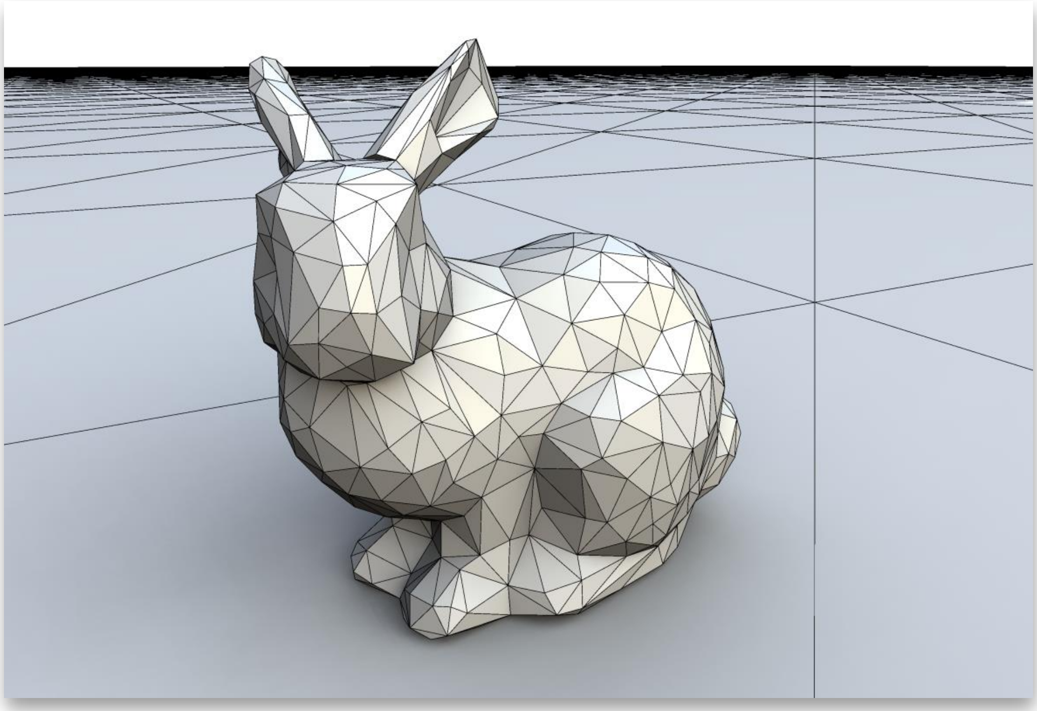
Graphics 2014



3D Modeling

Addendum

[Faculty of Science]
Information and
Computing Sciences

Universiteit Utrecht

Motivation

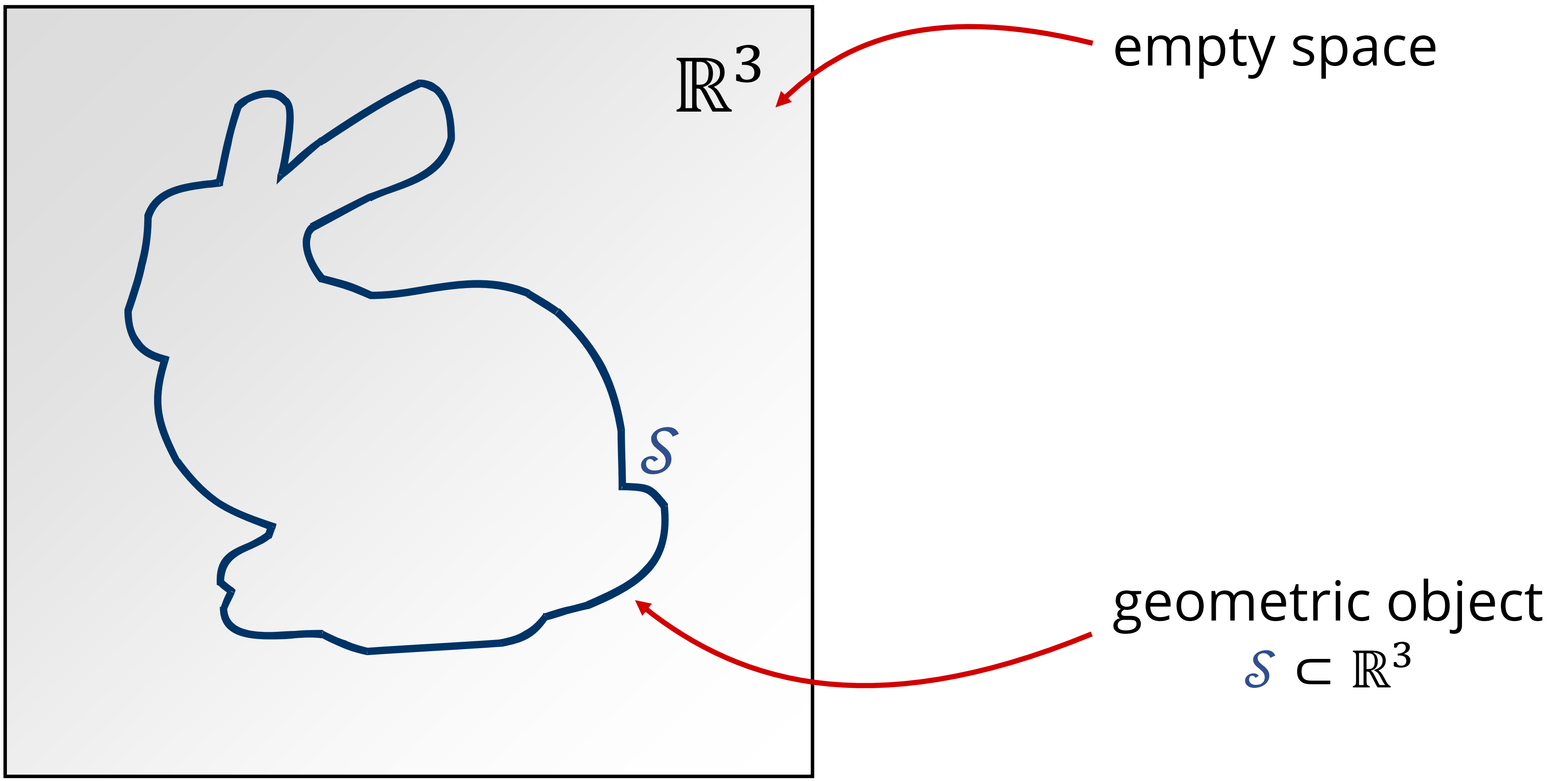
BASIC

basic topics

study completely

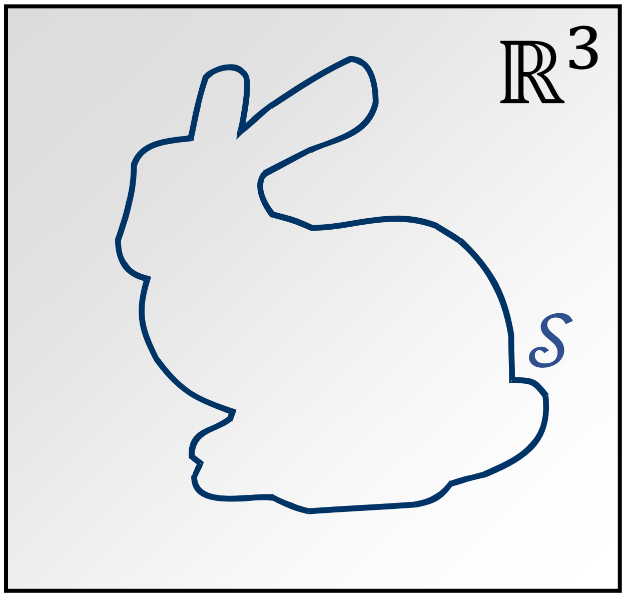
Geometric Modeling

What do we want to do?

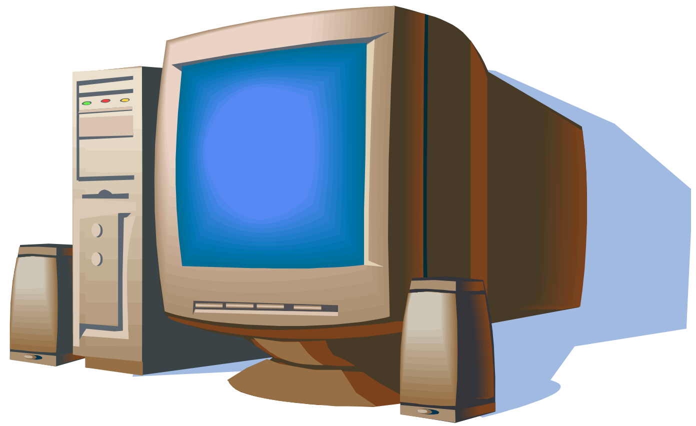


Fundamental Problem

The Problem:



infinite number of points



my computer: 16GB of memory

Encode a continuous model with
finite amount of information

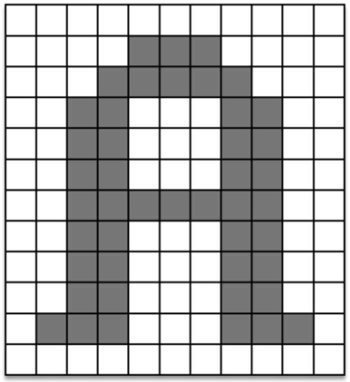
Modeling Zoo

CORE

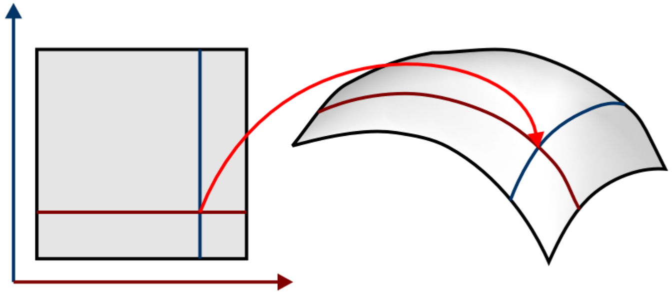
core topics
important

Modeling Zoo

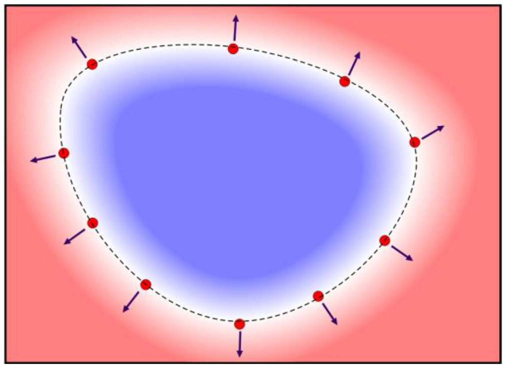
Space Discretization



Parametric Models



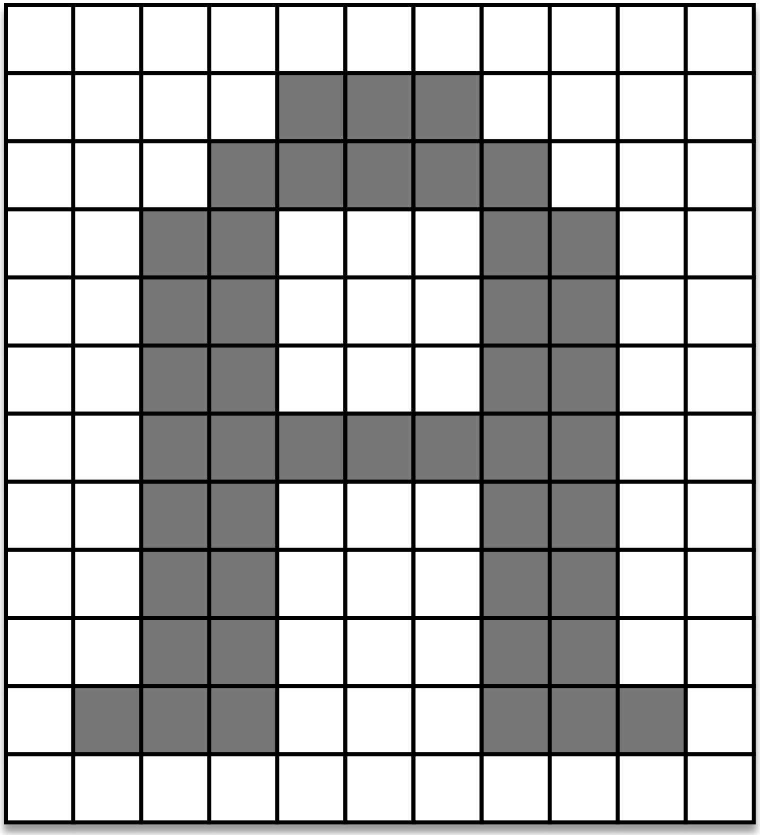
Implicit Models



Discrete Representations

You know this…

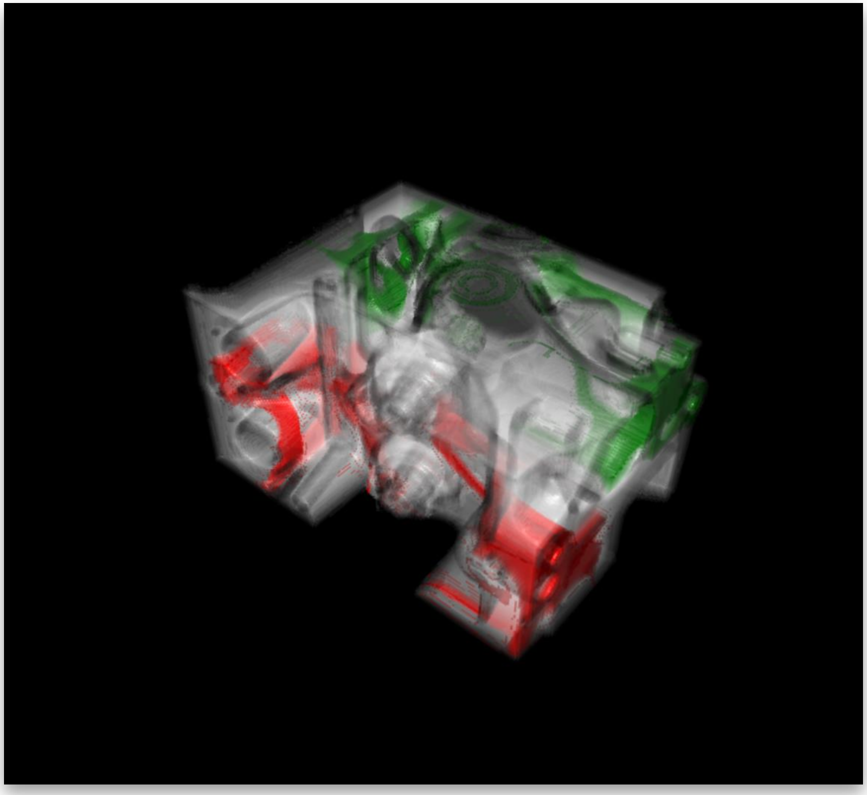
- Fixed Grids
  - 2D: Images
  - 3D: “Voxels”

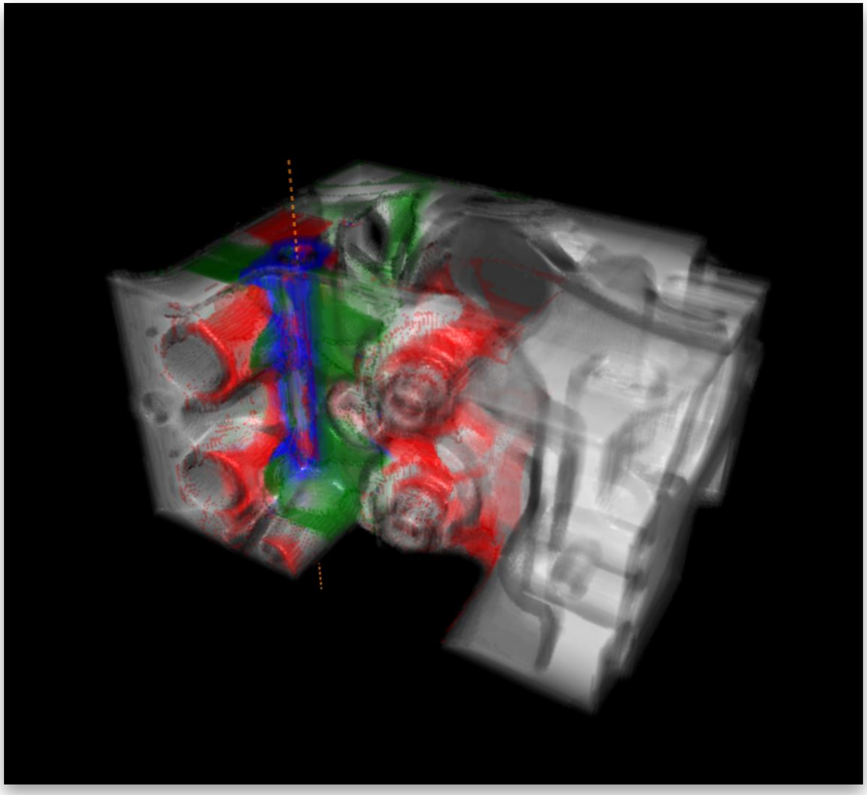


Voxel Grid

Voxels (Direct Volume Rendering)

- Input: CT-Scan





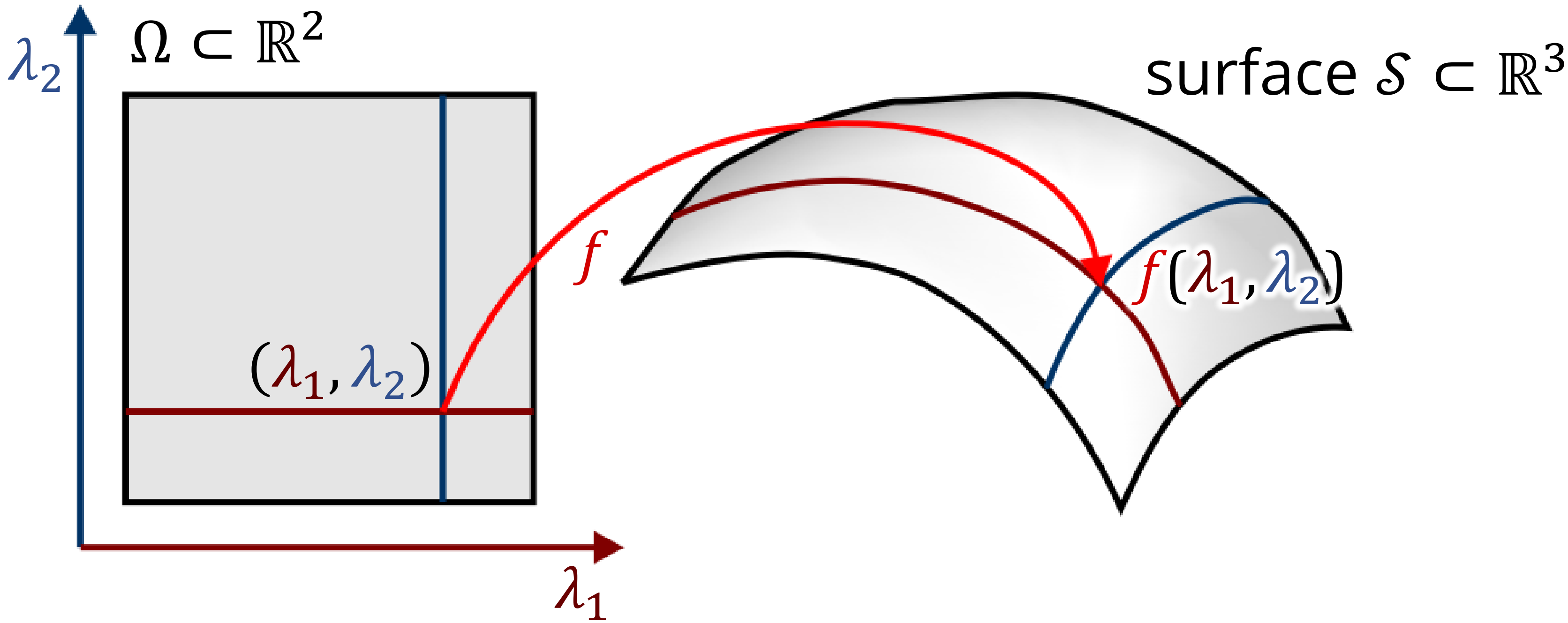
Parametric Models

$$\mathbf{x}(\lambda_1, \ldots, \lambda_k) = f(\lambda_1, \ldots, \lambda_k)$$

CORE

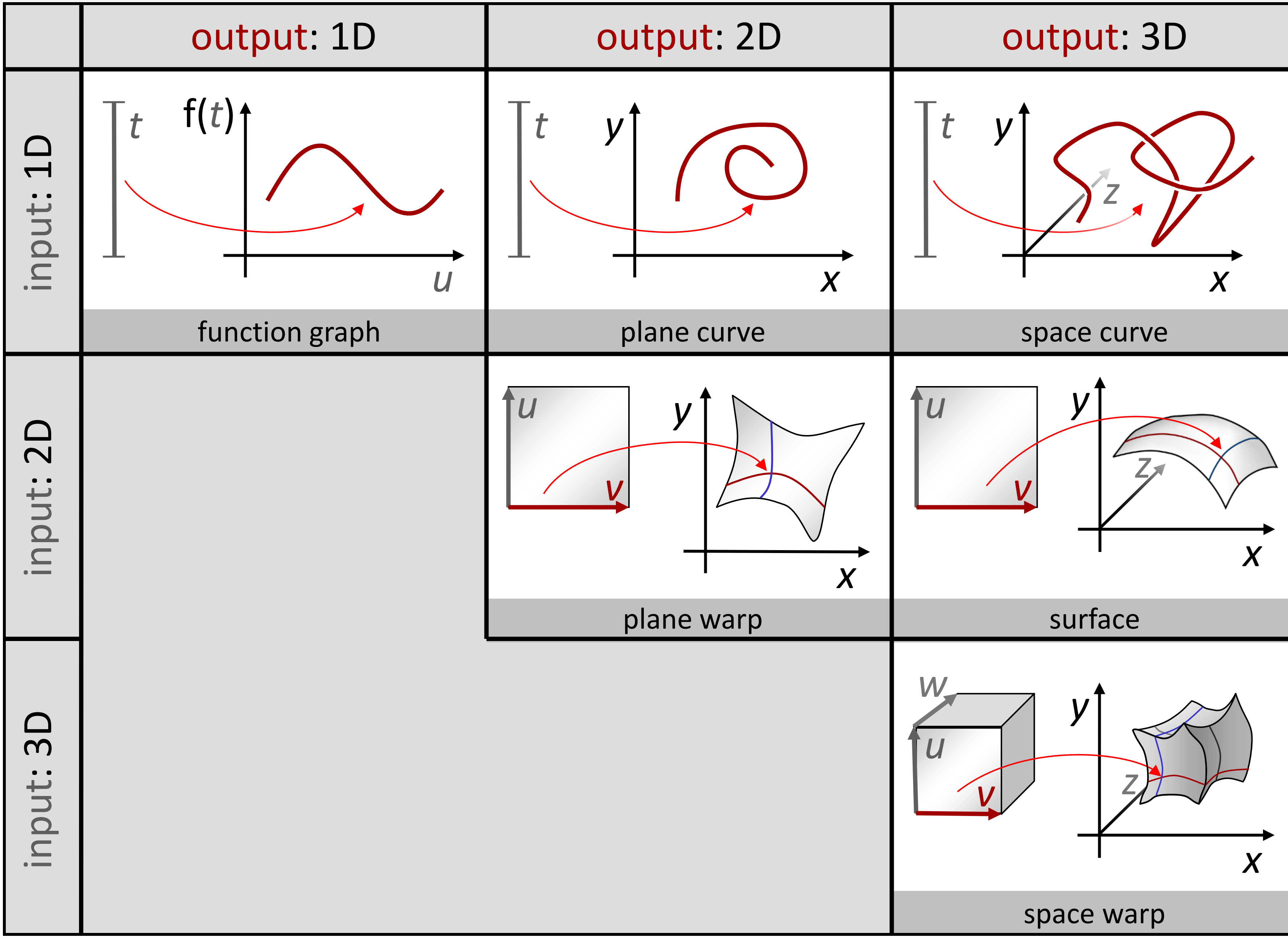
core topics
important

Parametric Models



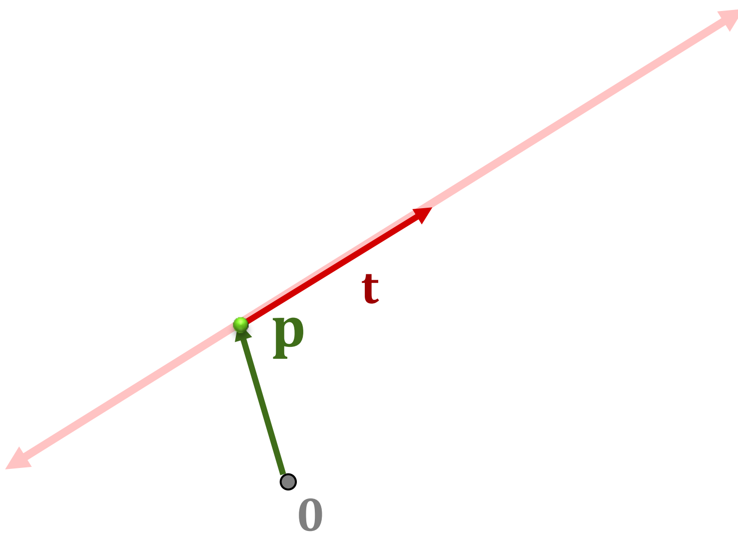
Parametric Models

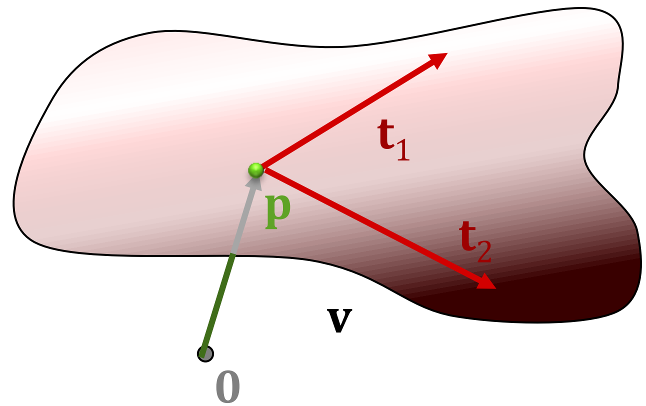
- Function f maps from *parameter domain* to target surface
- Evaluation of f gives one point on the model



output: 1D
output: 2D
output: 3D
input: 1D
input: 2D
input: 3D
t
f(t)
u
function graph
t
y
x
plane curve
t
y
z
x
space curve
u
v
y
x
plane warp
u
v
y
z
x
surface
w
u
v
y
z
x
space warp

Affine Parametric Functions





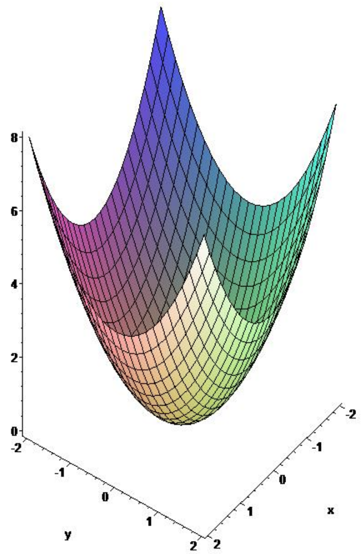
Parametric line equation:

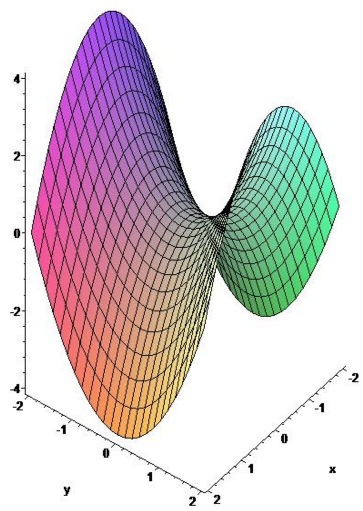
$$\mathbf{x}(\lambda) = \mathbf{p} + \lambda \cdot \mathbf{t}$$

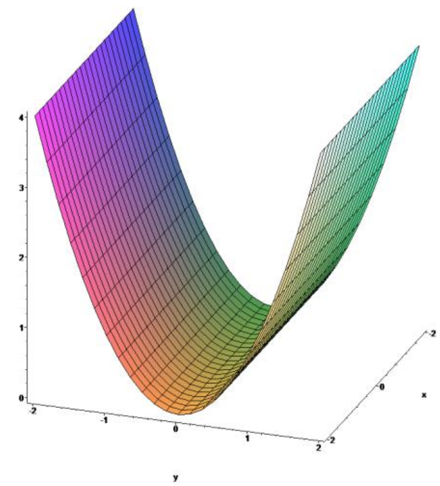
Parametric plane equation:

$$\mathbf{x}(\lambda_1, \lambda_2) = \mathbf{p} + \lambda_1 \mathbf{t}_1 + \lambda_2 \mathbf{t}_2$$

Quadratic Parametric Functions







Quadratic Surface:

$$\mathbf{x}(\lambda_1, \lambda_2) = \mathbf{p} + \lambda_1 \mathbf{t}_1 + \lambda_2 \mathbf{t}_2 + (\lambda_1, \lambda_2)\mathbf{Q}\begin{pmatrix}\lambda_1 \\ \lambda_2\end{pmatrix}$$

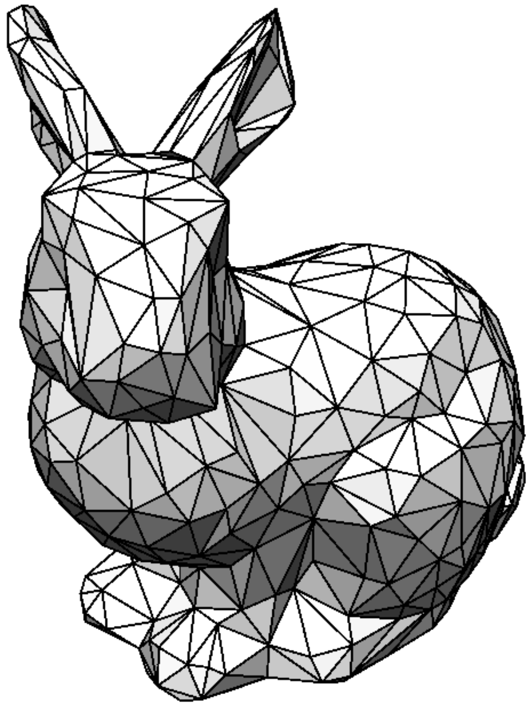
Non-linear curves & surfaces:

- Polynomial surfaces, splines
- More in “three-dimensional modeling” lecture

Primitive Meshes

Primitive Meshes

- Collection of geometric primitives
  - Triangles
  - Quadrilaterals
  - More general primitives
    (e.g. spline patches)
- Primitives are parametric surfaces
- We have used this already!



Implicit Models

$$f(\mathbf{x}) = 0$$

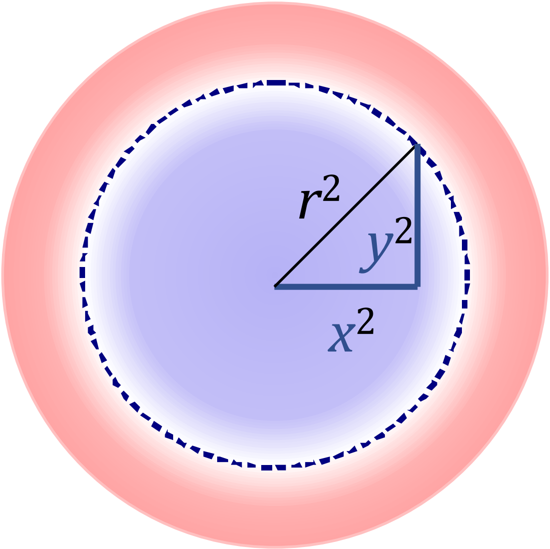
CORE

core topics
important

Implicit Modeling

Implicit formulation:

- Need to solve equation
  $$S = \{\mathbf{x} \mid f(\mathbf{x}) = 0\}$$
- $\mathbf{x} \in \mathbb{R}^2$ or $\mathbb{R}^3$
- S is usually $d - 1$ dimensional



Signed distance function

- Special case
- Function value
  = distance to surface
- Negative ≙ inside, positive ≙ outside

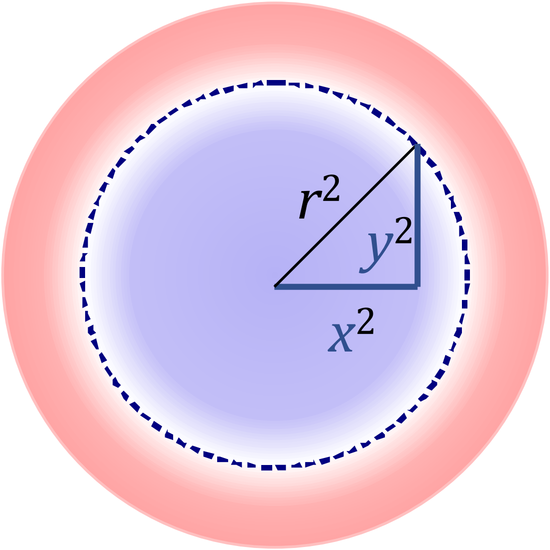
Circle:

$$f(x, y) = x^2 + y^2 - r^2 = 0$$

SDF:

$$f(x, y) = \sqrt{x^2 + y^2} - r$$

Implicit Modeling



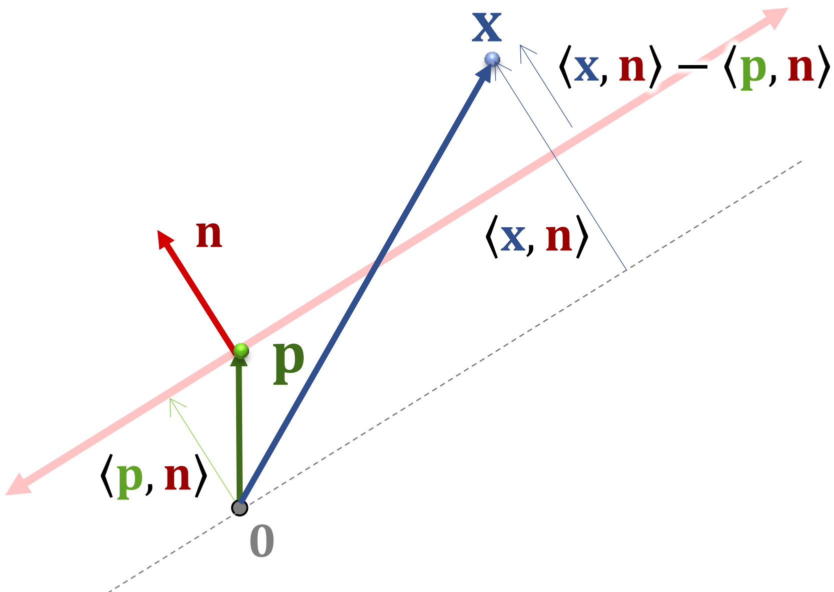
Example:

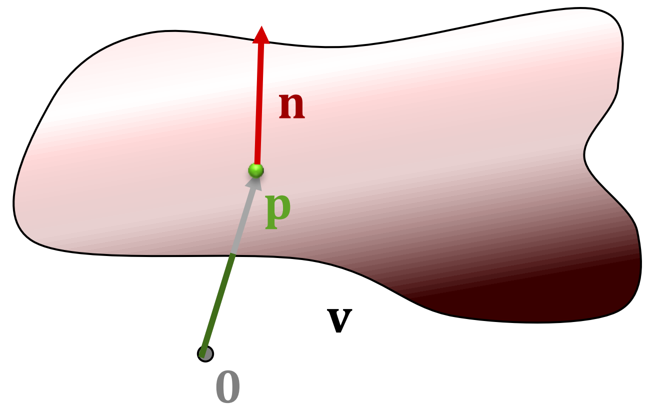
- Circle: $x^2 + y^2 = r^2$
- Implicit function:
  $$f(x, y) = x^2 + y^2 - r^2$$
- Sphere: $x^2 + y^2 + z^2 = r^2$

Special Case:

- Signed distance field
- Function value is signed distance to surface
- Negative means inside, positive means outside

Affine Implicit functions





Parametric line equation:

$$\langle \mathbf{x}, \mathbf{n} \rangle - \langle \mathbf{p}, \mathbf{n} \rangle = 0$$

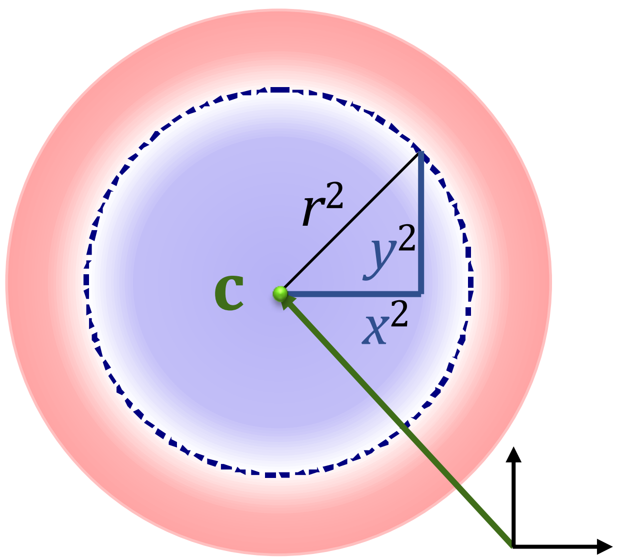
$$\|\mathbf{n}\| = 1$$

Parametric plane equation:

$$\langle \mathbf{x}, \mathbf{n} \rangle - \langle \mathbf{p}, \mathbf{n} \rangle = 0$$

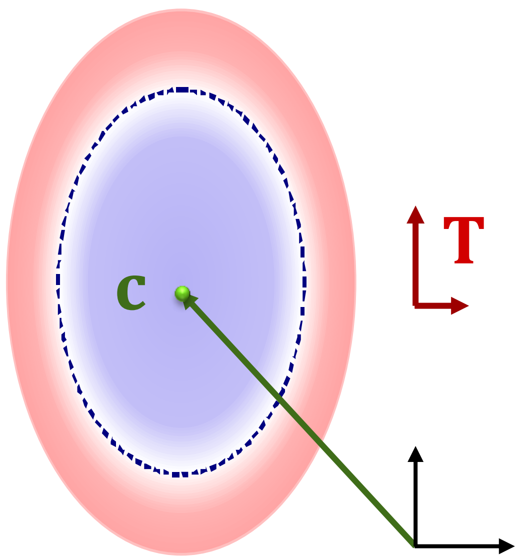
$$\|\mathbf{n}\| = 1$$

Quadratic Implicit Functions



Circle / Sphere:

$$\langle \mathbf{x} - \mathbf{c}, \mathbf{x} - \mathbf{c} \rangle = r^2$$



Ellipse / Ellipsoid

$$\langle \mathbf{T}\mathbf{x} - \mathbf{c}, \mathbf{T}\mathbf{x} - \mathbf{c} \rangle = r^2$$

$\mathbf{T}$: new coordinate system

Implicit Modeling

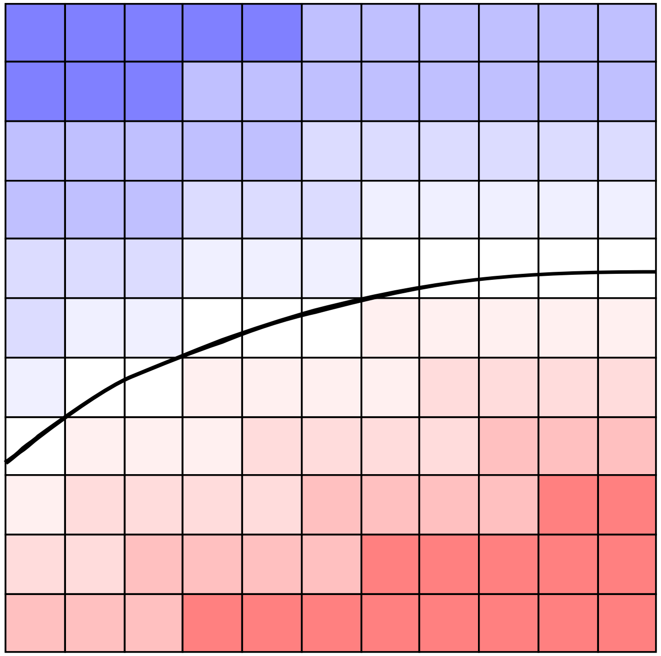
Applications:

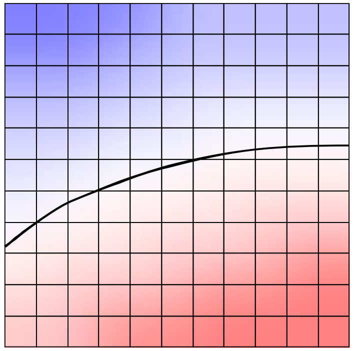
- Fluid simulation
- Surface fitting
- Free-form surfaces (“blobbies”)

Regular Grids

Discretization:

- Regular grid of values
- Interpolate smoothly

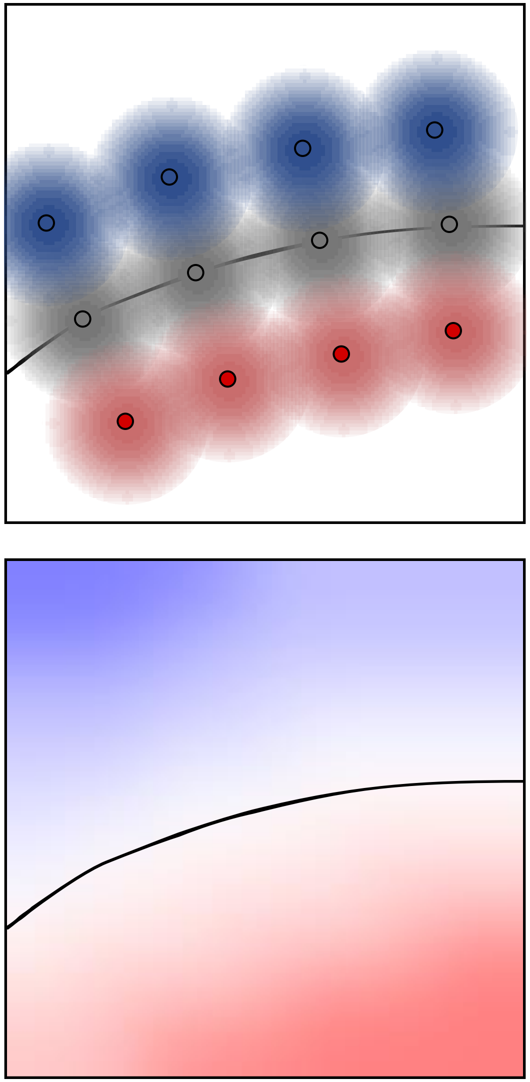




RBF Methods

Radial Basis Functions:

- Particles with smooth radial basis functions



ADV

advanced topics

main ideas