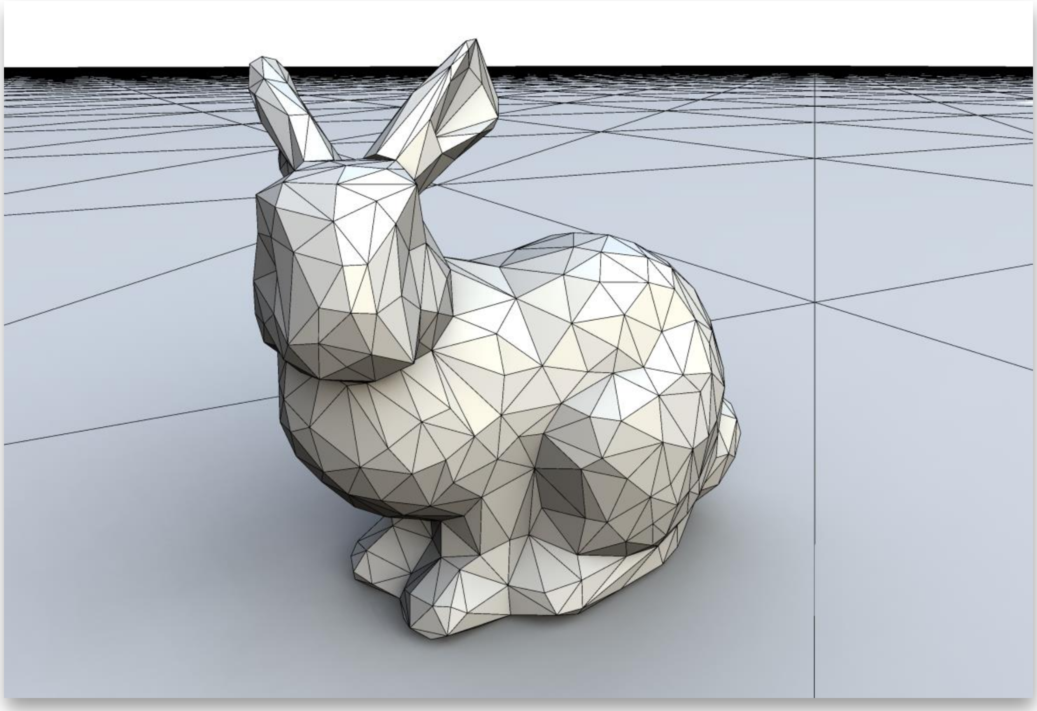
Graphics 2014



Global Illumination

Introduction to the basic concepts

[Faculty of Science]
Information and Computing Sciences

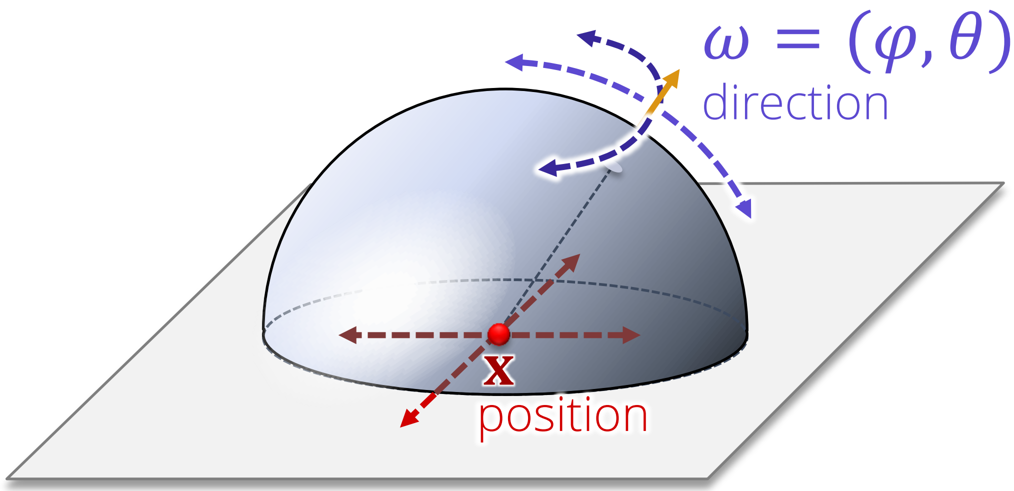
Universiteit Utrecht

Radiance & Light Fields

ADV *)

advanced topics
main ideas

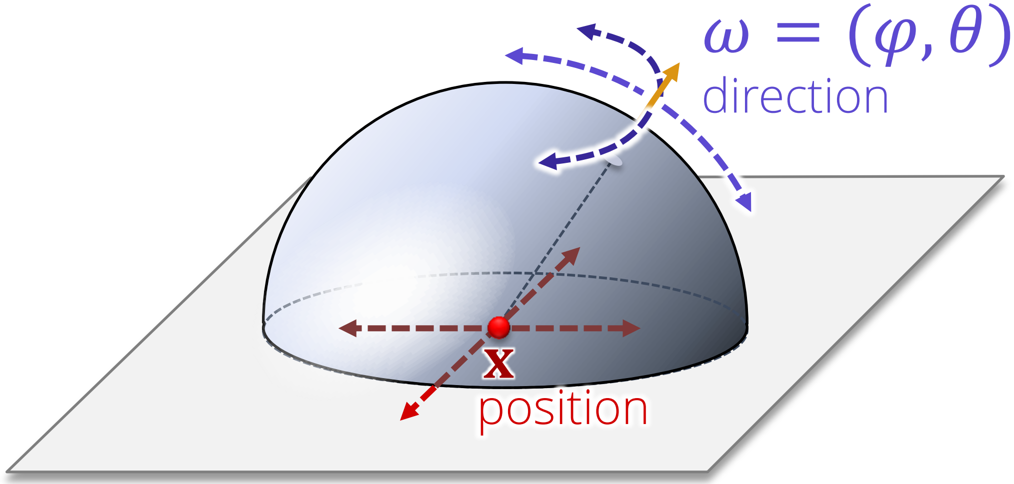
Ray Power Density: Radiance



Radiance

- Light (power) transmitted through a ray
  - Formally: energy density w.r.t. area, solid angle, time
  - For this lecture: think of
    - All rays in space
    - Each ray has an RGB values

Ray Power Density: Radiance



Radiance

- At each point $\mathbf{x} \in \mathbb{R}^3$:

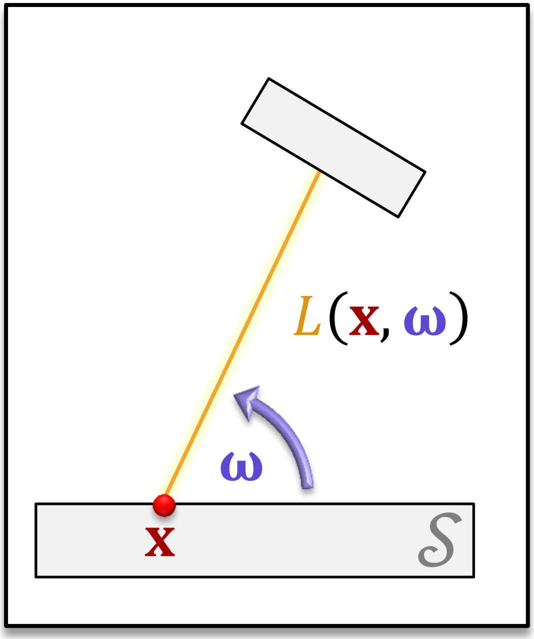
$$L(\varphi, \theta) = L(\boldsymbol{\omega}) \rightarrow \mathbb{R}^3\ (RGB)^{*)}$$

- Light field: all rays in space

$$L(\mathbf{x}, \boldsymbol{\omega}): \mathbb{R}^5 \rightarrow \mathbb{R}^3$$

*) slightly simplified, usually defined for continuous wavelength

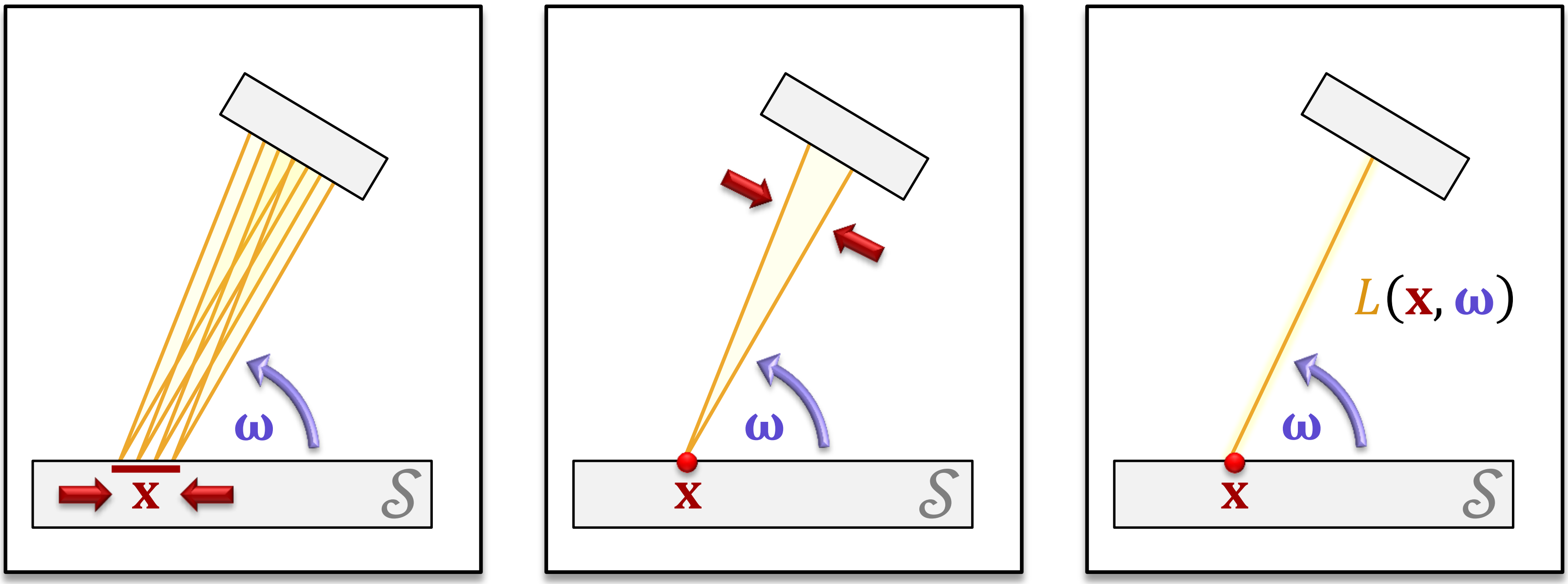
Radiance



Radiance

- Function $L: (\mathcal{S}, \Omega) \rightarrow \mathbb{R}^{\geq 0}$
  - Input variables: surface point $\mathbf{x}$, angle ω
  - Output: power transported through ray

Radiance



Radiance

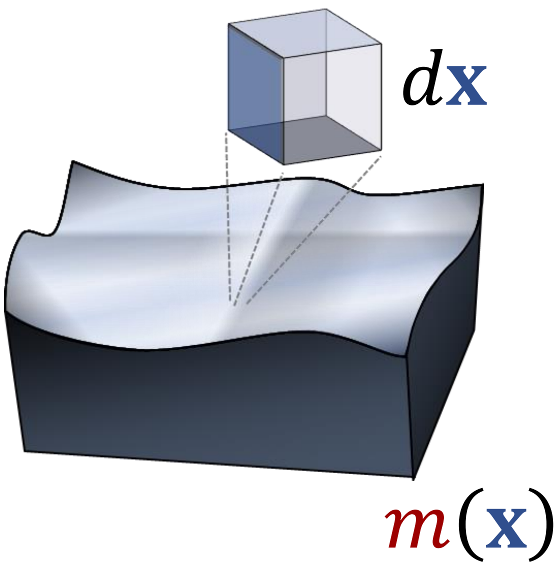
- Average over
  - Area
  - Angle
- Then: take the limit $\text{area} \to 0, \text{angle} \to 0$

Densities

Densities

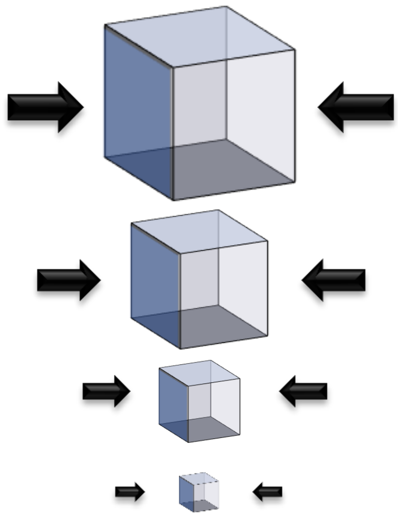
- Densities can be w.r.t. multi-dimensional quantities:

$$\rho(\mathbf{x}) = \frac{dm(\mathbf{x})}{d\mathbf{x}}$$



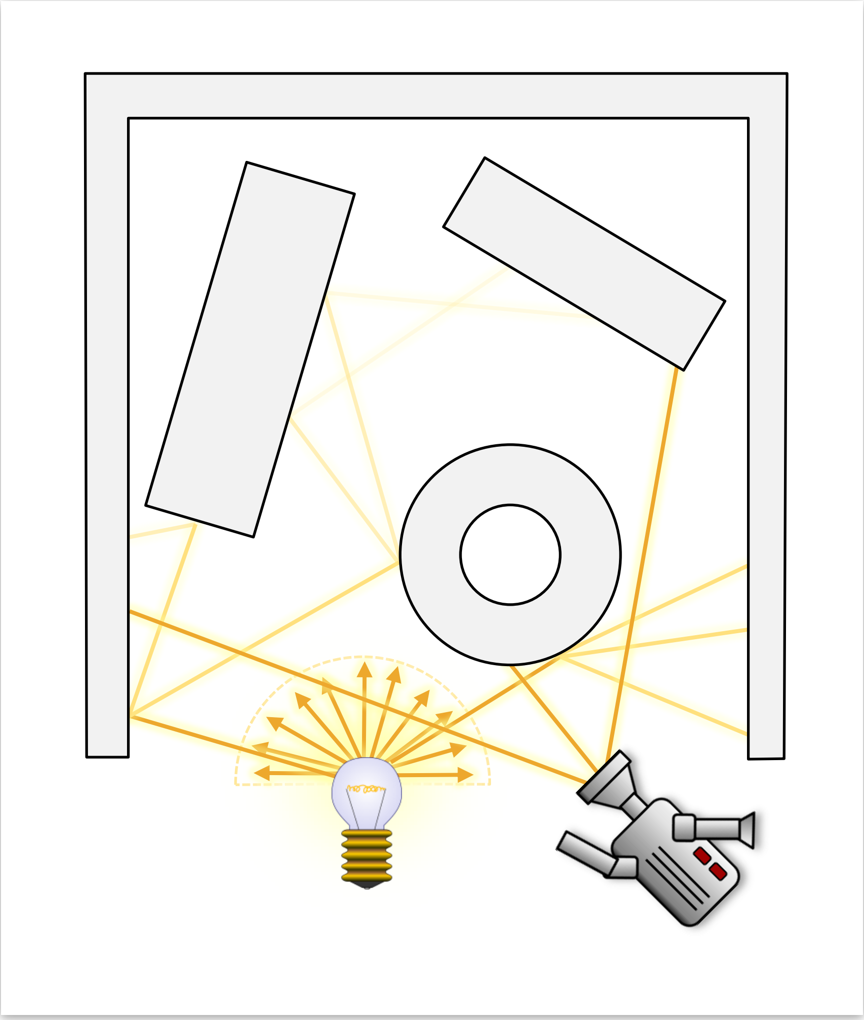
- Integrate to get back actual mass:

$$m_\Omega = \int_\Omega \rho(\mathbf{x}) d\mathbf{x}$$



Radiance: Light Field

Rays fill up space

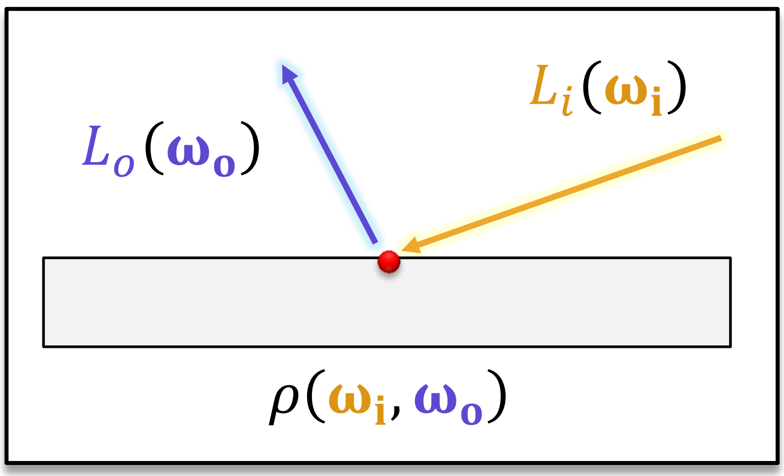


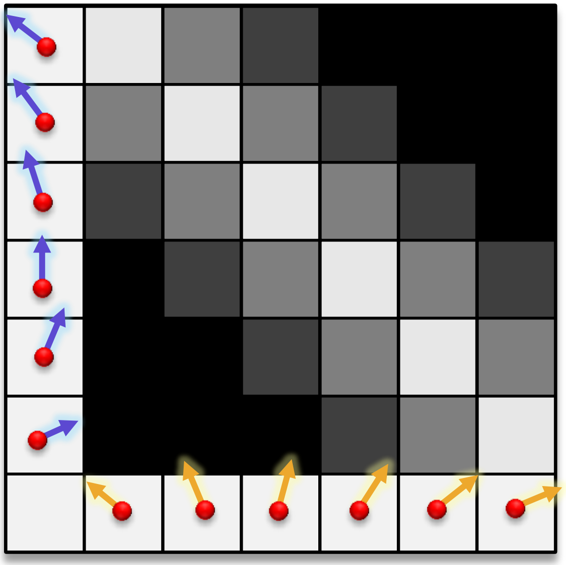
BRDFs

ADV

advanced topics
main ideas

Interaction with Surfaces

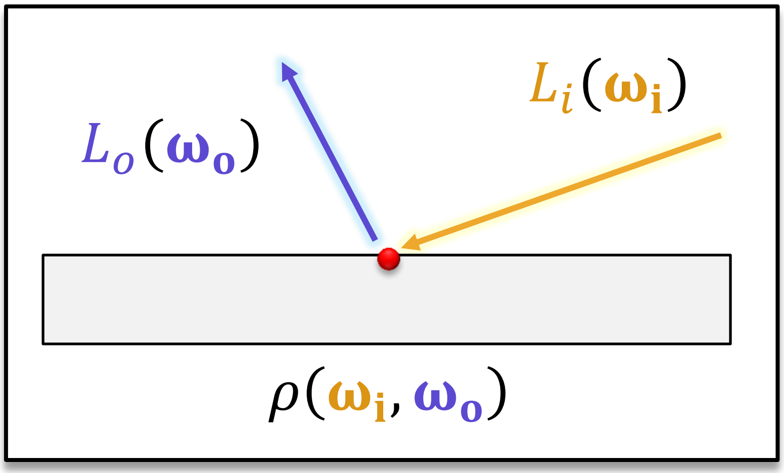


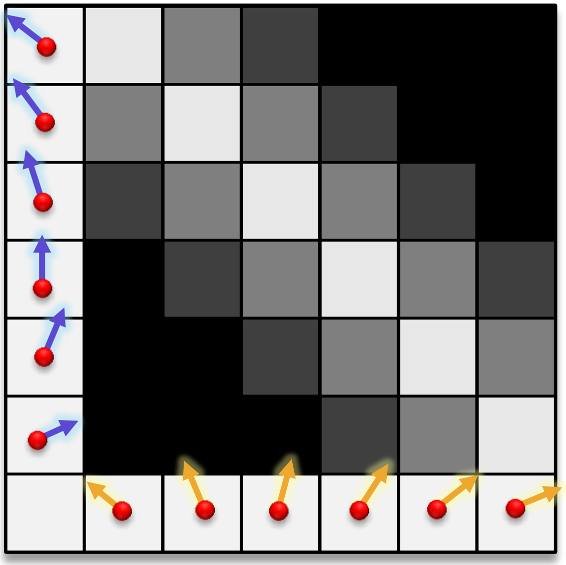


Bidirection Reflectance Distribution Function (BRDF)

- $\rho(\boldsymbol{\omega}_\mathbf{i}, \boldsymbol{\omega}_\mathbf{o})$
- Input: radiance (2D), Output: radiance (2D)
  - Think of a matrix that reshuffles power between rays

Interaction with Surfaces



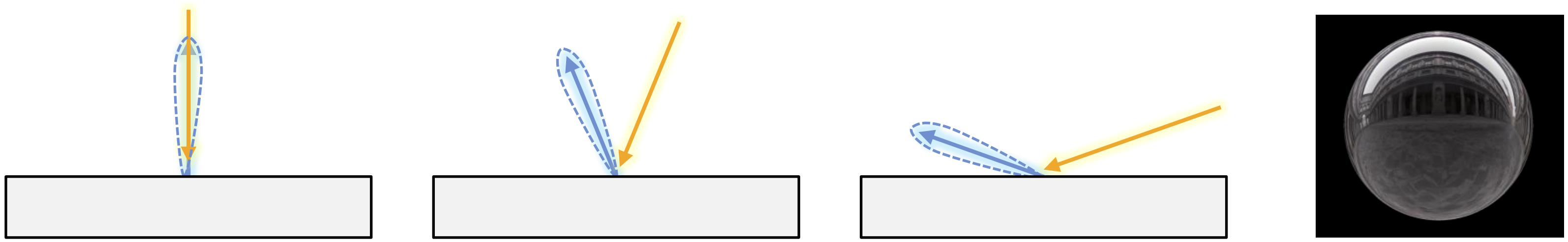


BRDF: Scale factor between input and output

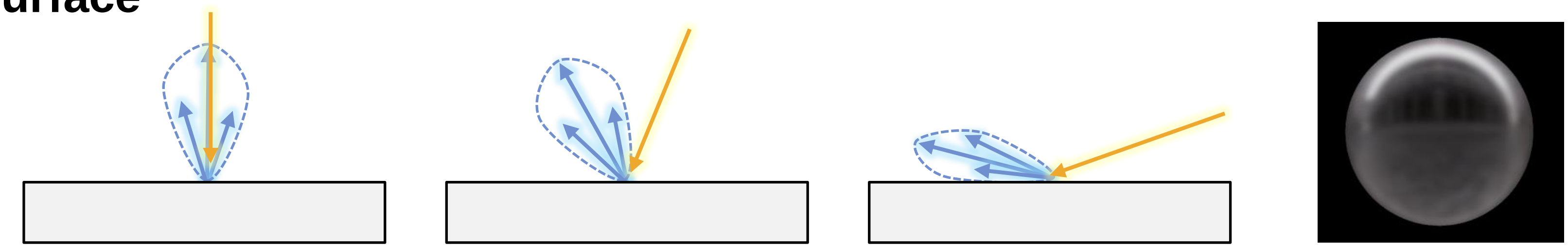
- Light is linear: 2× input → 2× output
- BRDF describes how light is distributed for unit input strength
  - Multiply with actual radiance to scale

Bidirectional Reflectance Distribution Function (BRDF)

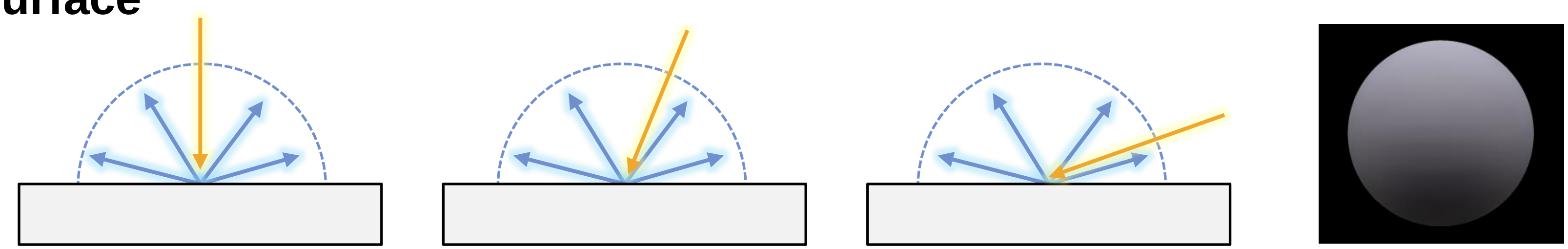
mirror



glossy surface



diffuse surface

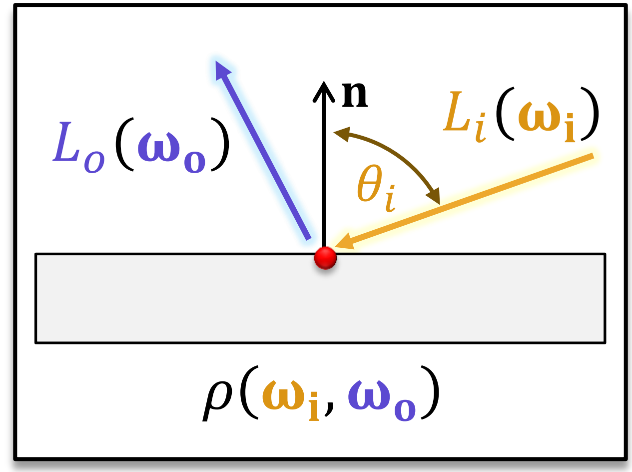


Definition

Technical Definition

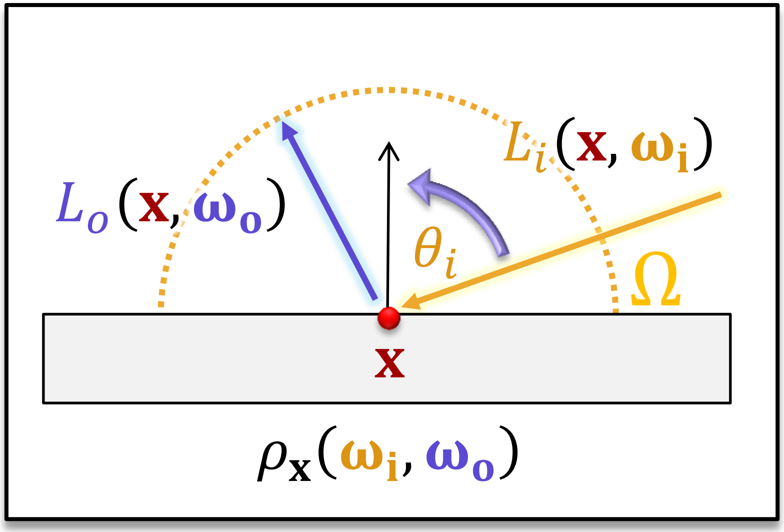
$$\rho(\boldsymbol{\omega}_i, \boldsymbol{\omega}_\mathbf{o}) = \frac{d}{d\boldsymbol{\omega}_i} \frac{L_o(\boldsymbol{\omega}_\mathbf{o})}{L_o(\boldsymbol{\omega}_\mathbf{i}) \cdot \cos\theta_i}$$

density quantity



- Cosine term in denominator
  - Not included in out direct shading computations!
  - Lambertian BRDF is constant (not $\equiv \cos\theta_i$)

Reflectance Equation



General Reflectance

$$L_o(\mathbf{x}, \boldsymbol{\omega}_\mathbf{o}) = \int_{\omega_i \in \Omega} \left[L_i(\mathbf{x}, \boldsymbol{\omega}_\mathbf{i}) \cdot \rho_\mathbf{x}(\boldsymbol{\omega}_\mathbf{i}, \boldsymbol{\omega}_\mathbf{o}) \cdot \cos \theta_\mathrm{i} \right] d\omega_i$$

Properties of the BRDF

BRDF properties

- Positivity:

$$\rho(\boldsymbol{\omega}_i, \boldsymbol{\omega}_\mathbf{o}) \geq 0$$

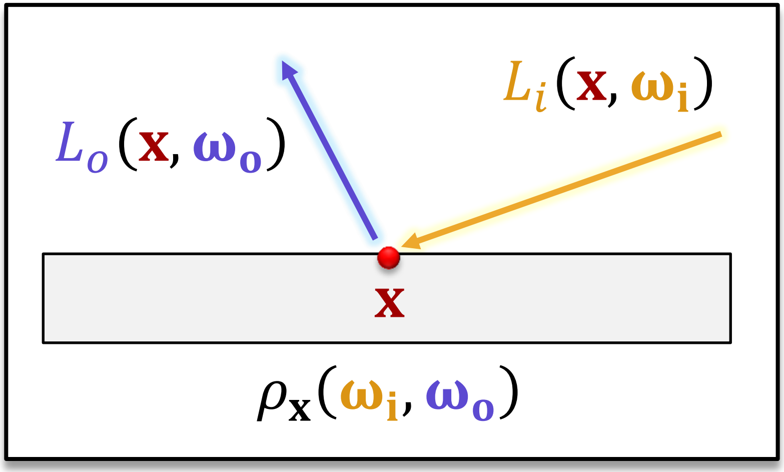
- Helmholtz reciprocity:

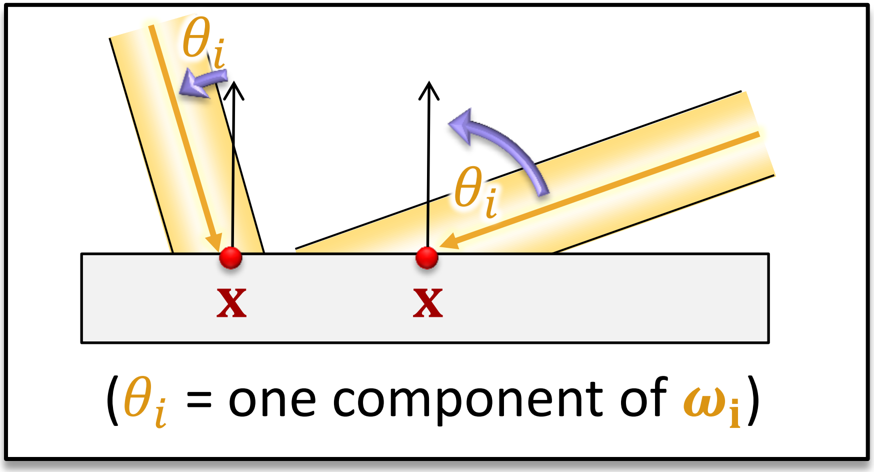
$$\rho(\boldsymbol{\omega}_i, \boldsymbol{\omega}_\mathbf{o}) = \rho(\boldsymbol{\omega}_\mathbf{o}, \boldsymbol{\omega}_i)$$

- Energy conservation:

$$\forall \boldsymbol{\omega}_i : \int_\Omega \rho(\boldsymbol{\omega}_i, \boldsymbol{\omega}_\mathbf{o}) \cos\theta_o \, d\boldsymbol{\omega}_\mathbf{o} \leq 1$$

Surface Interaction

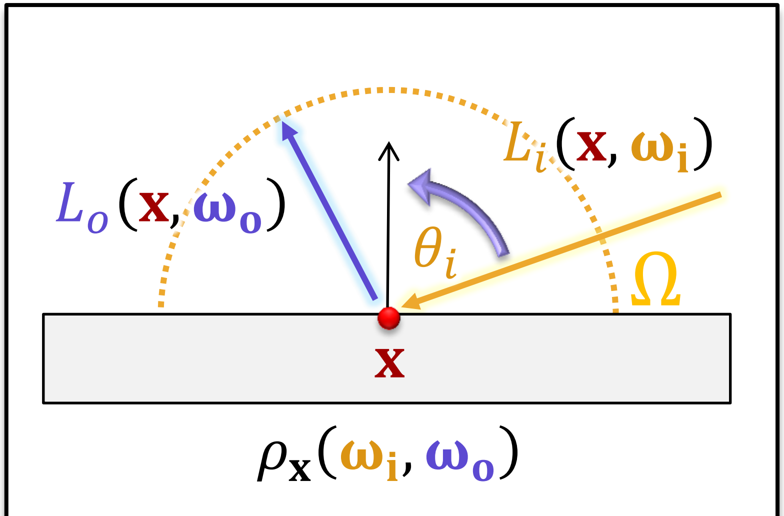


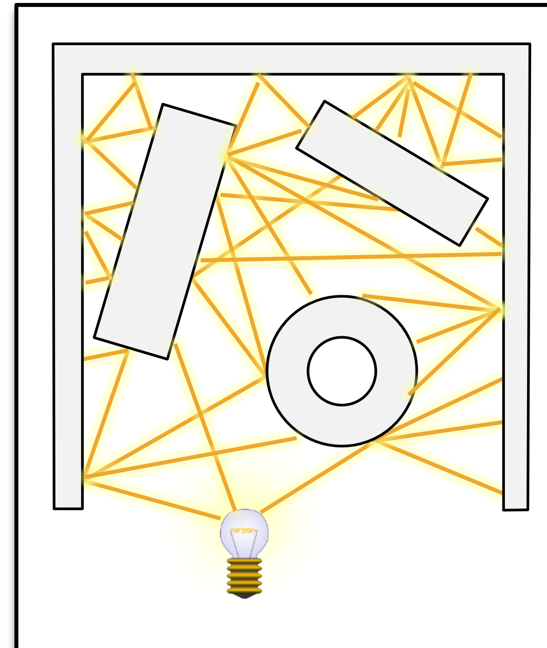


Coupling outgoing and incoming rays

- $L_o(\mathbf{x}, \boldsymbol{\omega}_\mathbf{o}) = L_i(\mathbf{x}, \boldsymbol{\omega}_\mathbf{i}) \cdot \rho_\mathbf{x}(\boldsymbol{\omega}_\mathbf{i}, \boldsymbol{\omega}_\mathbf{o}) \cdot \cos \theta_\mathrm{i}$
- Shallow angle: less power to pass on for each area element

Rendering Equation



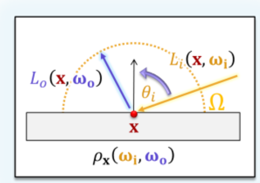


Rendering Equation

$$L_o(\mathbf{x}, \boldsymbol{\omega}_\mathbf{o}) = E_o(\mathbf{x}, \boldsymbol{\omega}_\mathbf{o}) + \int_{\omega_i \in \Omega} \left[L_i(\mathbf{x}, \boldsymbol{\omega}_\mathbf{i}) \cdot \rho_\mathbf{x}(\boldsymbol{\omega}_\mathbf{i}, \boldsymbol{\omega}_\mathbf{o}) \cdot \cos \theta_\mathrm{i} \right] d\omega_i$$

emission

reflection



Structure

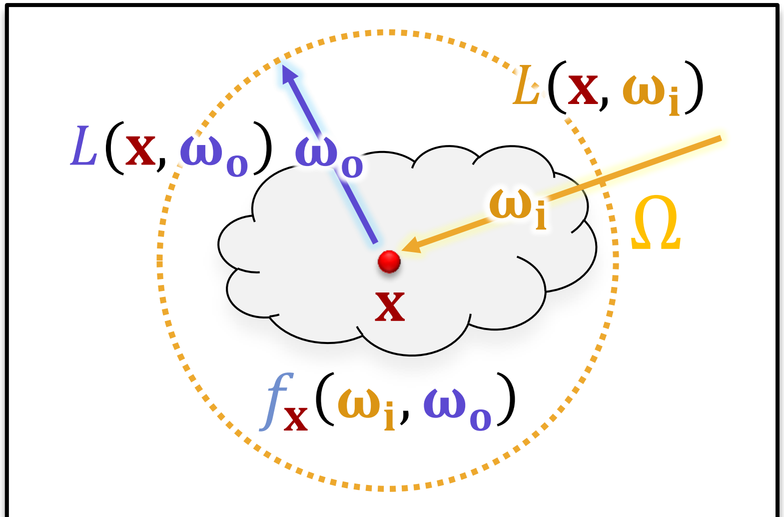
Rendering equation

$$L_o(\mathbf{x}, \boldsymbol{\omega}_\mathbf{o}) = E_o(\mathbf{x}, \boldsymbol{\omega}_\mathbf{o}) + \int_{\omega_i \in \Omega} L_i(\mathbf{x}, \boldsymbol{\omega}_\mathbf{i}) \cdot \rho_\mathbf{x}(\boldsymbol{\omega}_\mathbf{i}, \boldsymbol{\omega}_\mathbf{o}) \cdot \cos \theta_\mathrm{i} \; d\omega_i$$

Abstract notation

- Unknown function L is related to linear operation on itself + emission function E
- Discrete:
  - Functions = vectors, linear operators = matrices
  - $L = E + \mathbf{K} \cdot L$
  - Linear system of equations
- Rendering: solving linear systems of equations

Volumetric Rendering Equation



Most general case: participating media

$$\frac{dL(\mathbf{x}, \boldsymbol{\omega}_\mathbf{o})}{ds} = -\kappa_a L(\mathbf{x}, \boldsymbol{\omega}_\mathbf{o})$$
$$+ \kappa_e E(\mathbf{x}, \boldsymbol{\omega}_\mathbf{o})$$
$$+ \kappa_s \int_{\omega_i \in \Omega} L(\mathbf{x}, \boldsymbol{\omega}_\mathbf{i}) \cdot f_\mathbf{x}(\boldsymbol{\omega}_\mathbf{i}, \boldsymbol{\omega}_\mathbf{o}) d\omega_i$$

$\mathbf{s}$ = tangent vector at $\mathbf{x}$ in direction $\boldsymbol{\omega}_\mathbf{o}$

phase function f

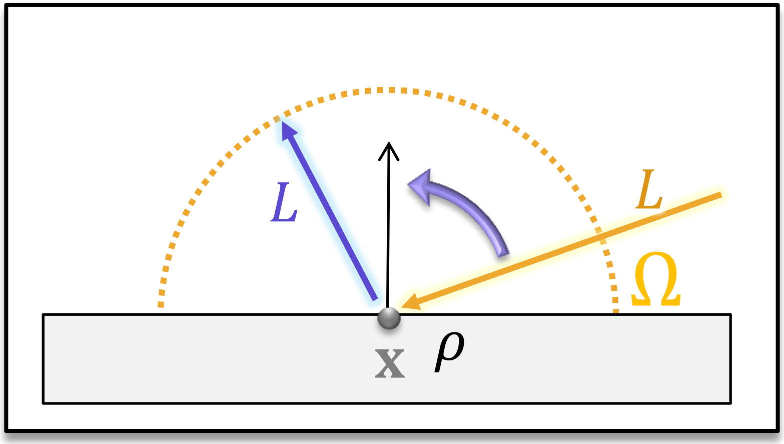
Impact

http://en.wikipedia.org/wiki/Trinity_test

Short Version (Caveats Omitted)

Simplified Story

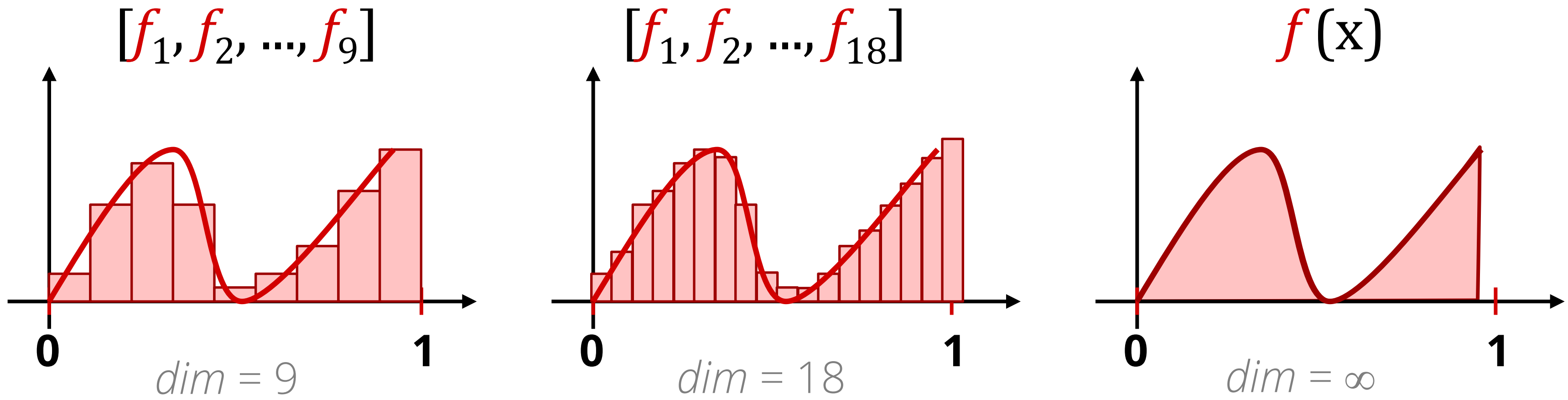
- We are looking for L
- L is a function
  - (discretized: array of numbers)
- Transport operator $\mathbf{K}$
  - Linear operator
  - Read: a big matrix
- Emission term E
  - The part of L that glows on its own



Operator $\mathbf{K}$:
One light bounce

$$L = E + \mathbf{K}L$$
$$\Rightarrow \quad L = (1 - \mathbf{K})^{-1}E$$

High Dimensional: Function Spaces

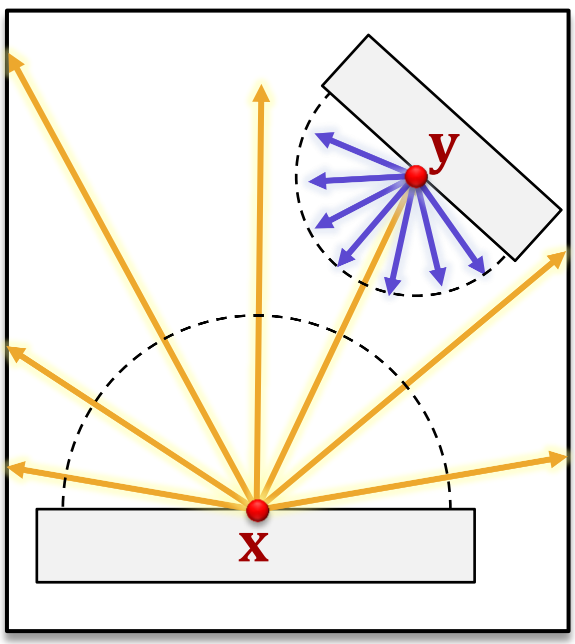


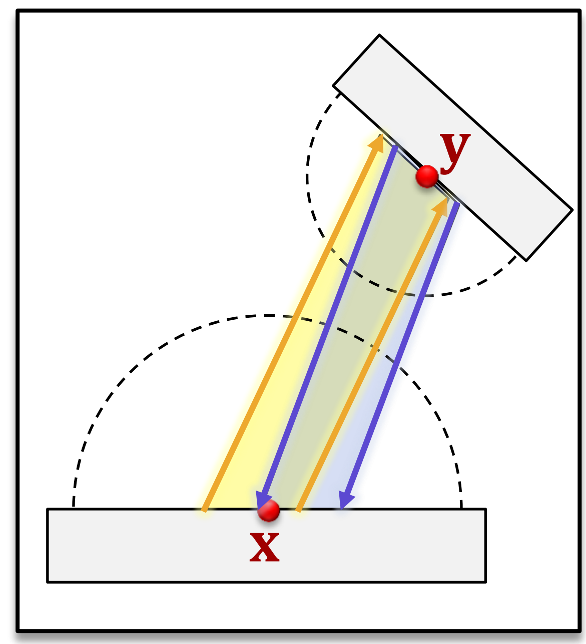
Application

- Approximate continuous functions
- Increase sampling density towards infinity

ADV
advanced topics
main ideas

Consider Ray Bundles (Finite Elements)





Efficiency

ADV

advanced topics

main ideas

List of Problems

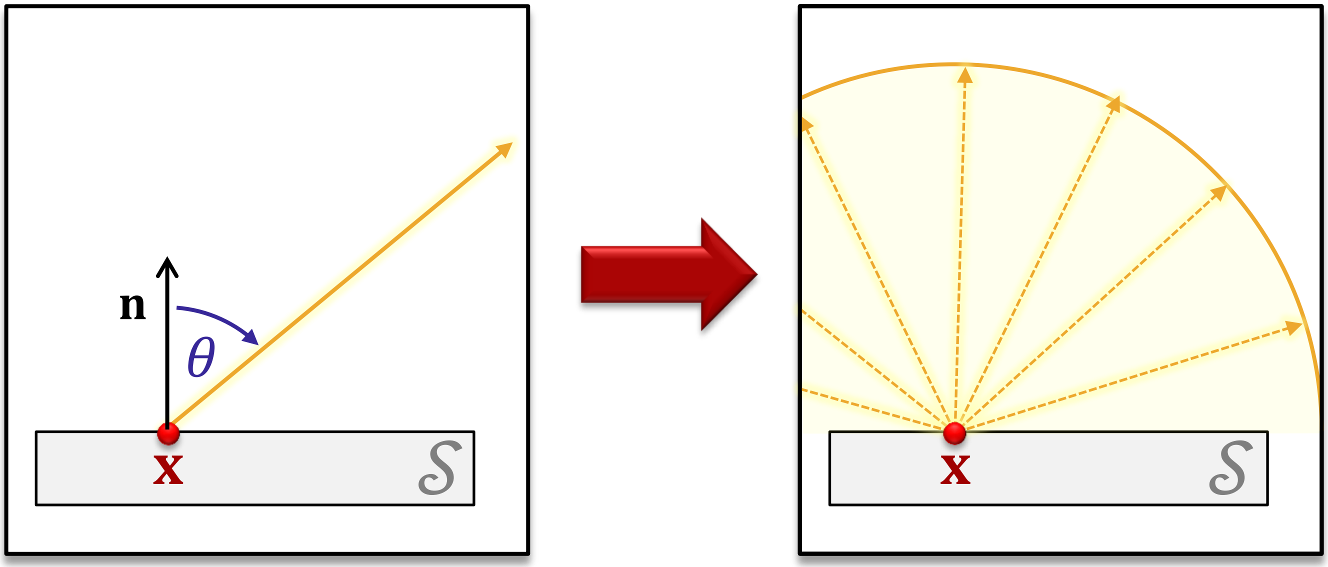
Problems

- 4-dimensional parameter domain ($\mathcal{O}(n^4)$)
  - Dense matrix: $\mathcal{O}\left((n^4)^2\right)$
- It’s polynomial time!
  - But that is never going to run on any realistic scene…

Solutions

- **Simplify:** radiosity method
- **Output-sensitive solution:** stochastic raytracing

Radiosity



Radiance

Radiosity

Radiosity

- *Overall power* exiting a point **x**

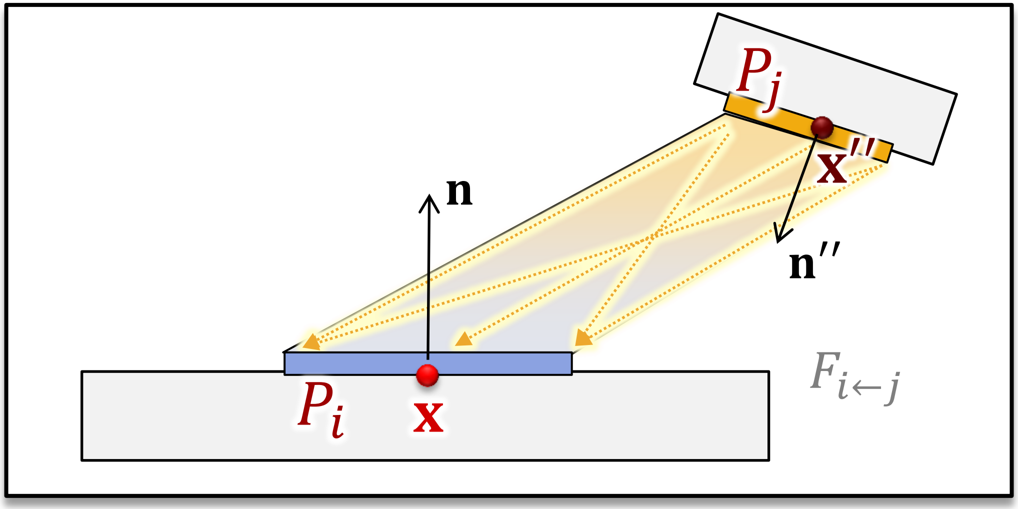
$$B(\mathbf{x}) = \int_{\Omega} L(\mathbf{x}, \boldsymbol{\omega}) \cdot \cos\theta \, d\boldsymbol{\omega}$$

Piecewise Constant Radiosity

Discretization: piecewise constant radiosity

- Patches P_i
- Scene $\mathcal{S} = P_1 \dot{\cup} \ldots \dot{\cup} P_k$
- Radiosity B is constant on each P_i

Piecewise Constant Radiosity



Form Factor:

$$F_{i\leftarrow j} = \int_{P_j}\int_{P_i} \underbrace{h(\mathbf{x},\mathbf{x}'')}_{\text{visibility}} \cdot \frac{\cos\angle(\mathbf{x}-\mathbf{x}'',\mathbf{n}) \cdot \cos\angle(\mathbf{x}-\mathbf{x}'',\mathbf{n}'')}{\|\mathbf{x}-\mathbf{x}''\|^2}\, d\mathbf{x}''\, d\mathbf{x}$$

Piecewise Constant Radiosity

Result: Linear System

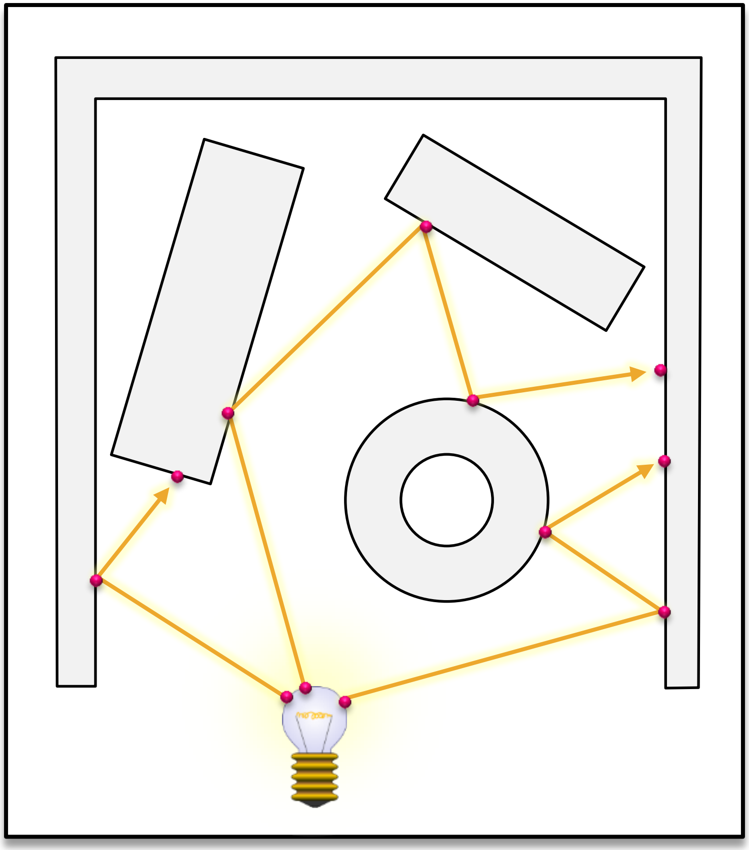
$$B_i = E_i + \sum_{j=1}^{k} \rho_j \cdot F_{i \leftarrow j} B_j$$

Solving the linear system

How to solve the linear system?

- Gaussian elimination?
  - No, not a good idea
  - Unstable & very slow [$\mathcal{O}(n^3)$]
- Iterative solvers
  - Gauss-Seidel (slow) [$\sim\mathcal{O}(n^2)$]
  - Southwell-relaxation (GS with shooting)
    - Memory advantages
    - Used to be the standard solution
  - Standard solvers: bi-CG, sparse Cholesky, …

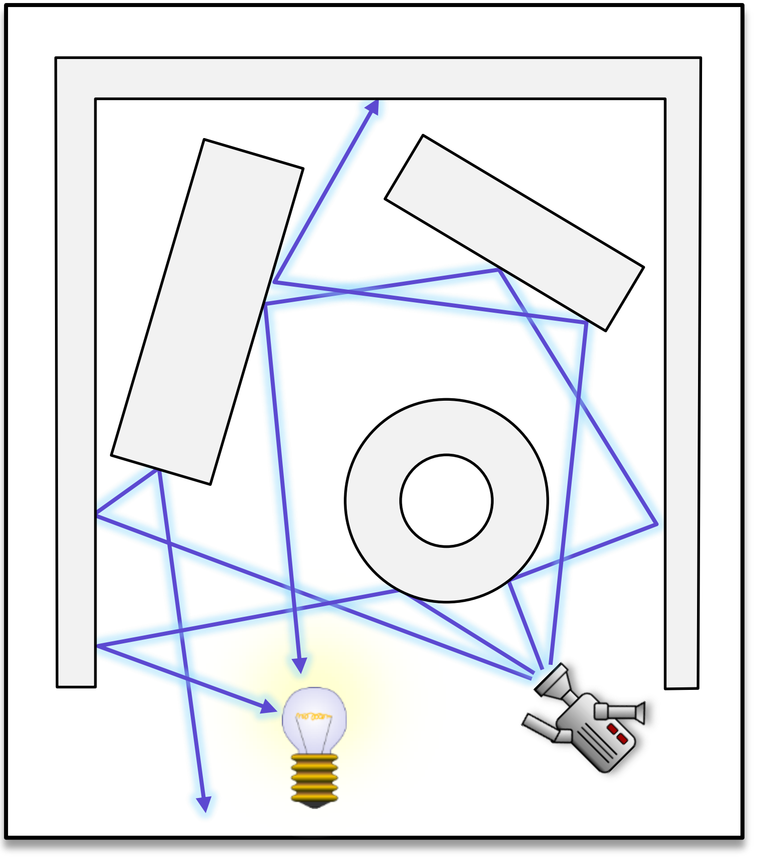
Photon Tracing



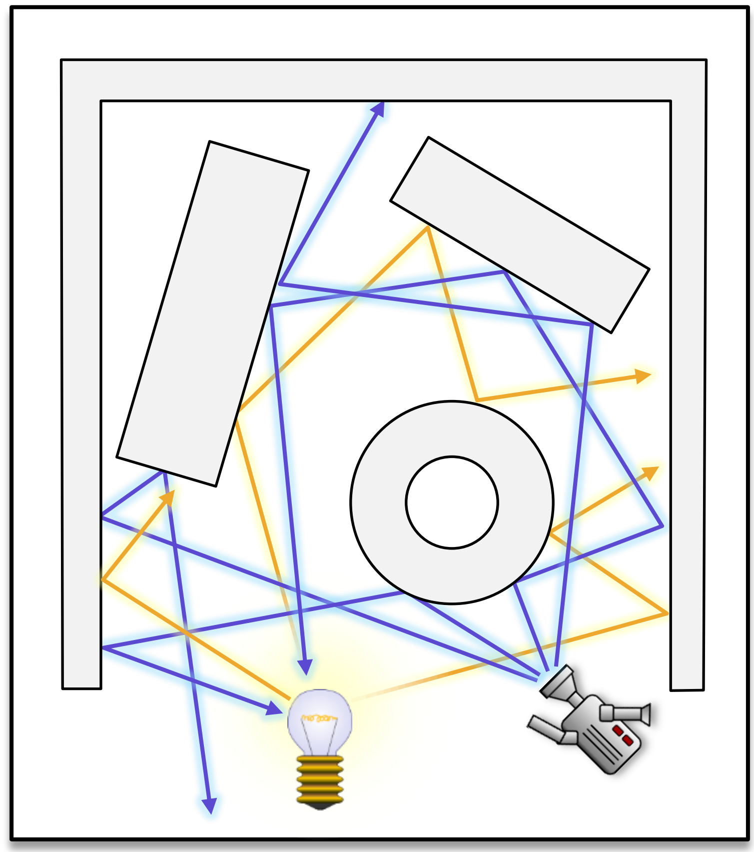
$$L = (1 - \mathbf{K})^{-1} E$$

$$= E + \mathbf{K}E + \mathbf{K}^2 E + \mathbf{K}^3 E + \cdots$$

Path Tracing



Bidirectional Path Tracing



bidirectional path tracing