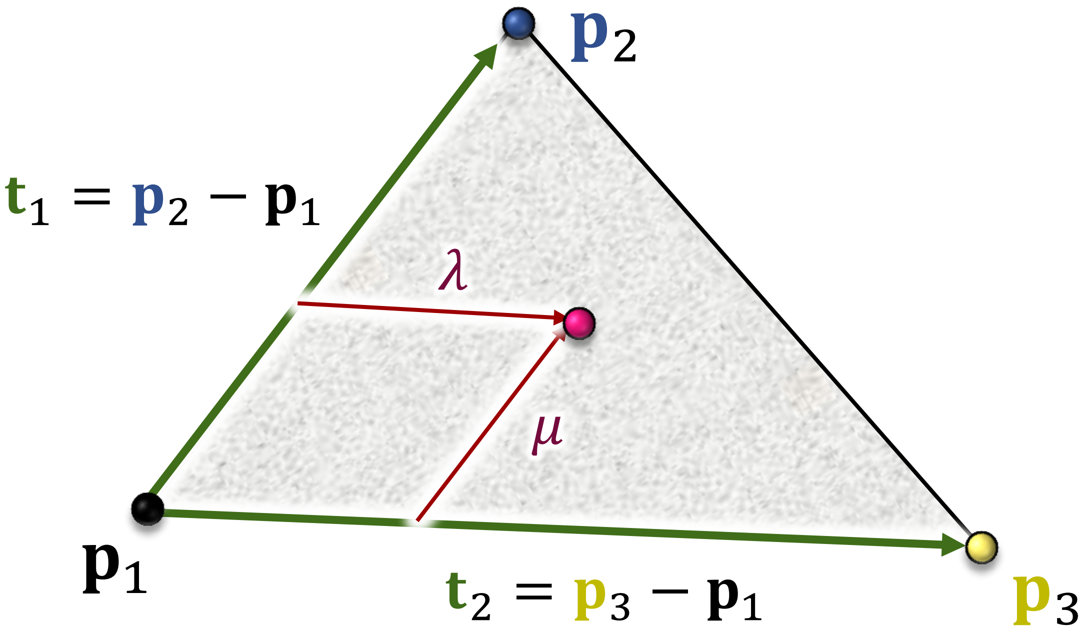
$$\mathbf{x} \to \mathbf{p}_1 + \begin{pmatrix} | & | \\ \boldsymbol{t}_1 & \boldsymbol{t}_2 \\ | & | \end{pmatrix} \cdot \begin{pmatrix} | & | \\ \mathbf{t}'_1 & \mathbf{t}'_2 \\ | & | \end{pmatrix}^{-1} (\mathbf{x} - \mathbf{p}'_1)$$

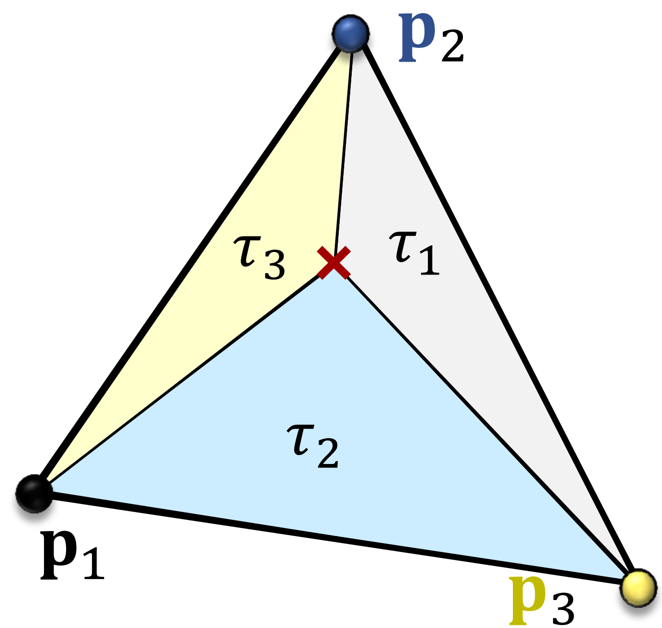
Screen space to world space:

$$\mathbf{x} \to \mathbf{p}'_1 + \begin{pmatrix} | & | \\ \mathbf{t}'_1 & \mathbf{t}'_2 \\ | & | \end{pmatrix} \cdot \begin{pmatrix} | & | \\ \boldsymbol{t}_1 & \boldsymbol{t}_2 \\ | & | \end{pmatrix}^{-1} (\mathbf{x} - \mathbf{p}_1)$$

*) **Formally:** this is a change of coordinate system

Barycentric Coordinates





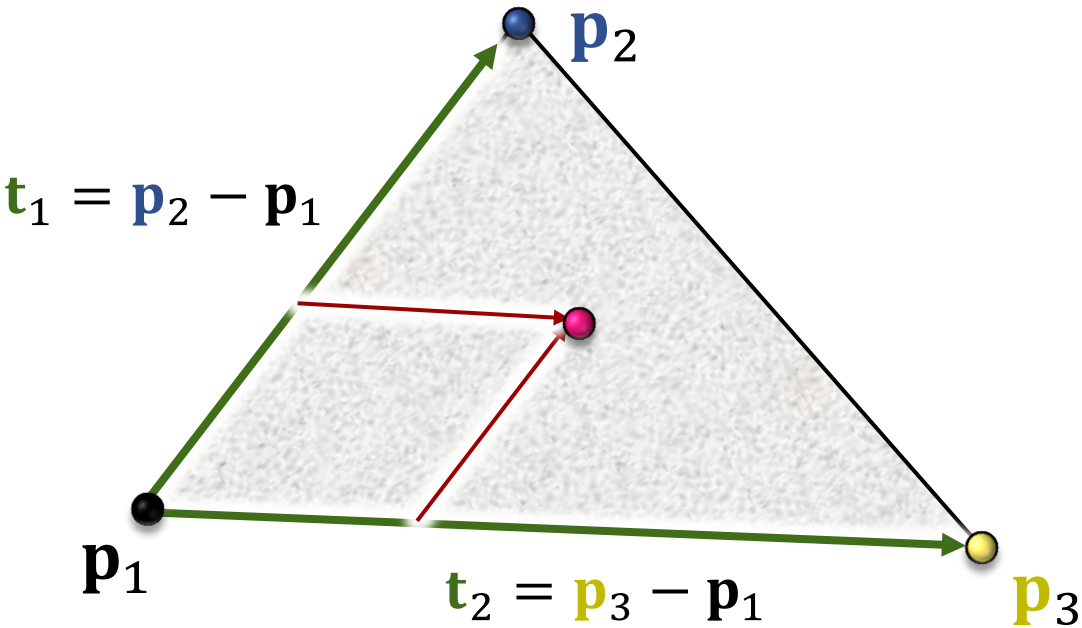
Barycentric coordinates

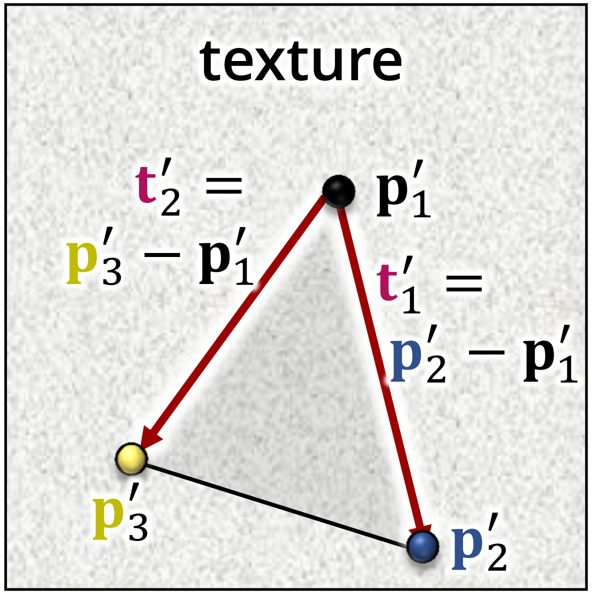
- 2D coordinate system (in plane)
- Triangle edge coordinates
- Same as ratio of area of opposing triangle to overall triangle area

$$\lambda = \frac{\tau_3}{\tau_1 + \tau_2 + \tau_3}$$

$$\mu = \frac{\tau_2}{\tau_1 + \tau_2 + \tau_3}$$

Barycentric Coordinates



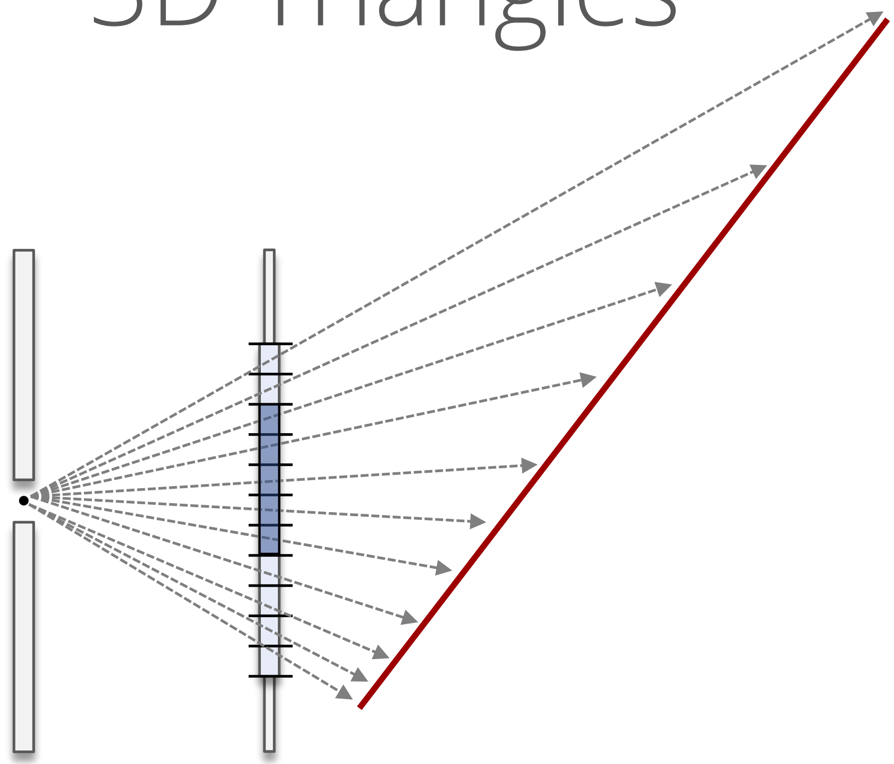


Interpretation:

$$\mathbf{x} \rightarrow \mathbf{p}'_1 + \begin{pmatrix} | & | \\ \mathbf{t}'_1 & \mathbf{t}'_2 \\ | & | \end{pmatrix} \cdot \begin{pmatrix} | & | \\ \boldsymbol{t}_1 & \boldsymbol{t}_2 \\ | & | \end{pmatrix}^{-1} (\mathbf{x} - \mathbf{p}_1)$$

- Transform to barycentric coordinates, then to texture coordinates

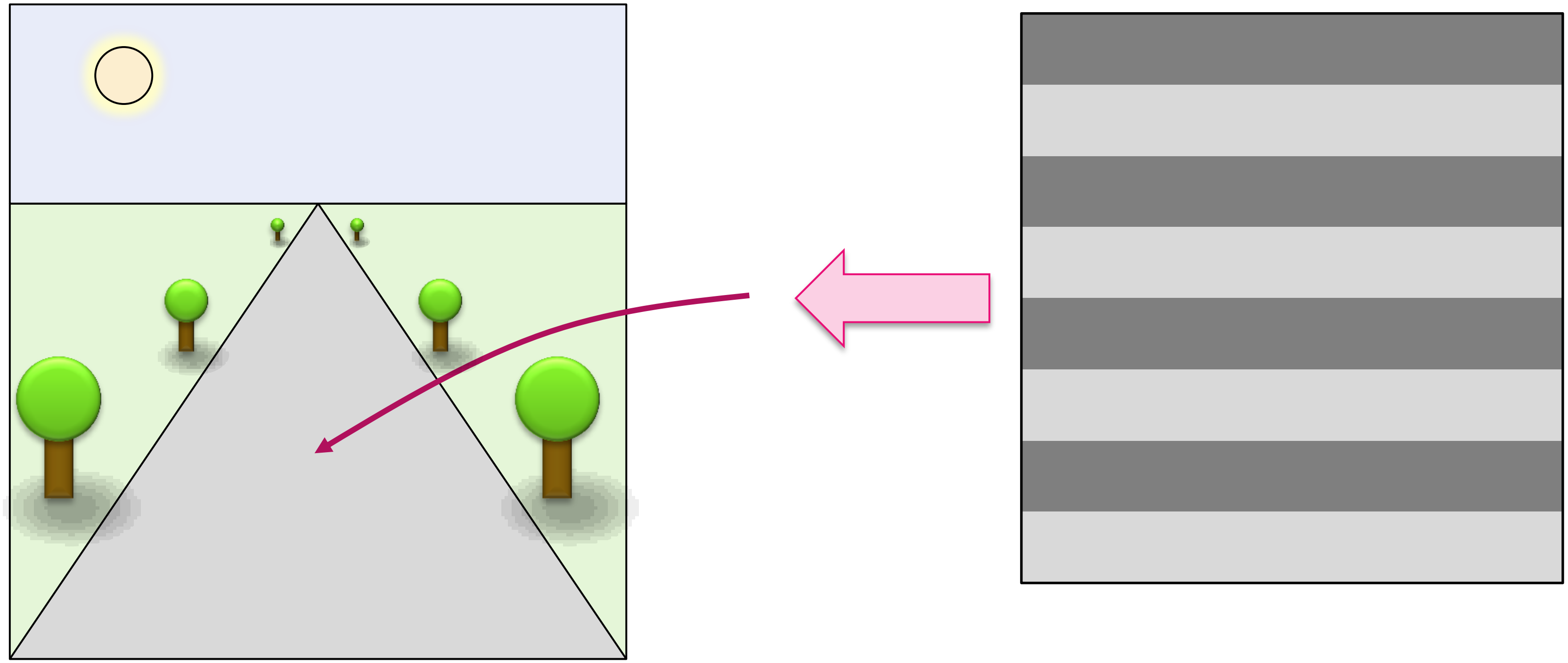
3D Triangles



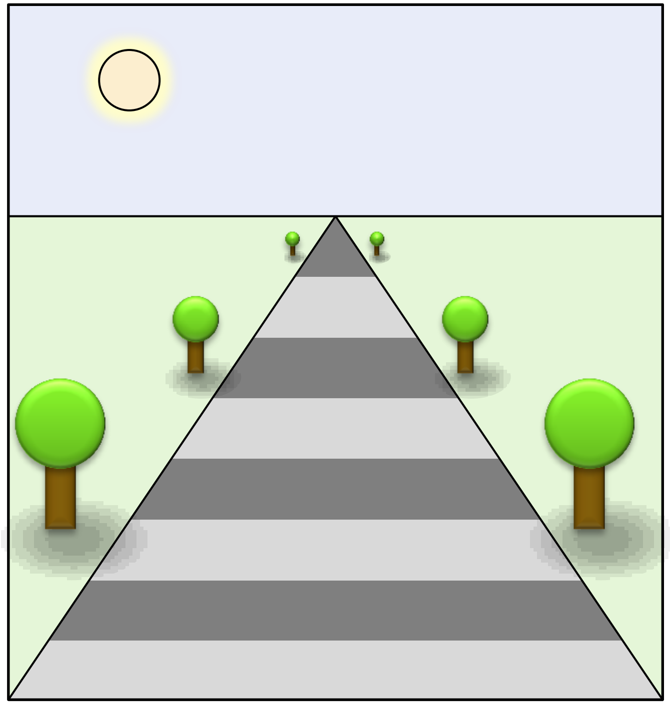
Non-uniform spacing!

- 2D texture mapping will create artifacts!

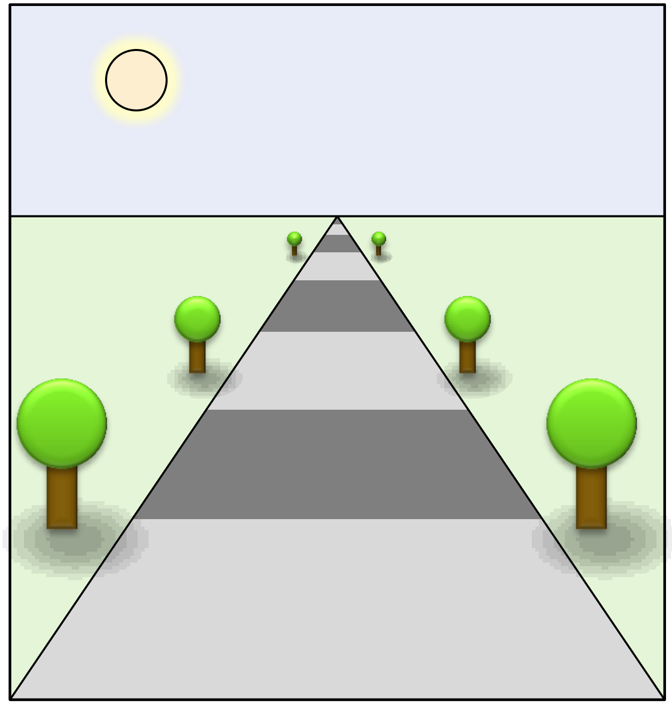
Example



Example



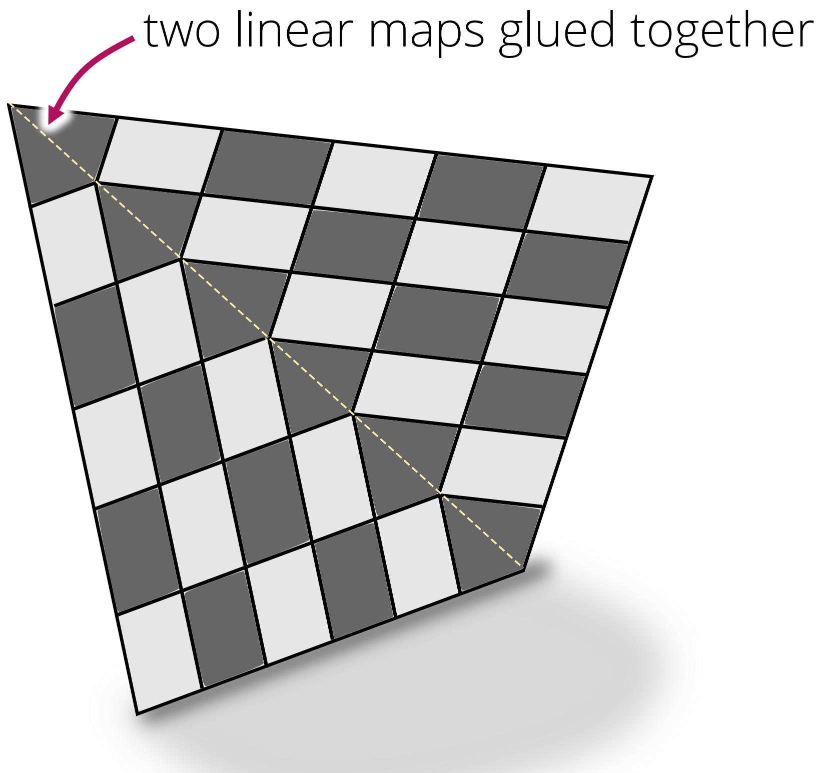
2D Texture Mapping



3D Texture Mapping

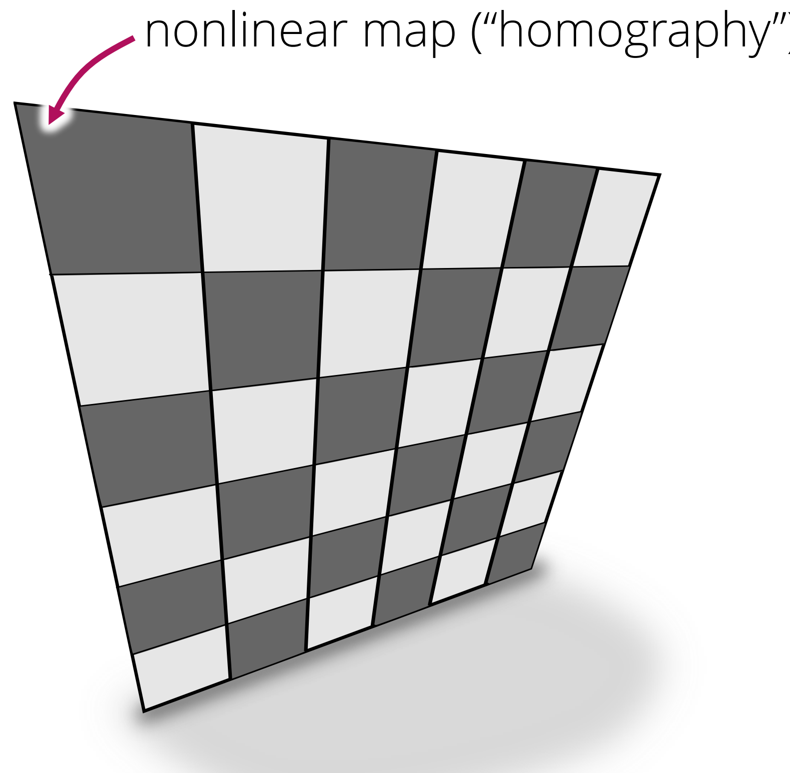
Obviously, we want this!

A Related Problem



2D Texture Mapping

triangulation dependent



3D Texture Mapping

triangulation independent

Perspective Correction

Incorrect results:

- Linear*) interpolation in screen space

Correct results

- Linear*) interpolation in object space
- Uneven steps in texture coordinates between pixels

How to compute?

*) again, this is actually an affine map, but the term “linear interpolation” is almost exclusively used here

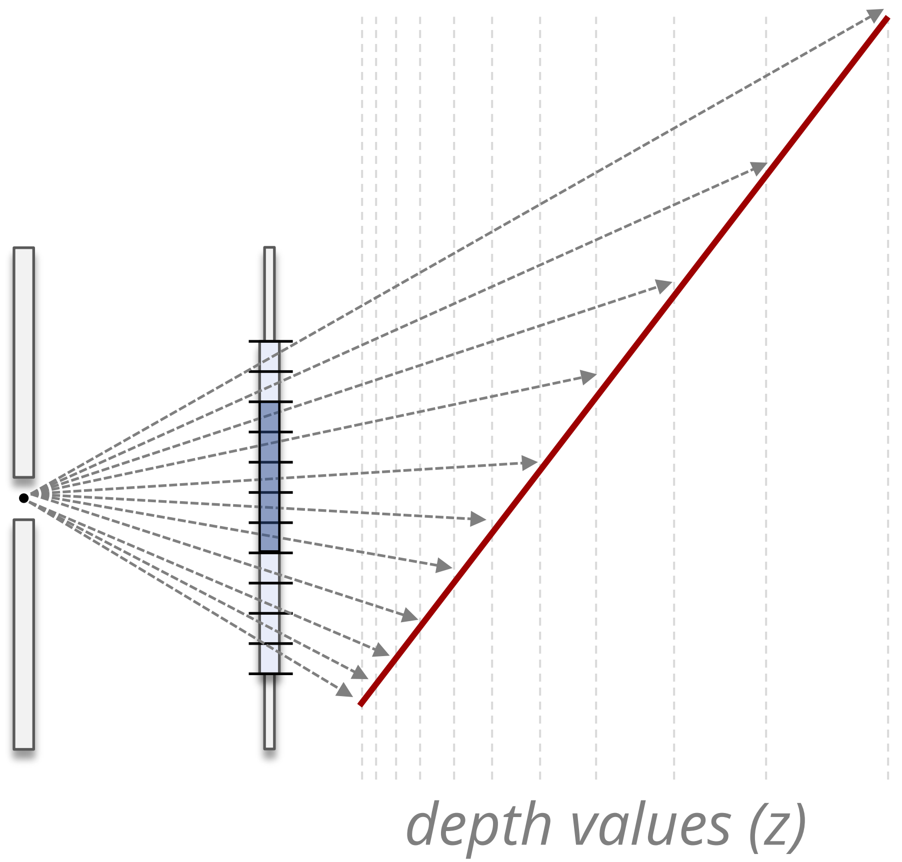
Correct Perspective Texturing

Two solutions

- Absolute computation
  - Setup ray equation for each pixel
  - Compute ray-triangle intersection
  - Possible and correct, but slow
- Incremental interpolation
  - Do not interpolate *u*,*v* but $\frac{u}{z}, \frac{v}{z}$
    - Gives correct results in screen space!
  - Multiply by *z* in the end
    - Interpolate $\frac{1}{z}$ in screen space (z also non-linear!)
    - Divide by $\frac{1}{z}$ (approximation strategies for speed)

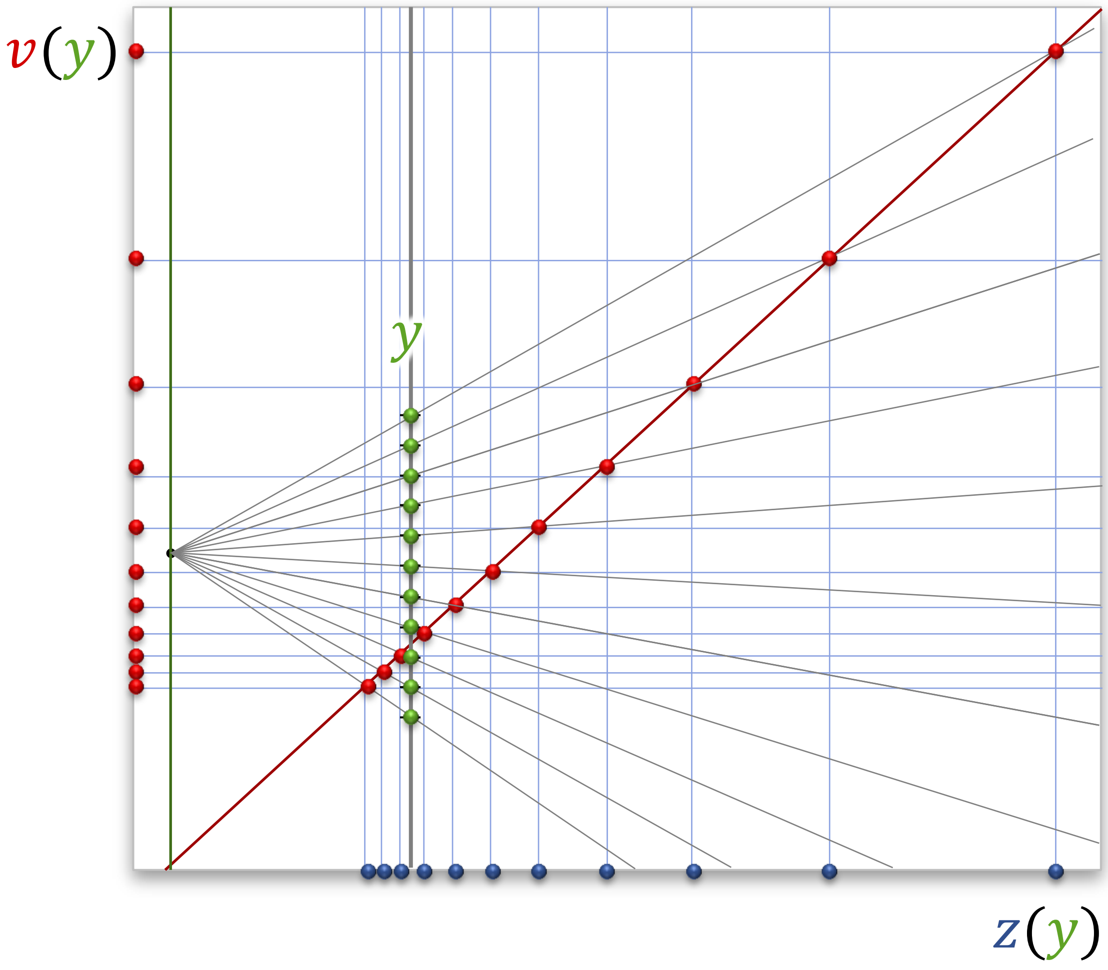
(Lazy "option" #3: use GPU/GfxLib and don't worry :-) **)**

3D Triangles



Non-uniform spacing!

3D Triangles

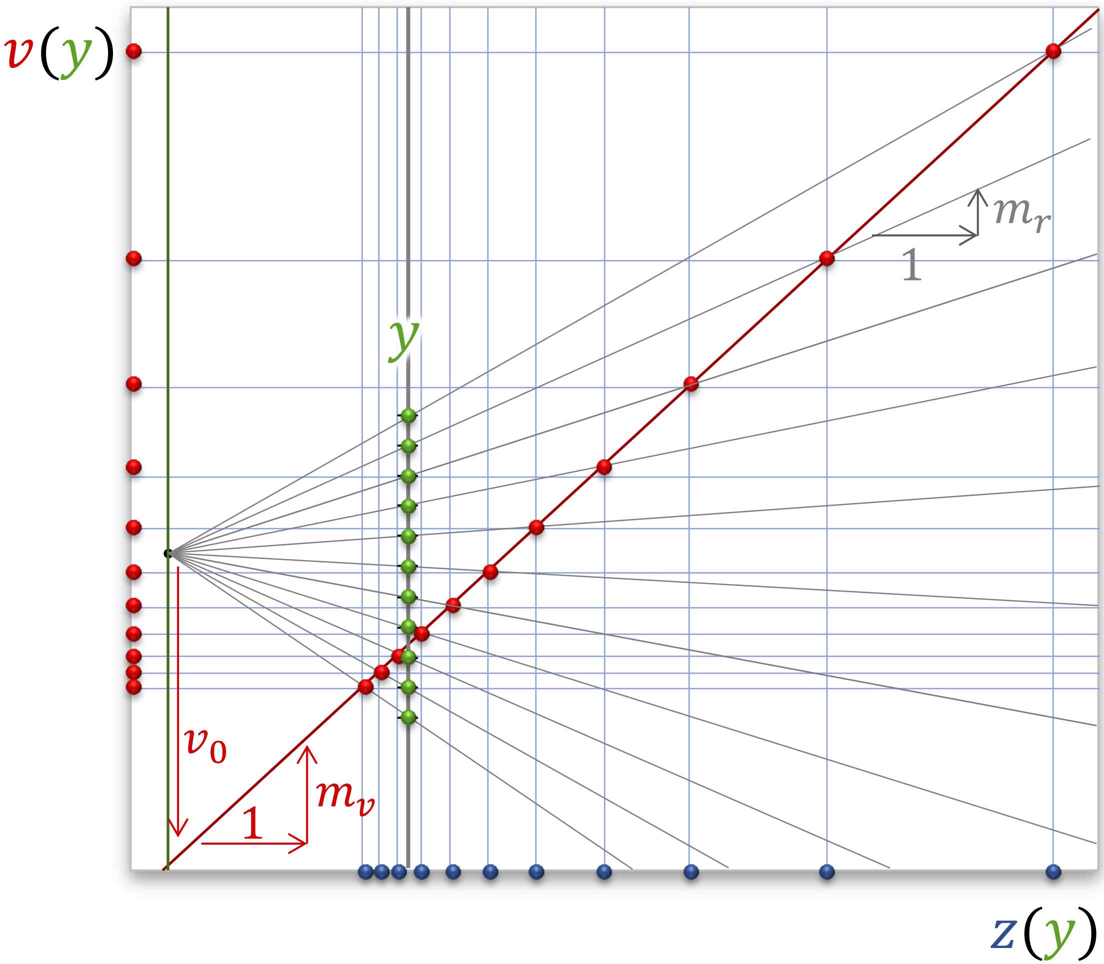


proportional:

$$z(y) \sim v(y)$$

$$\frac{1}{z} \sim y + c$$

3D Triangles



$$v(z) = m_v \cdot z + v_0$$
$$r(z) = m_r \cdot y \cdot z$$

intersect plane / view ray

$$z = \frac{v_0}{(m_r \cdot y - m_v)}$$

$$v(y) = \frac{m_v \cdot v_0}{(m_r \cdot y - m_v)} + v_0$$

$$\left[\frac{1}{z}\right](y) = \frac{1}{v_0}(m_r \cdot y - m_v)$$

affine

$$\left[\frac{v}{z}\right](y) = m_r \cdot y$$

linear

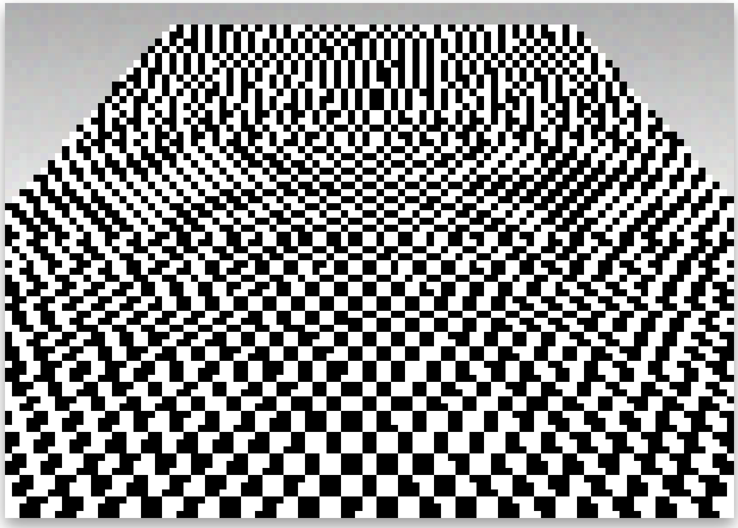
Aliasing and Anti-Aliasing

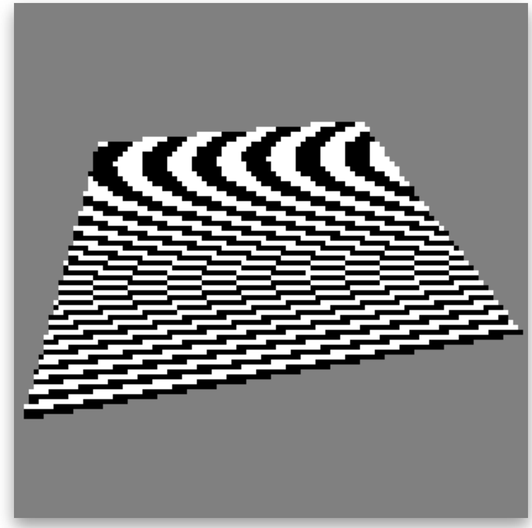
ADV

advanced topics
main ideas

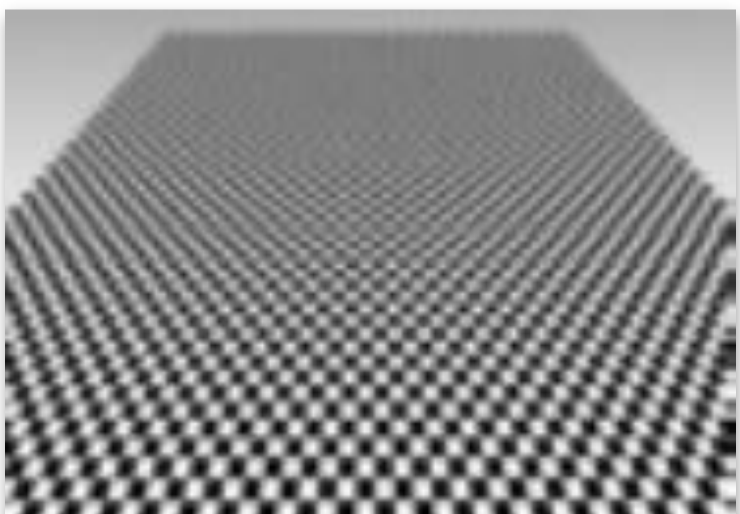
Aliasing

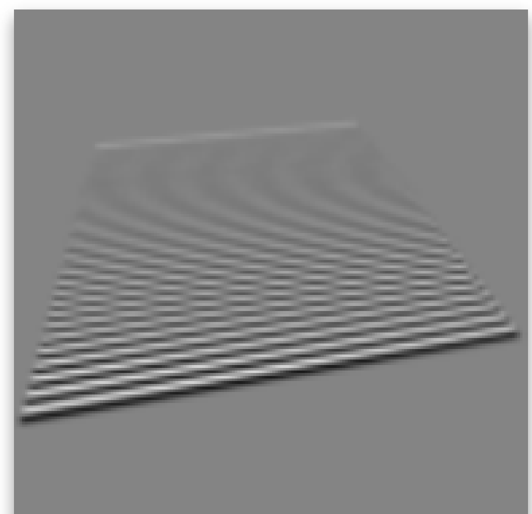
simple
sampling



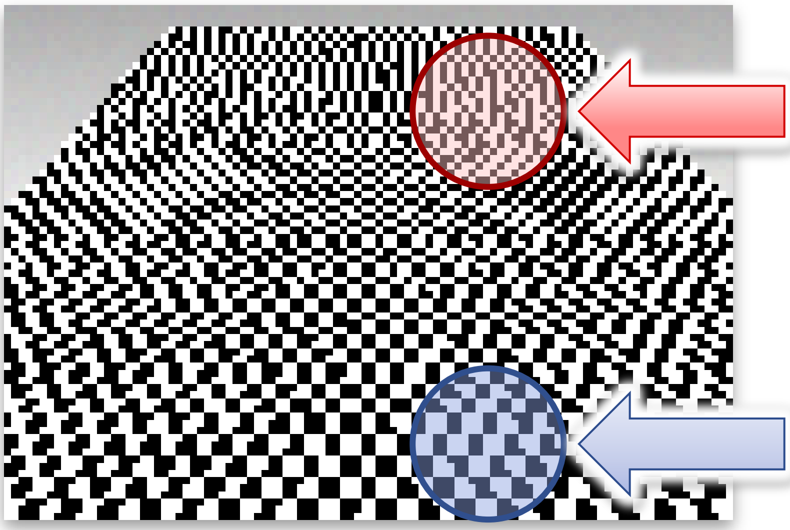


anti-
aliasing
(Gaussian)





Aliasing

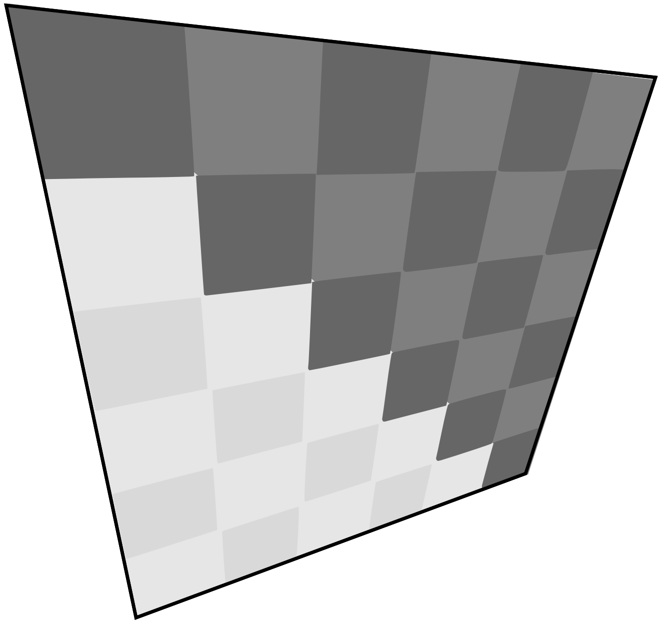


Minification: Moiré
Sampling aliasing

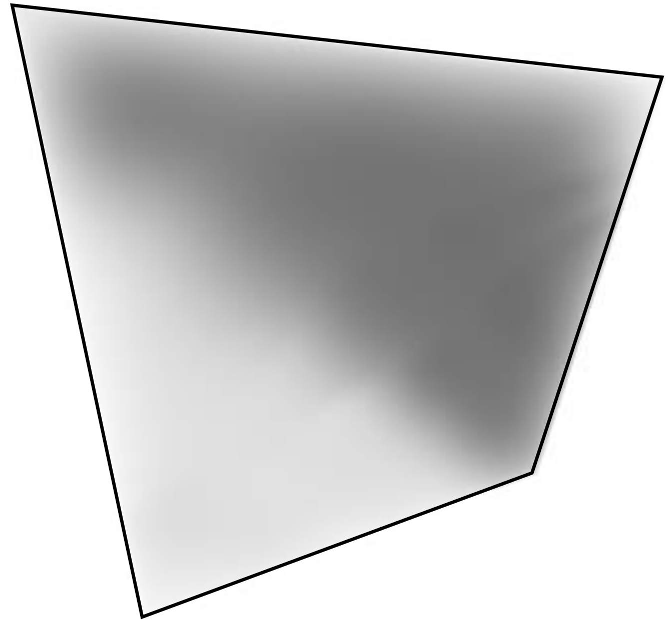
Magnification: “Staircasing”
Reconstruction aliasing

Magnification

*) same here, it is a (bi-) affine map, but called "(bi-) linear" in literature / APIs



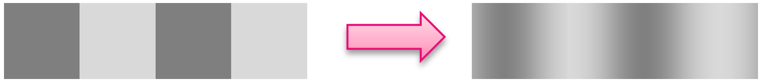
pixel sampling



Interpolation

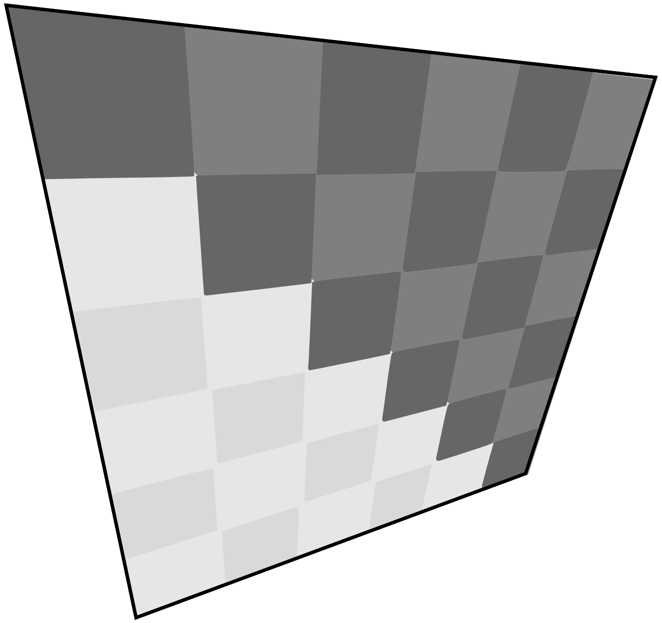
In hardware:

- Bi-linear*) interpolation
- Linear blend in *u*- and *v*-direction

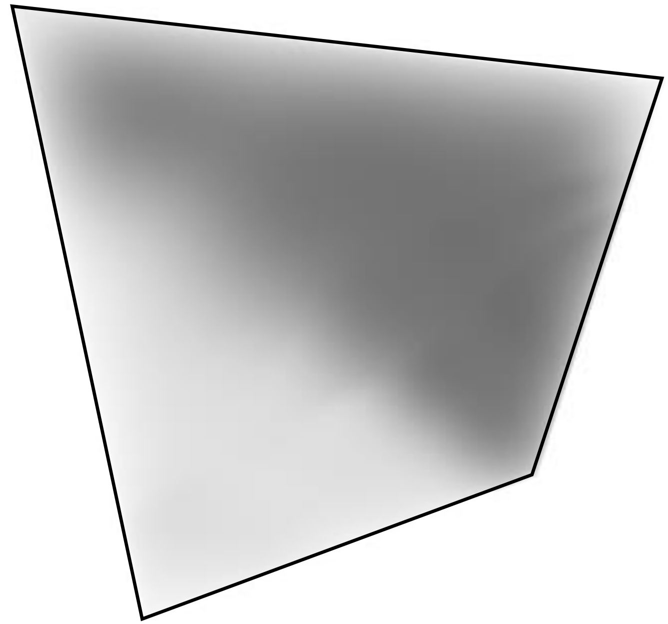


*) you know, linear ~ affine...

Magnification



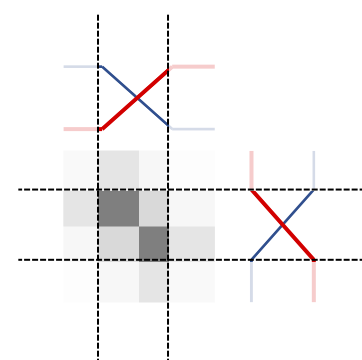
pixel sampling



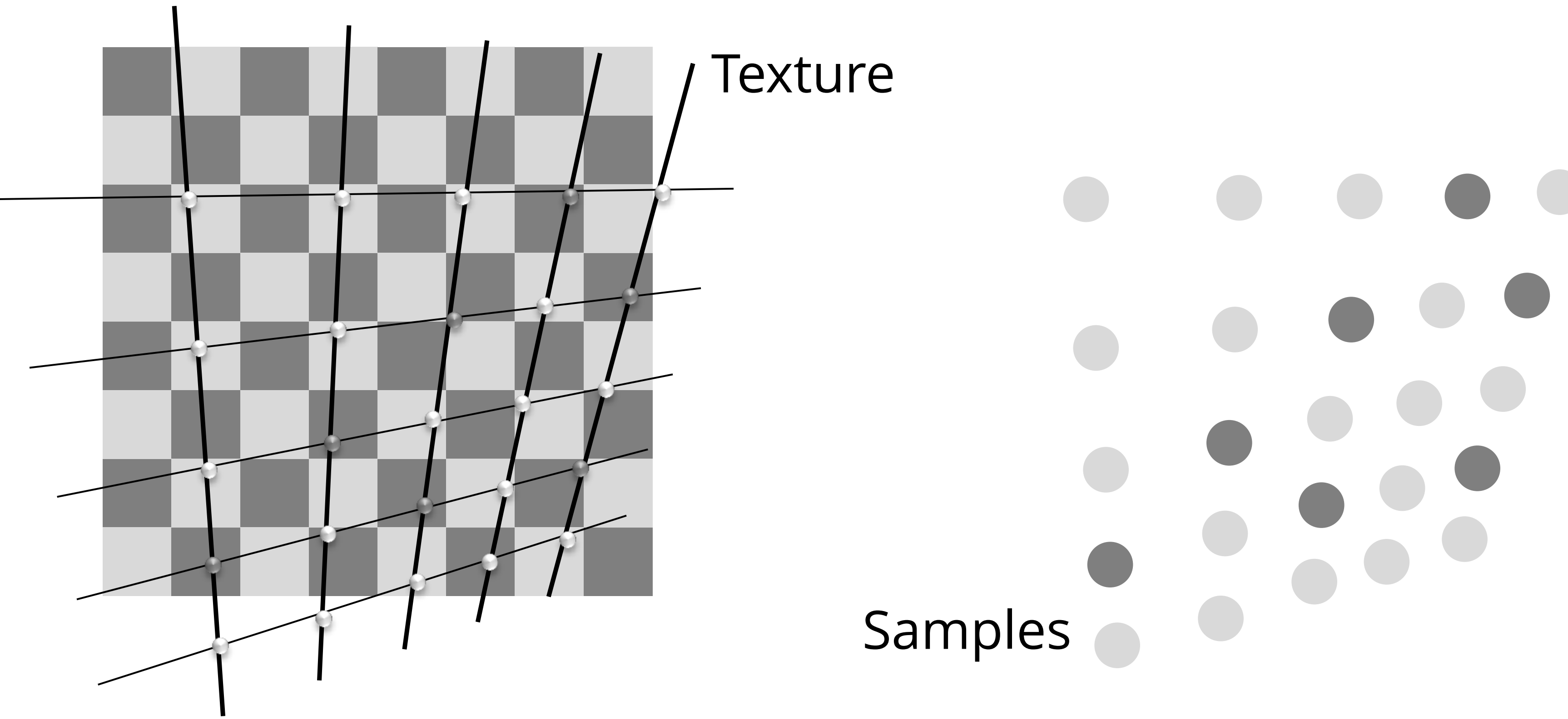
Interpolation

In hardware:

- Bi-linear*) interpolation
- Linear blend in *u*- and *v*-direction



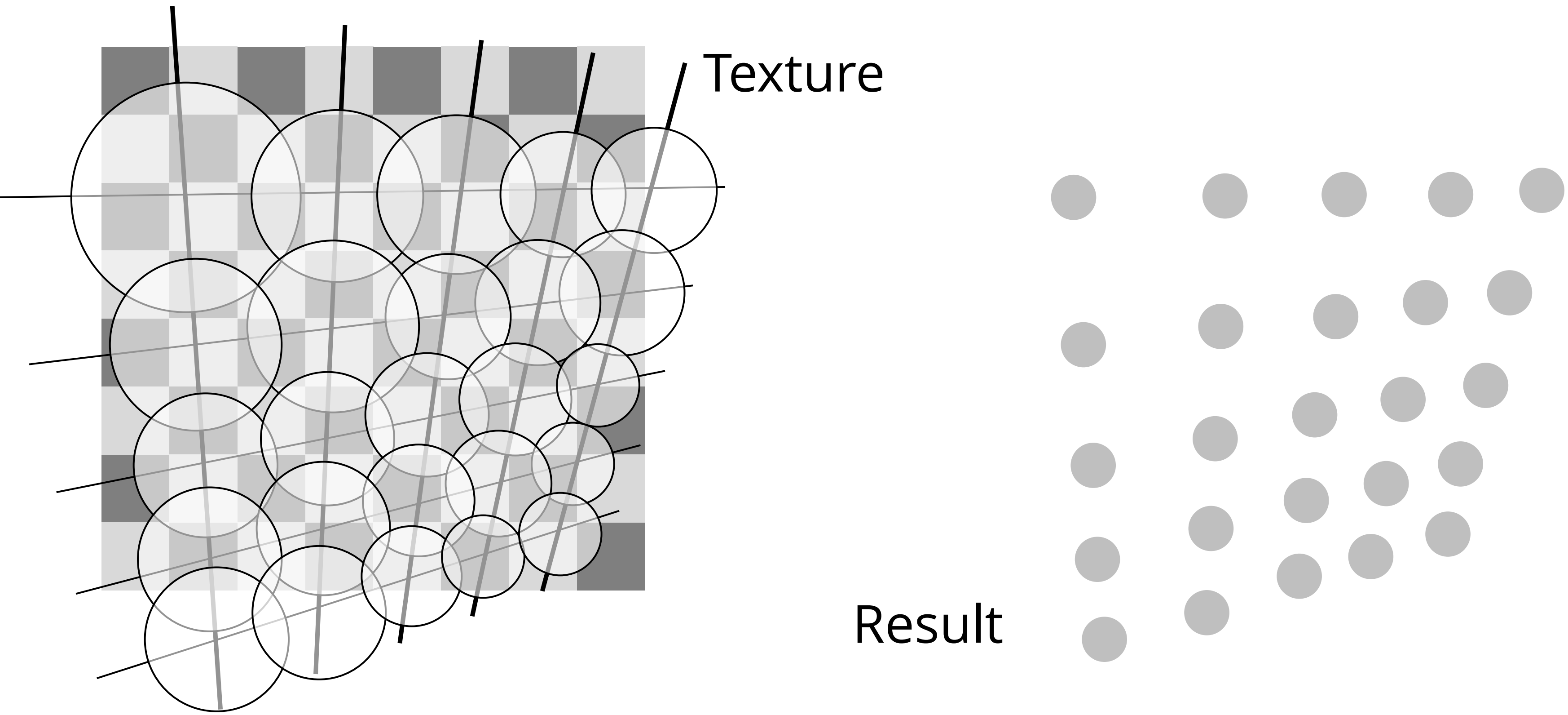
Minification: Interfering Grids



Sampling Aliasing

- Sampling grid misses information, spacing too big
- Creates Moiré (or noise, if unstructured)

Solution



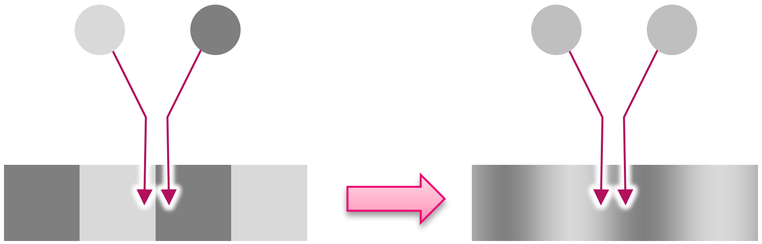
Solution

- Average over neighborhood
  - Heuristic: “leave no free space”
- Best: weighted filters with overlap (e.g. Gaussians)

*) same here

Graphics Hardware

Two Steps

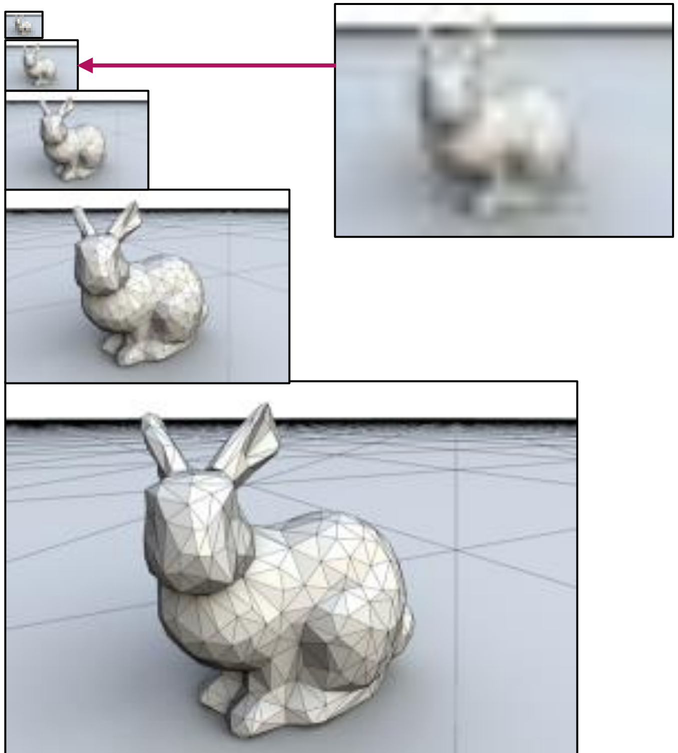


Moderate Minification

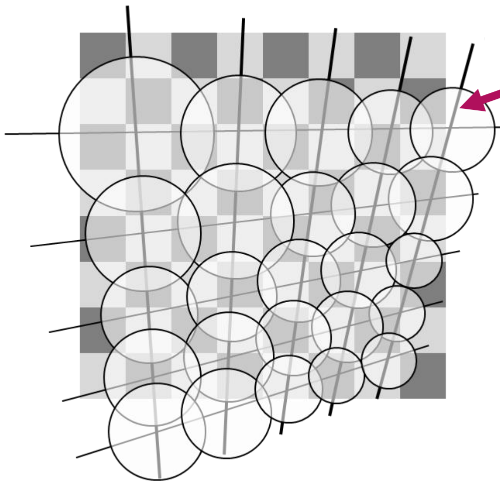
- Bilinear*) interpolation

Strong Minification

- Mip-mapping
  - Average 2 × 2 pixels
  - Store reduce size by 1/2
  - Iterate
- Trilinear interpolation

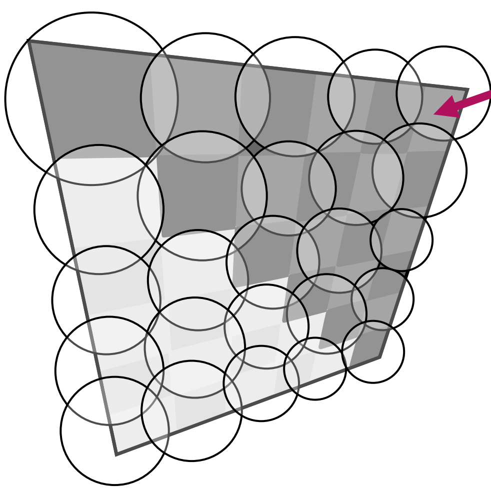


Summary



Minification

- Average multiple texels

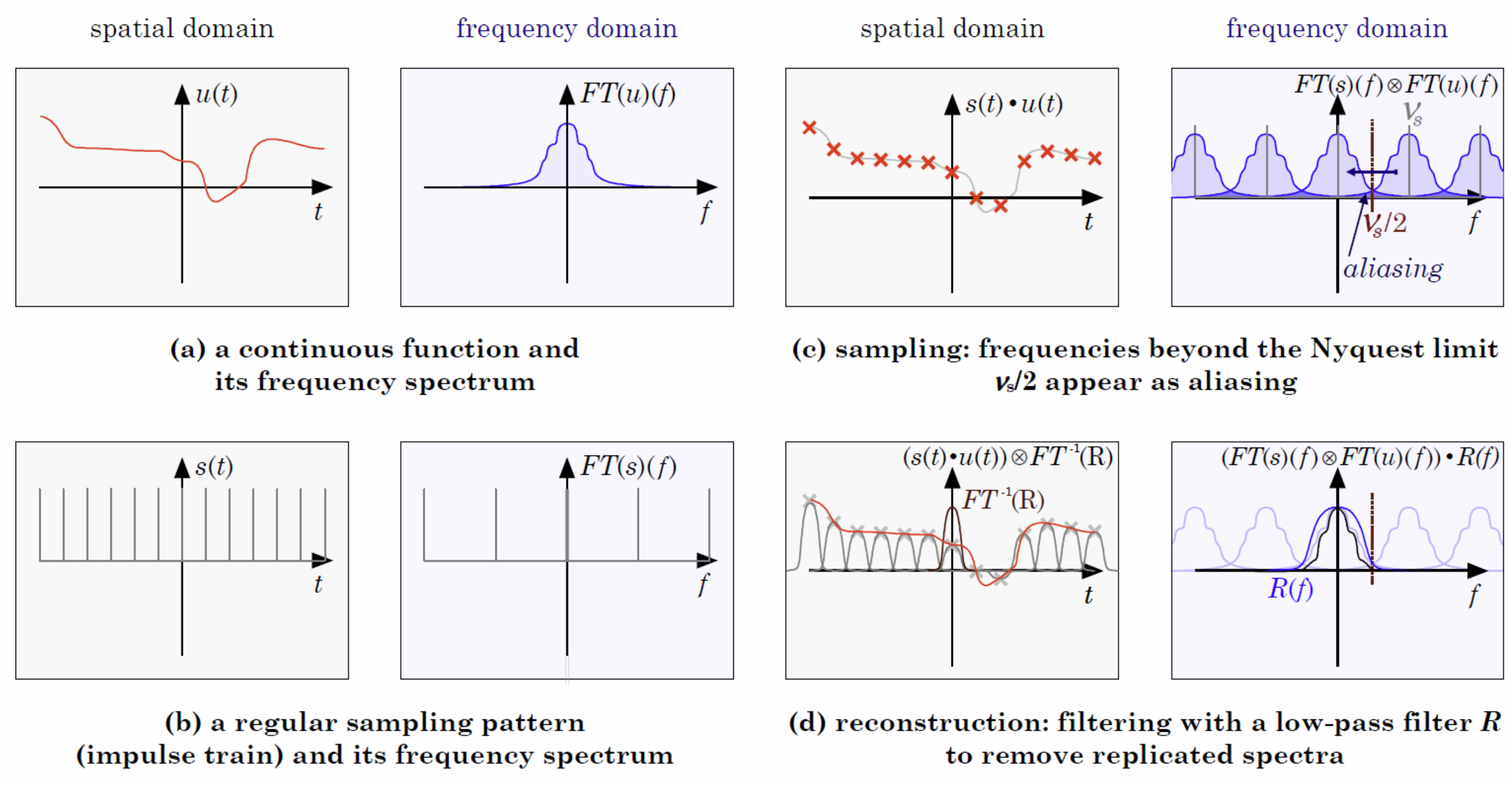


Magnification

- Average over multiple pixels

Swap screen & texture

The Full Story: Fourier Transforms



(a) a continuous function and its frequency spectrum

(c) sampling: frequencies beyond the Nyquest limit $\nu_s/2$ appear as aliasing

(b) a regular sampling pattern (impulse train) and its frequency spectrum

(d) reconstruction: filtering with a low-pass filter R to remove replicated spectra

- more in "advanced graphics" -

Topics

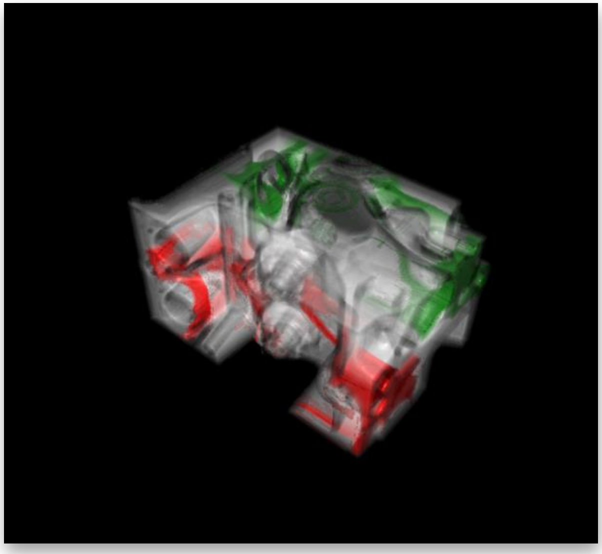
Texture mapping variants

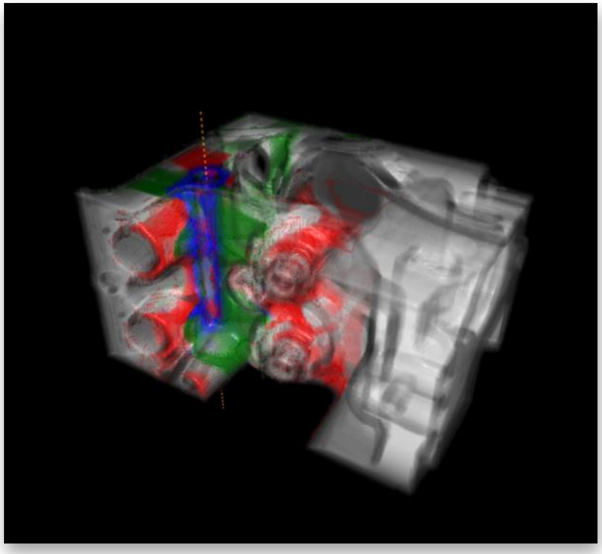
- 3D Textures
- Shadow maps
  - Ambient Occlusion
- Environment Maps
  - Image-based Lighting
- Bump mapping / normal maps
- Displacement maps

3D Textures

3D Textures

- Use 3D array of “voxels”
- u,v,w-coordinates
- Texture space itself





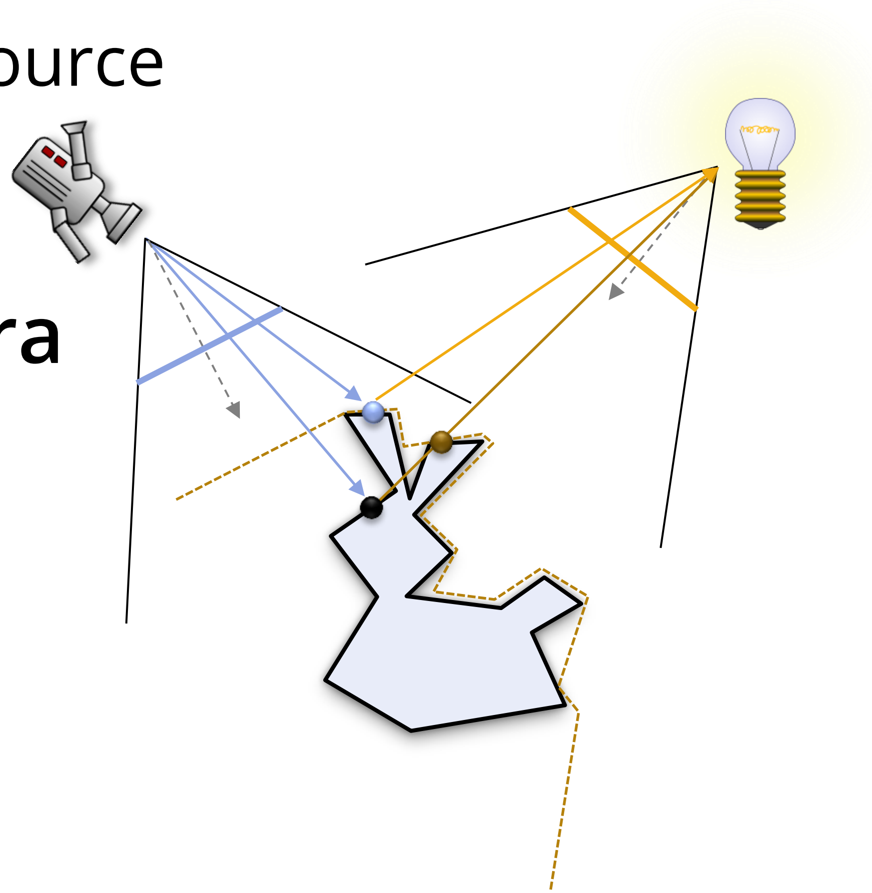
Shadow Maps

Create shadow map

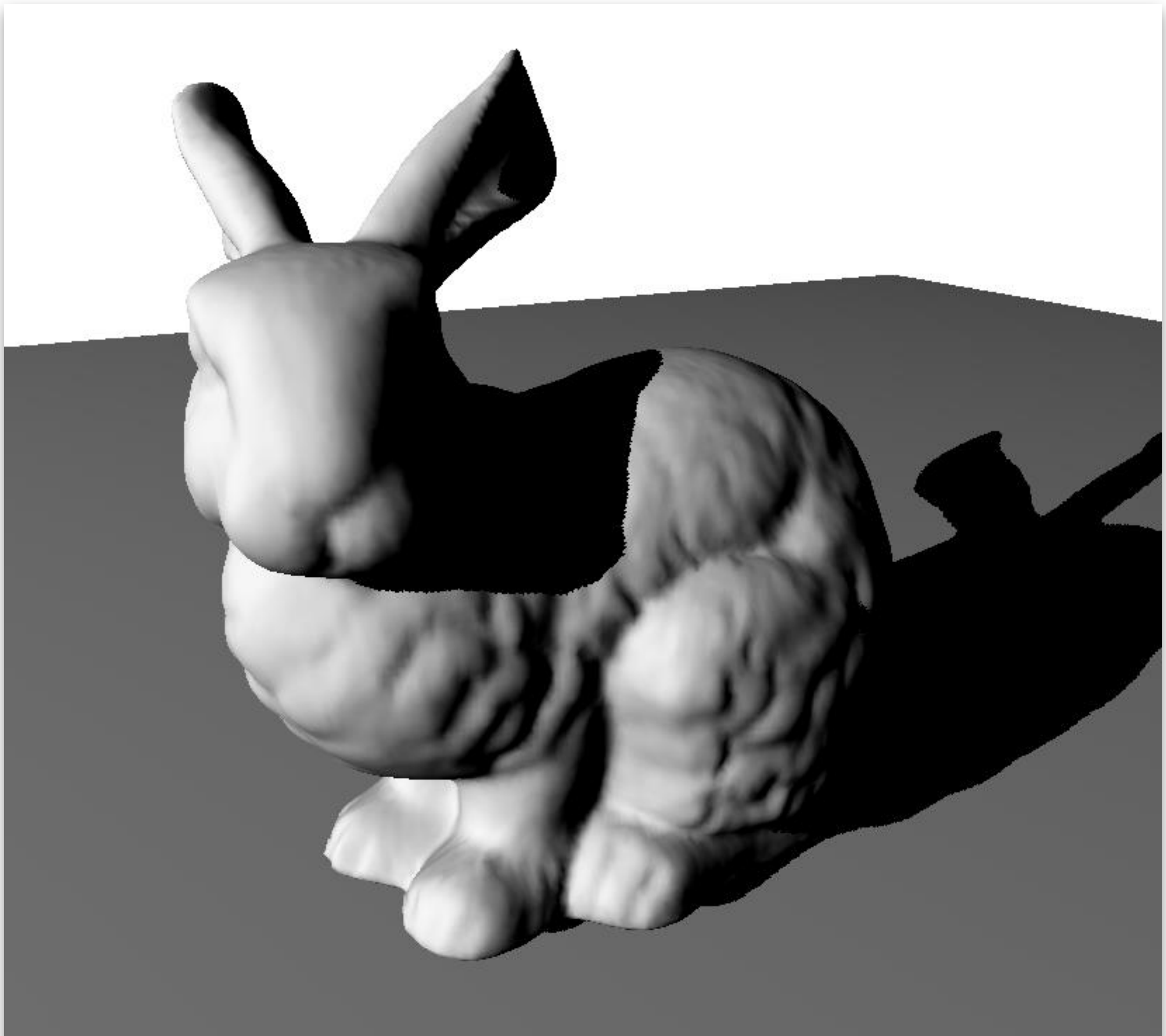
- Render scene from light source
- Store depth buffer

Render scene from camera

- Project fragment to depth buffer/light source
  - If occluder in front → dark
  - Otherwise → bright



Example Result



Shadow Maps Pitfalls

Offset problem

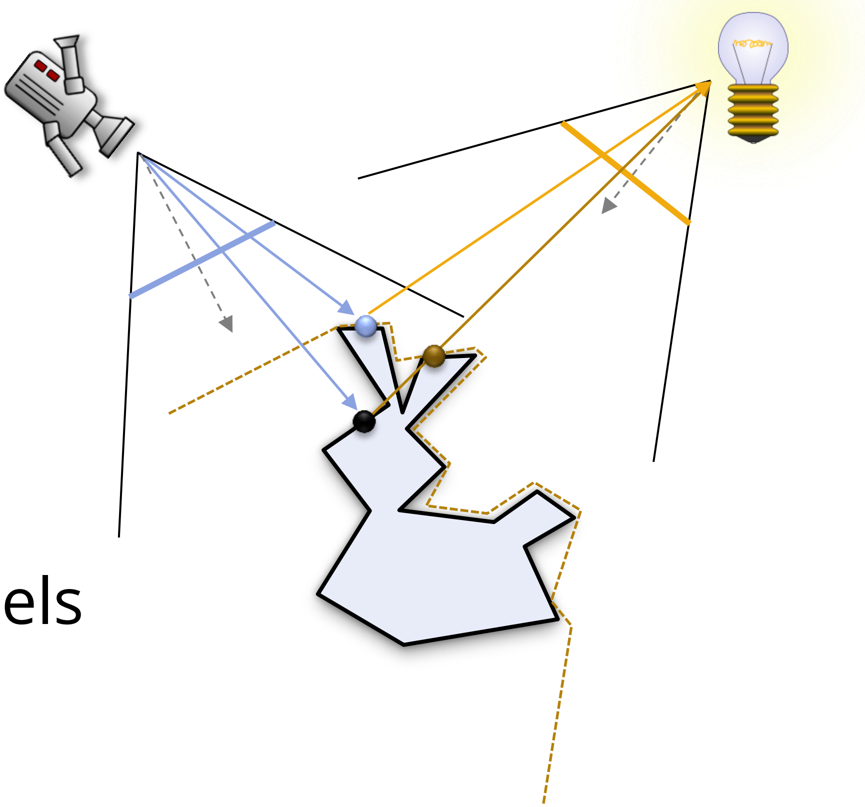
- Camera pixels (slightly) different from light pixels
- Need small offset for depth comparison

Aliasing

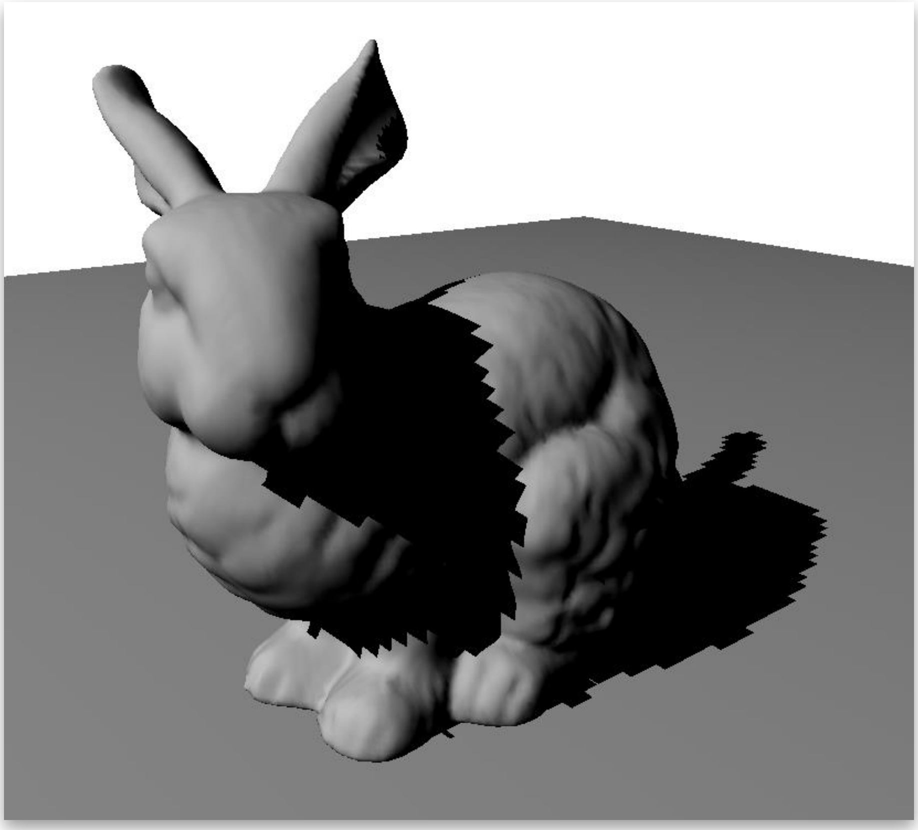
- Visible staircasing
- Light projection ≠ screen pixels

Spot-lights only

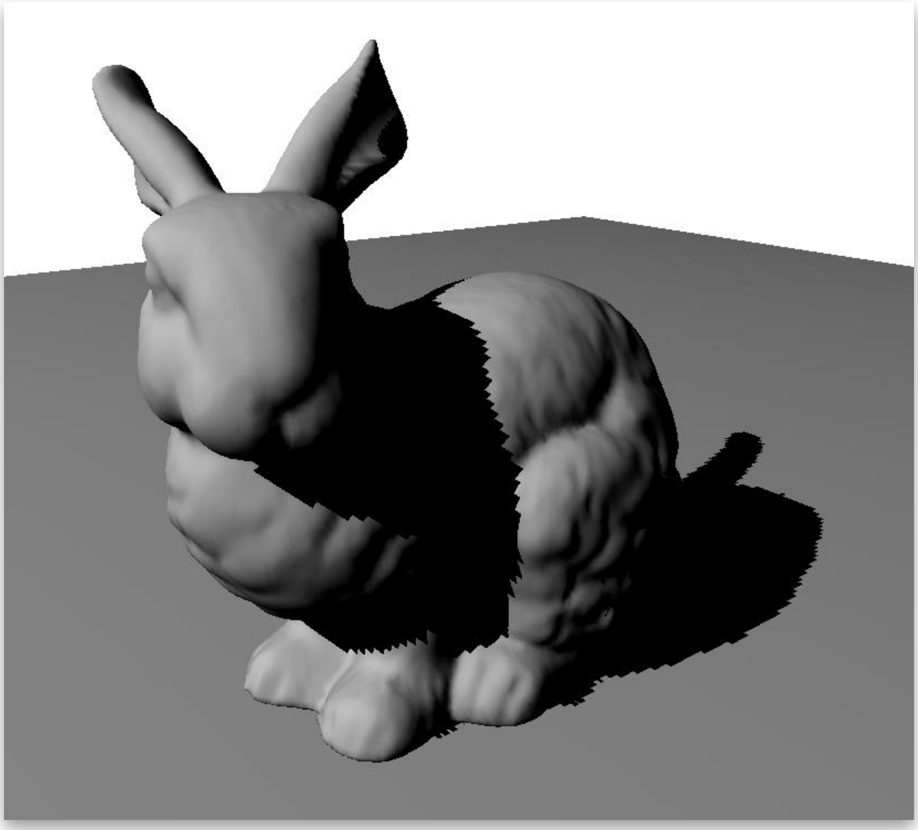
- → Cubemaps (later)



Resolution

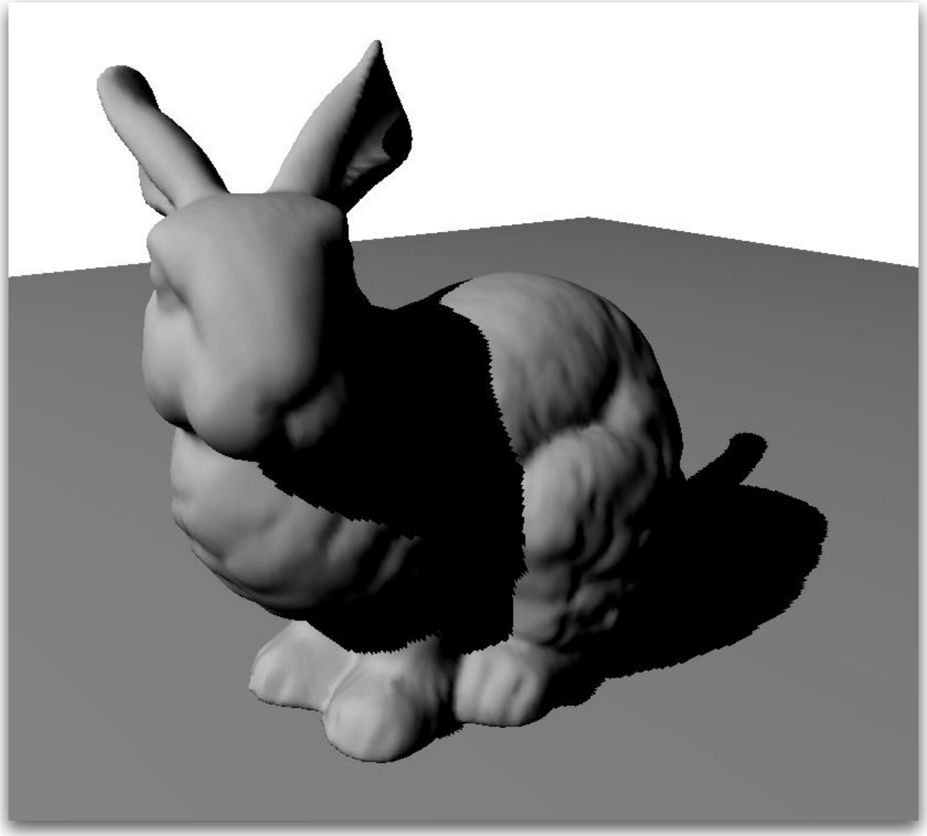


low resolution

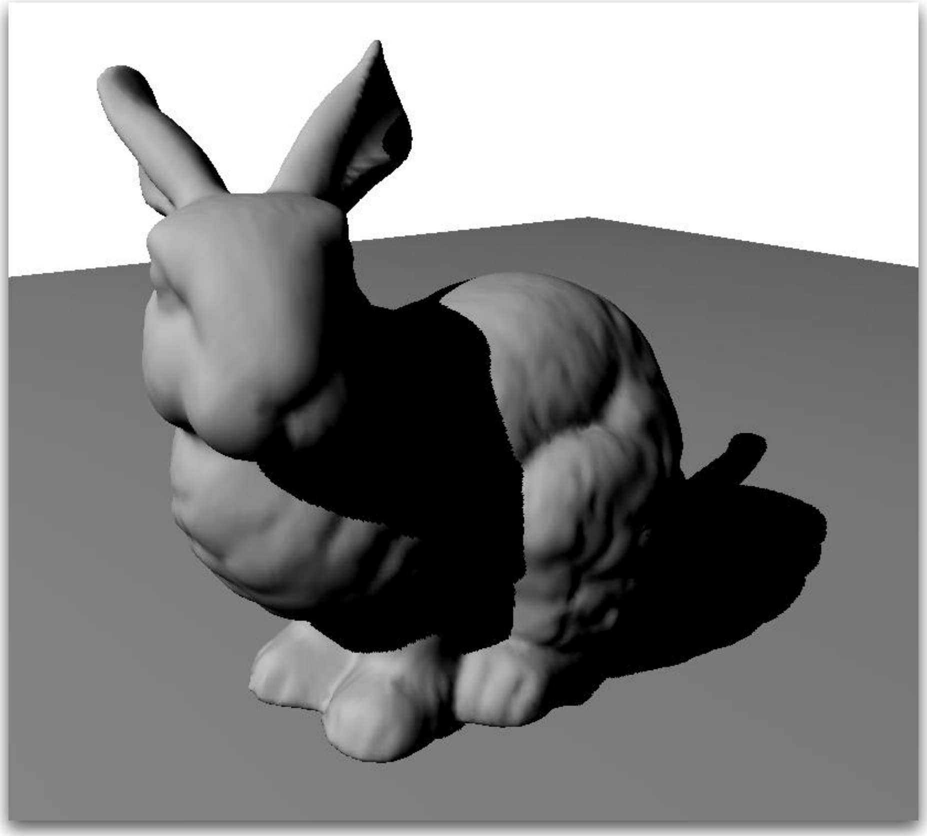


medium resolution

Resolution

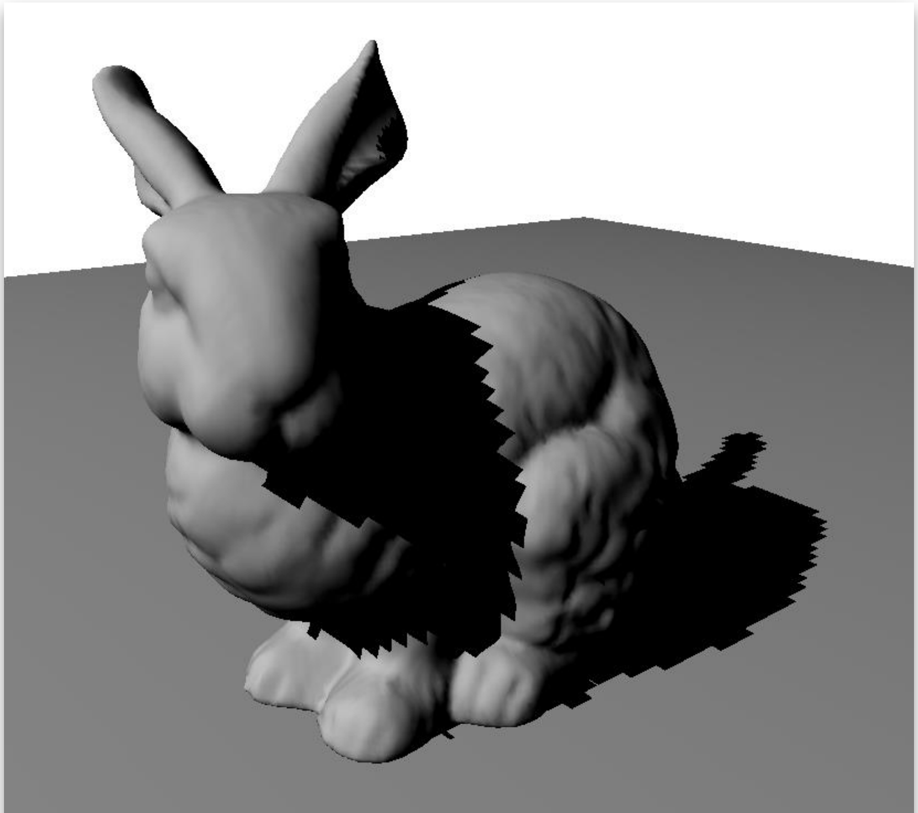


high resolution

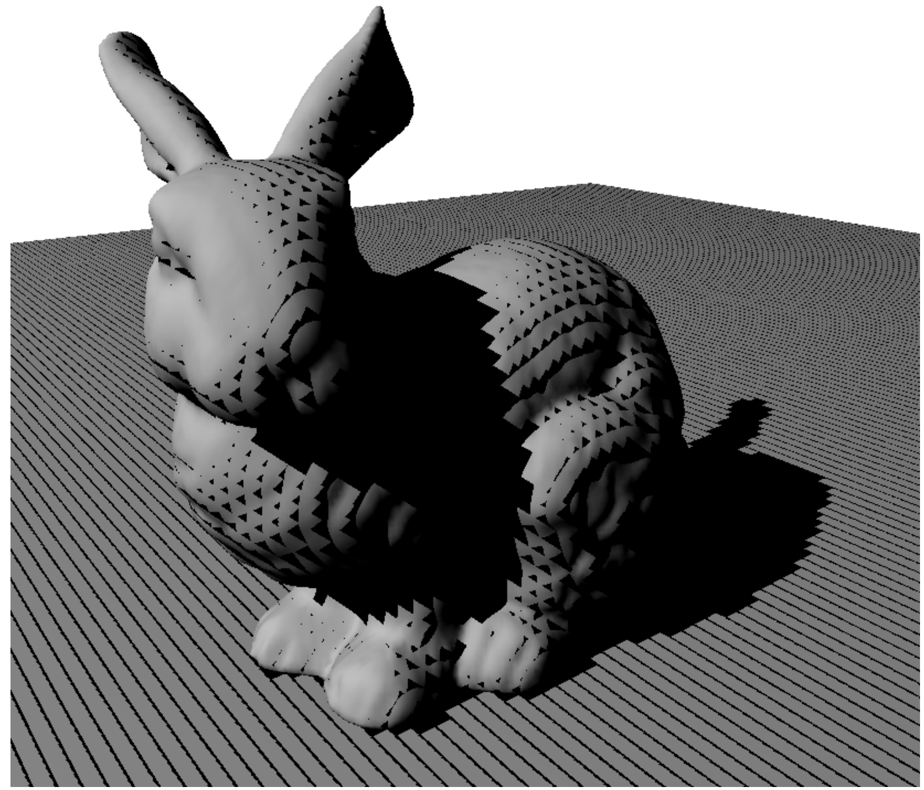


very high resolution

Offset Problem

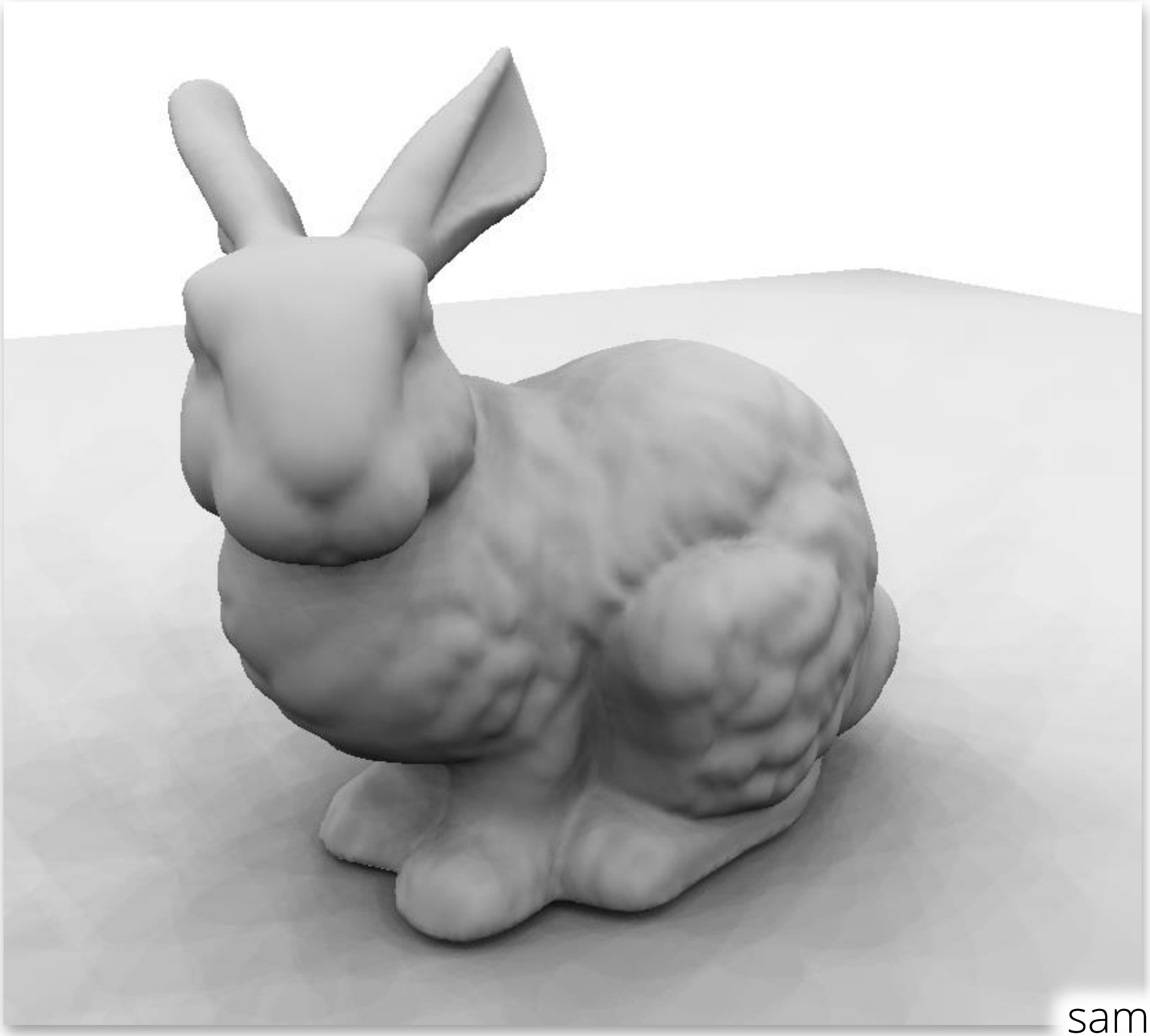


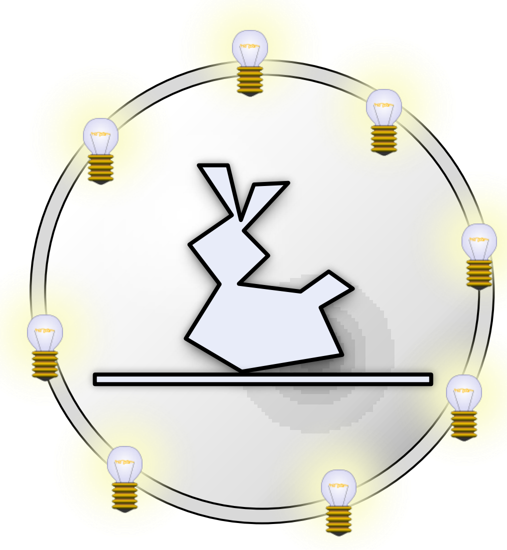
good offset



bad offset

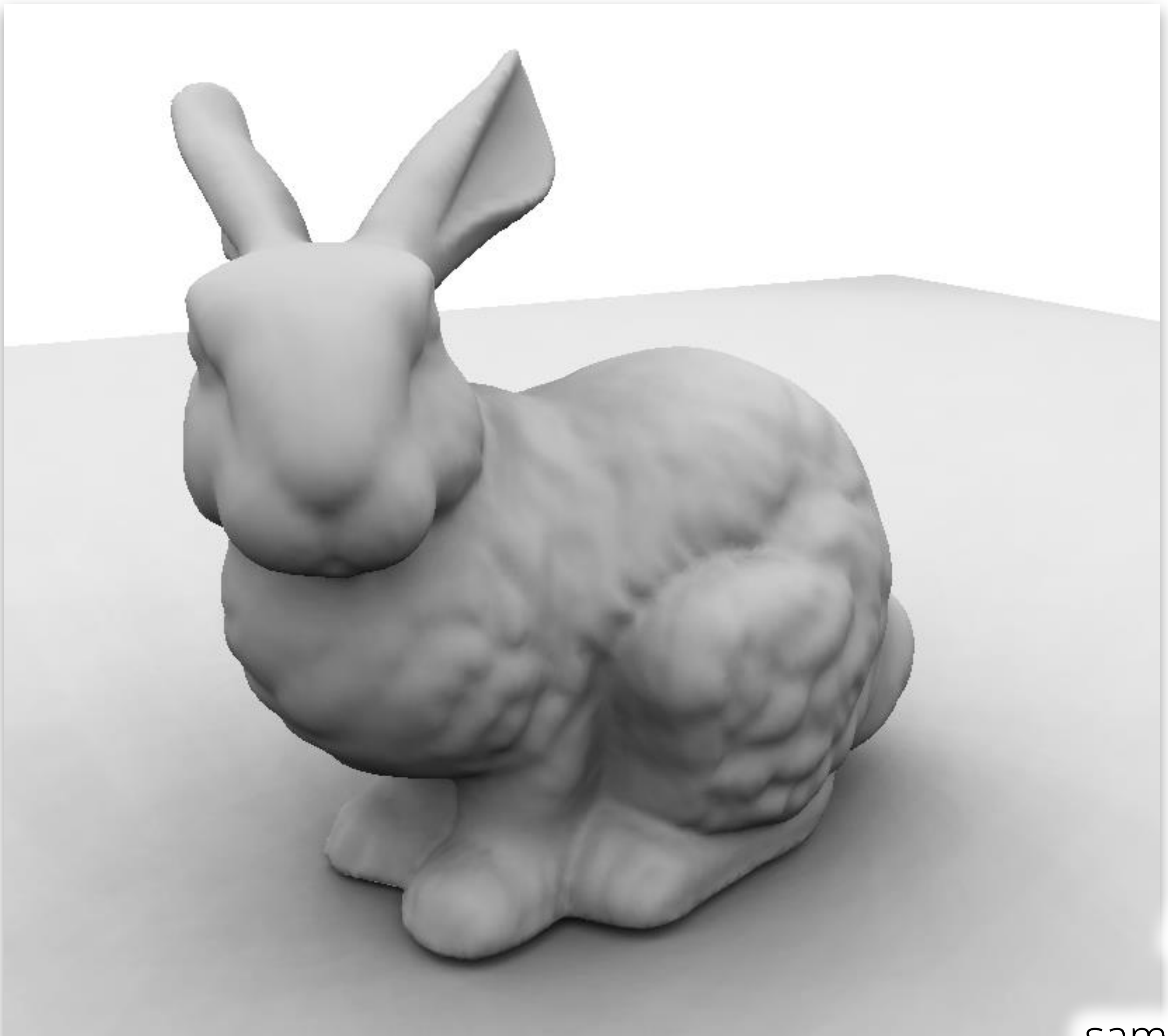
Ambient Occlusion

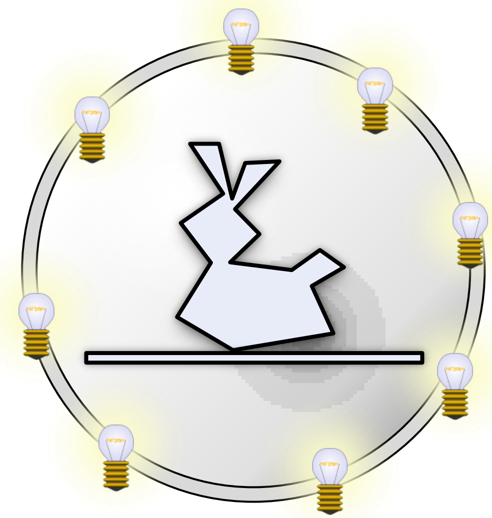




Average of 256 Images
light sources randomly
sampled on enclosing sphere

Ambient Occlusion





Average of 2560 Images
light sources randomly
sampled on enclosing sphere

Environment Maps

Approximate Reflections

- Store panoramic image ("360°") of environment
- Use for reflection

Approximation

- Far away environment
- Single bounce
- No occlusion in path
- Refraction less accurate (single bounce?)

Reflective Bunny
(Environment Mapping)

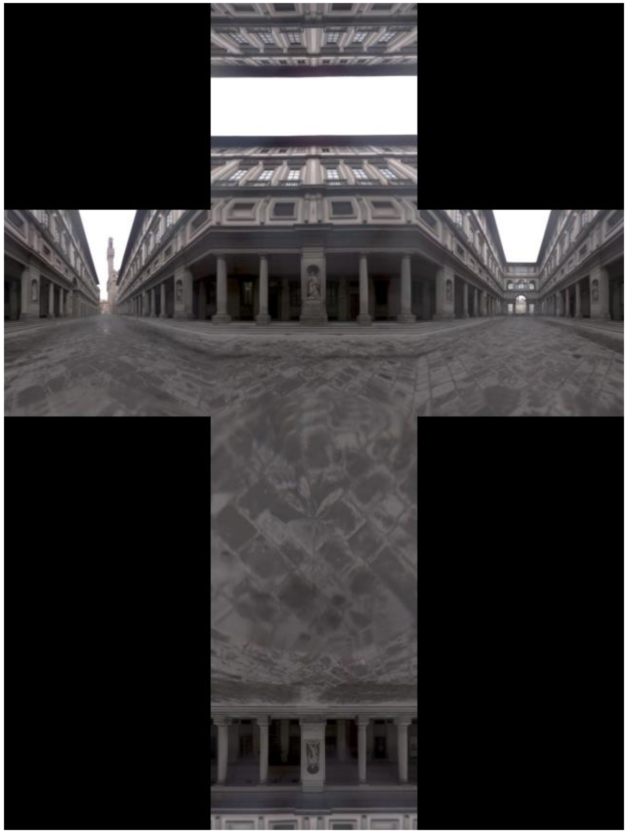
Implementation: Cube-Maps

Cube maps

- Cube with 6 textured faces
- “Infinite size”

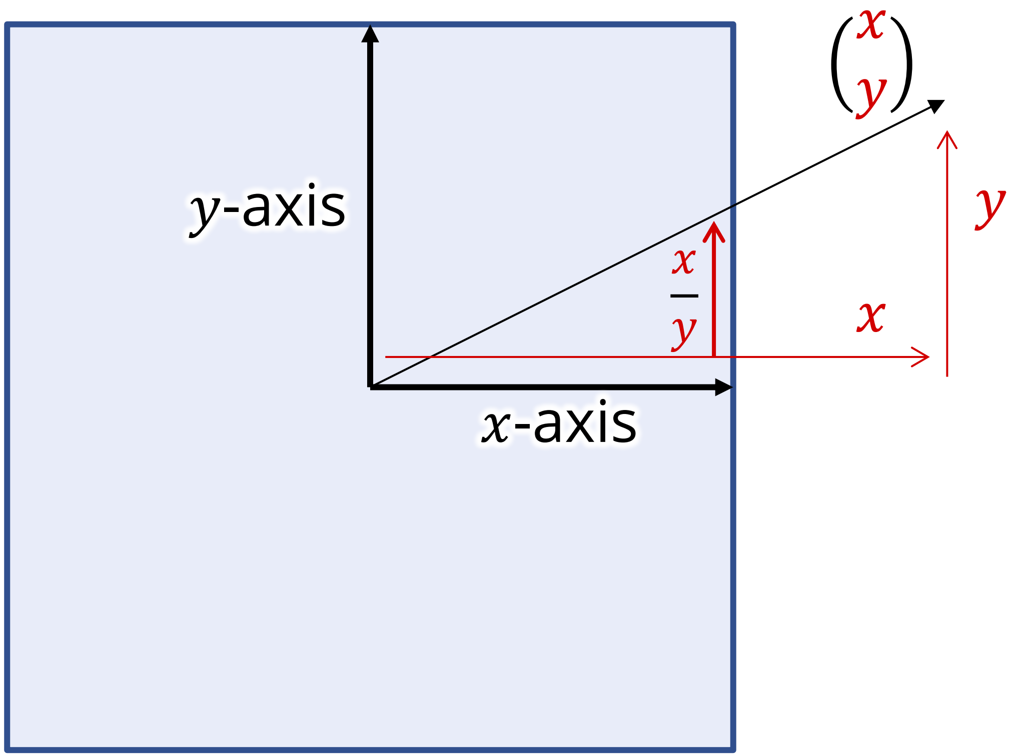
Cube-map texture lookup

- Very easy!
- Arbitrary vector
- texcube $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$
  1. arg max $\{|x|, |y|, |y|\}$
  2. sign
  3. divide by $|\cdot|$
  - Select face: largest entry, sign
  - 2D coordinates: divide by maximum entry



texture (c) Paul Debevec, USC

Cube Map Lookups



$|y|$ largest → horizontal

$y > 0$ → right face

$\frac{x}{y}$ → texture coordinate

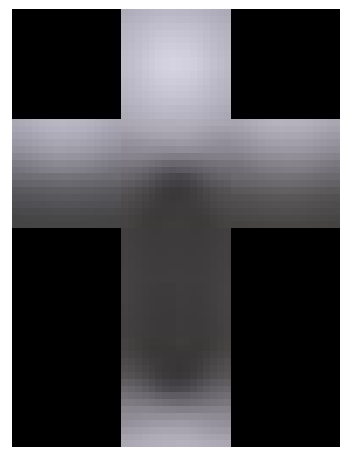
Rendering with Natural Light

Traditional Cube Maps

- Created using rendering
- 6 passes with different camera settings
- Latest hardware: “render to cubemap”

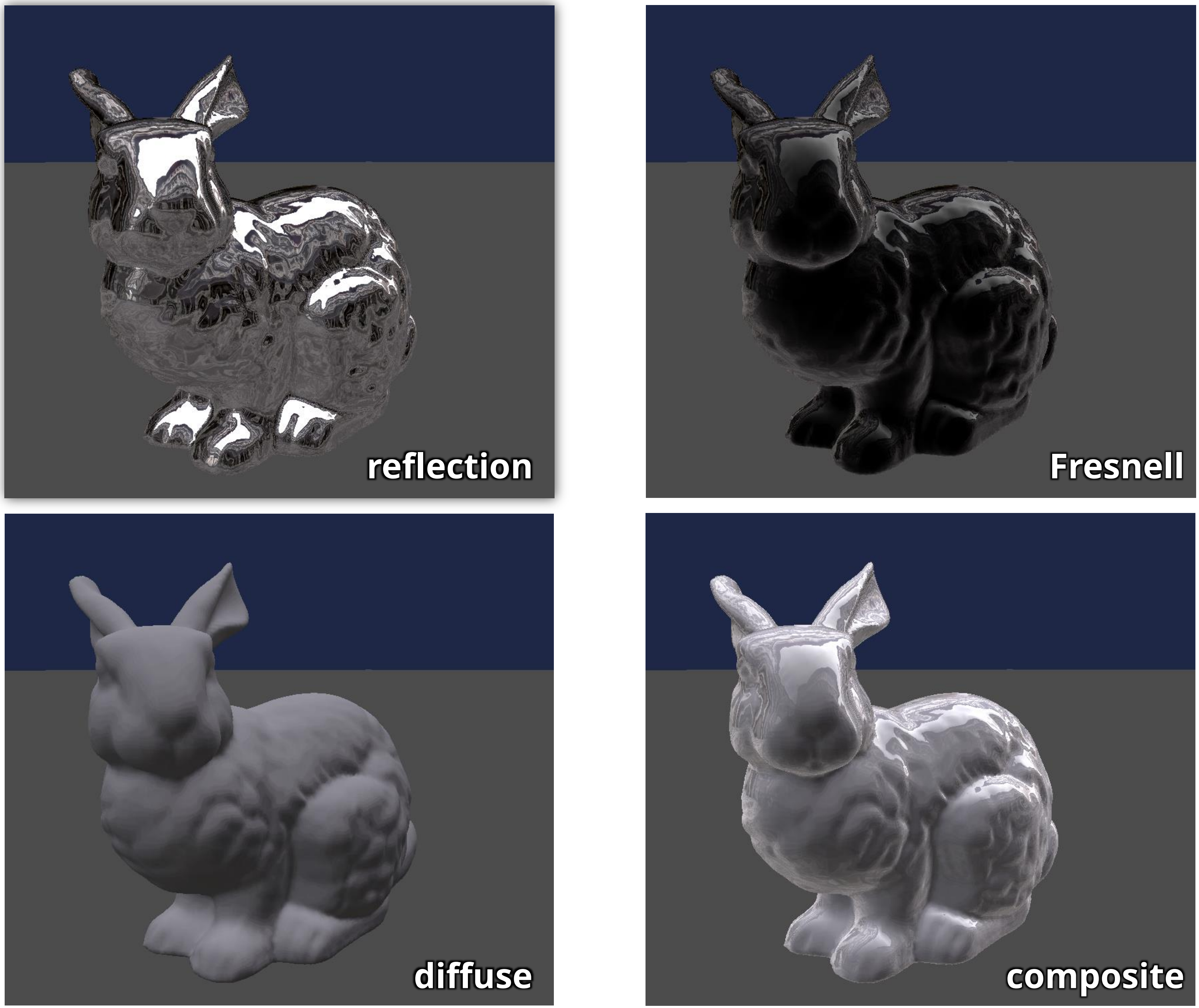
Rendering with Natural Light

- Use HDR photographs for cube maps
  - High-dynamic range is important
- Diffuse, specular, glossy $\Rightarrow$ filter (blur) cube map



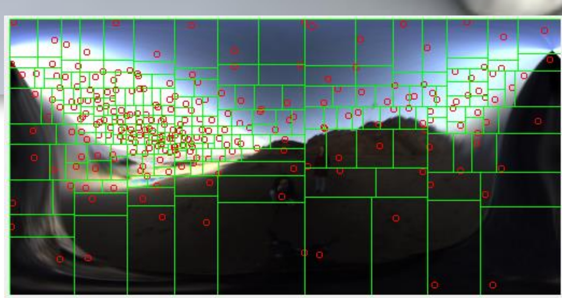
diffuse

specular


reflection
Fresnell
diffuse
composite

Title-Bunny

Ambient Occlusion with Natural Light



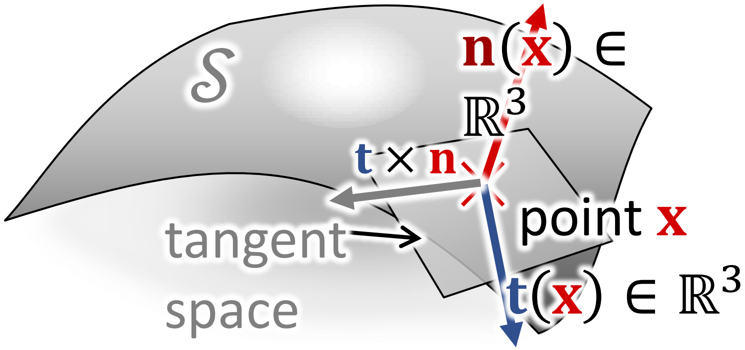
adaptive sampling
density ~ intensity

texture (c) Paul Debevec, USC

Bump-Maps / Normal Maps

Normal Maps

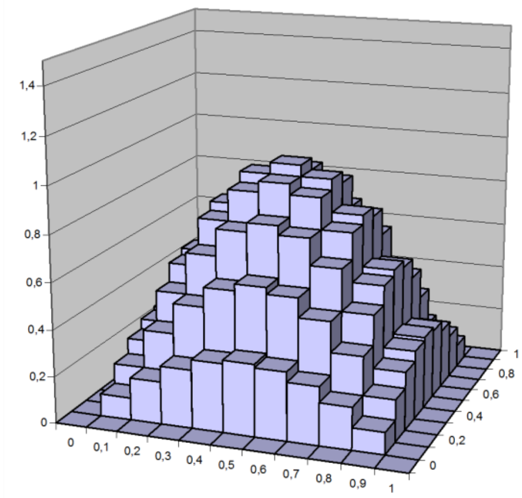
- Store normal in texture
- Map to tangent coordinate frame
- Need normals **n**
  and tangent field **t**
- Then: coordinate transform



Bump Maps

Bump Maps

- Same as normal maps
- But only height field given
- Need to precompute normals



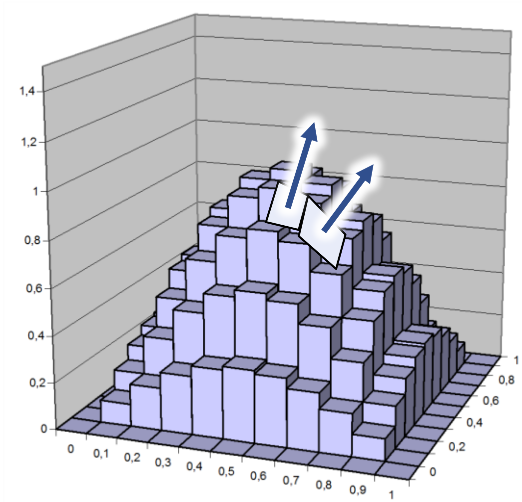
Precompute Normals

- Discrete partial derivatives

$$\Delta x := \frac{f(x+1,y) - f(x)}{h}$$

$$\Delta y := \frac{f(x,y+1) - f(x)}{h}$$

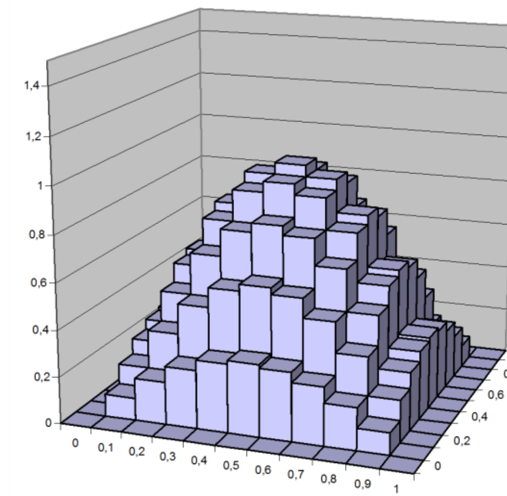
$$n \approx \begin{pmatrix} -\Delta x \\ -\Delta y \\ 1 \end{pmatrix}$$



Displacement Maps

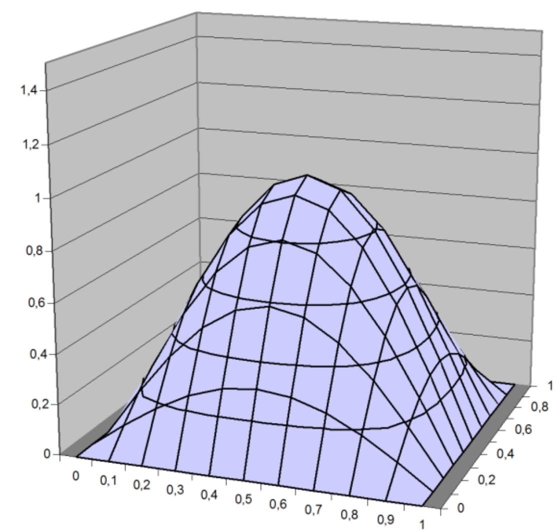
Simplest Method

- Start with Bump Map
- Create actual mesh by displacement in normal direction



Needs powerful hardware

- Fancy in the past few years for real-time / games
- Offline rendering used this ever since

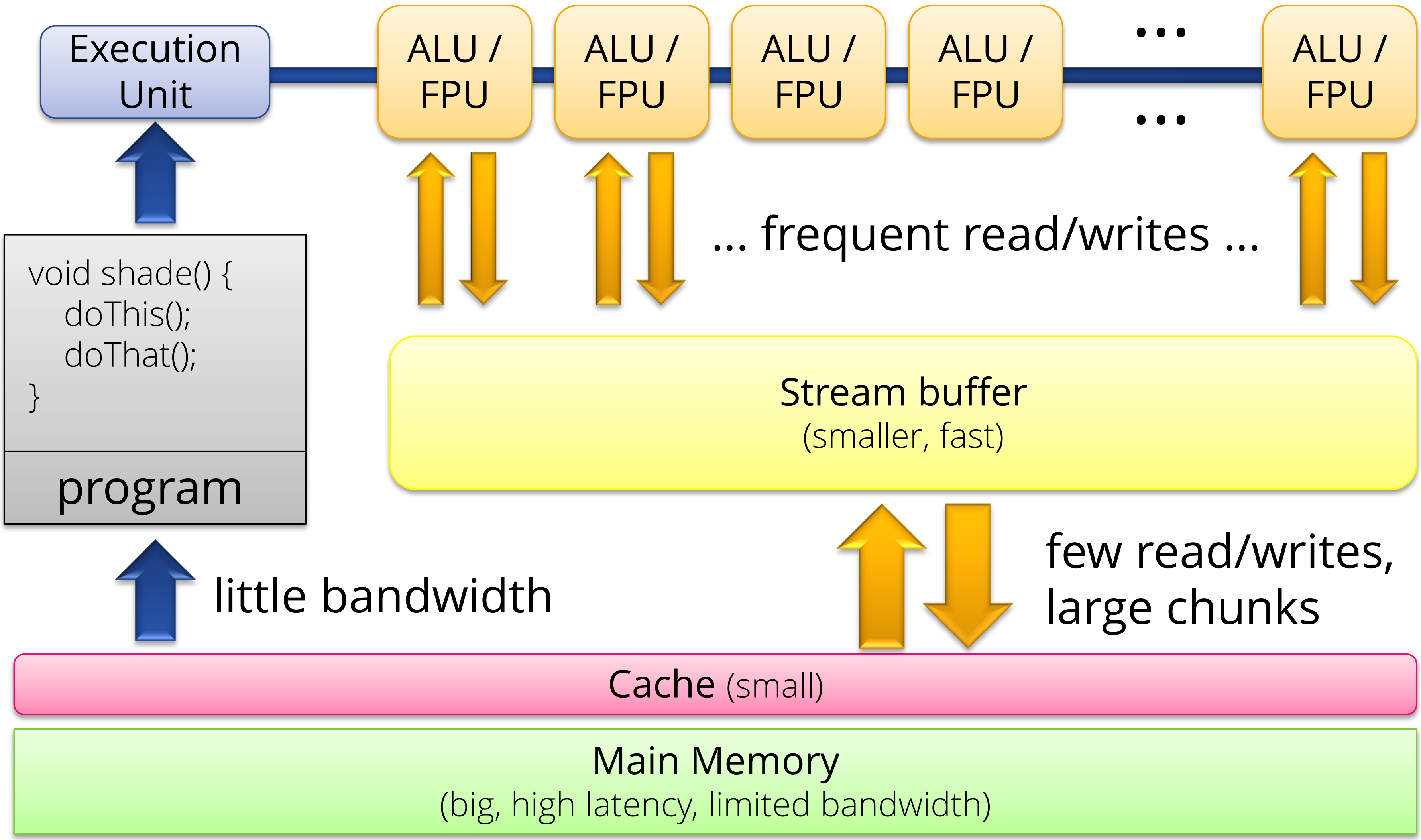


Hardware Architecture

CORE

core topics
important

Simplified GPU Model



Simplified GPU Model

A GPU is a vector processor

- SIMD – single instruction, multiple data
- “Branching” possible at costs
  - Divergent instructions executed serially (write masks)

Specifically: Stream processor

- Memory interface main problem
- Idea: big on-chip buffer, work within buffer
- Write to/from memory more rarely
  - Direct access still possible (“texture fetches”)
  - Caching, hide latency using multi-threading

Advantages

High throughput

- Geforce GTX Titan Z: 8 TFlops (32-bit SP - 2 chips)
- Radeon R9 295x2: 11 TFlops (32-bit SP - 2 chips)
- Xeon Phi 7120: 2.5 TFlops (32-bit SP)
- Dual Xeon E5-2697: 500 GFlops (32-bit SP)

*) example numbers! architectures have different advantages

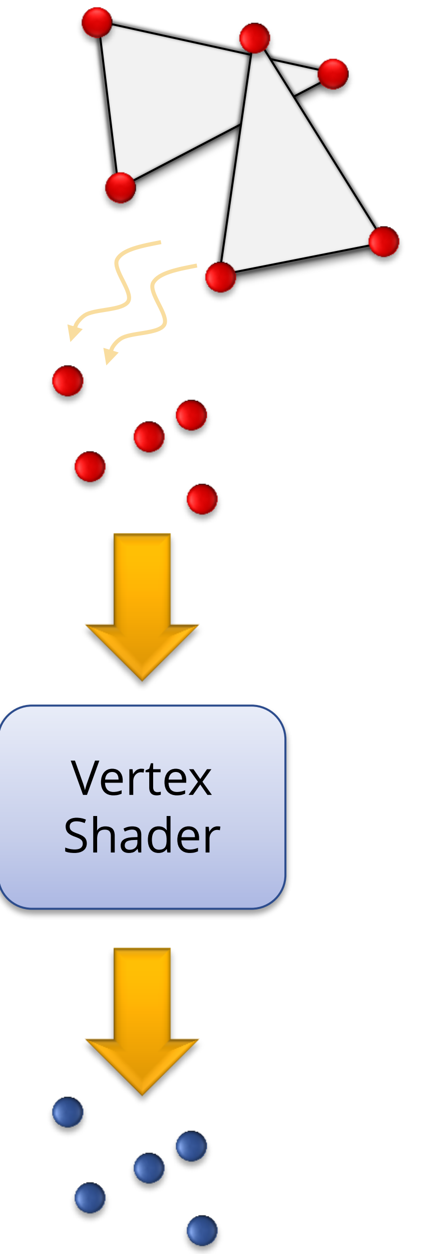
Not all workloads

- Parallel problems
  - Not all problems are parallelizable!
- Instruction stream coherence
  - Additional constraints over MIMD computers!

Hardware Architecture

Programmable Vertex Shaders

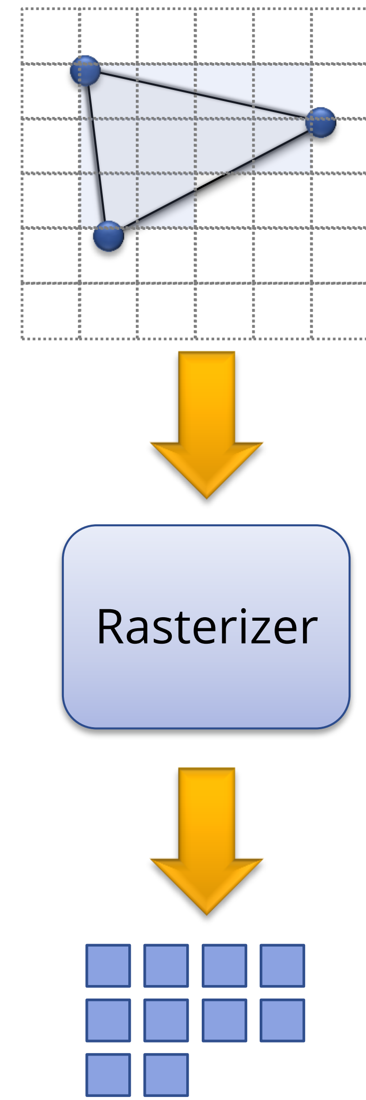
- Input: Vertex Buffer
  - Multiple attributes
  - Position, color, normals, texture coordinates, etc…
- Execute program
  - Texture reads possible
- Output: one vertex per vertex
  - One-in-one-out
  - Internal queue (efficiency)



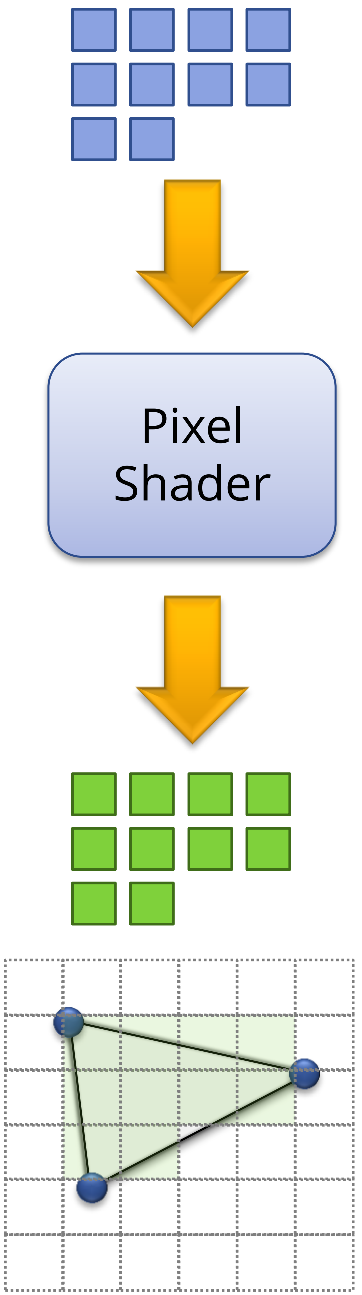
Rasterizer

Rasterizer

- Clipping of triangles
- Creation of fragments
- Interpolate attributes
  - With perspective correction!
- Hard-wired for efficiency
- Output: Long sequence of fragments



Pixel Shader



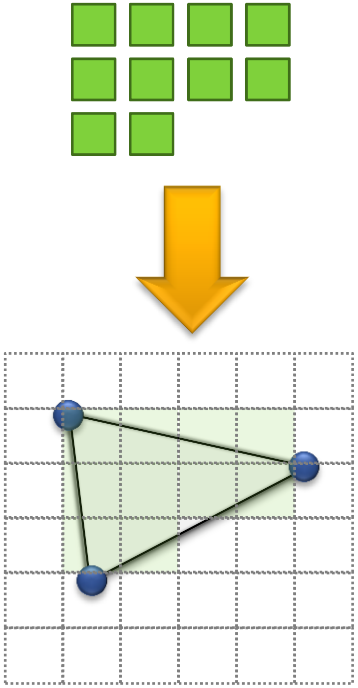
Pixel shader

- Input: Fragment
  with interpolated attributes
- Perform computation
  - Arithmetics
  - Texture reads:
    tex2D, tex3D, texCube,...
  - Includes filtering (anti-aliasing)
- Output:
  - Color (always)
  - Alpha-value (optional)
  - Depth (optional, reduces efficiency;
    no early fragment rejection)

Final Combiner

Write to frame-buffer

- Depth test and update
- Color update
  - Overwrite,
    additive, subtractive,
    alpha-blending
    (and a few more)
- (Usually) hardwired



Recent Extensions

Geometry shader

- Between vertex shader and rasterizer
  - Convert single primitive into a small number of additional ones
  - Amount limited (e.g., 32 vertices output)

Hull shader / tessellation shader

- Better adaptive subdivision
  - Spline surfaces
  - Displacement mapping