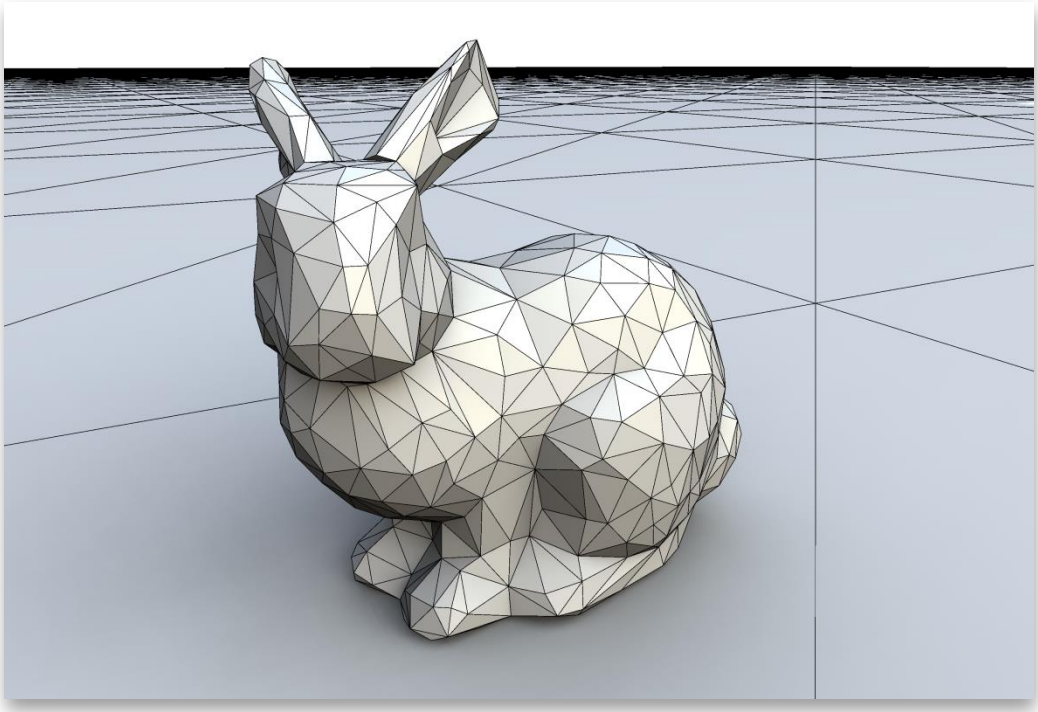
Graphics 2014



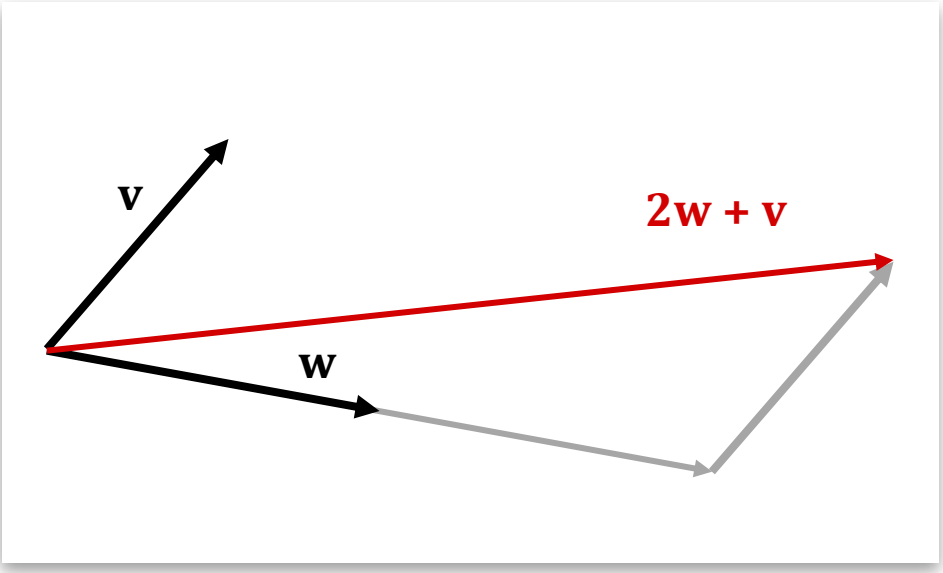
Summary

[Faculty of Science]
Information and
Computing Sciences

Universiteit Utrecht

Linear Algebra

Vector Spaces

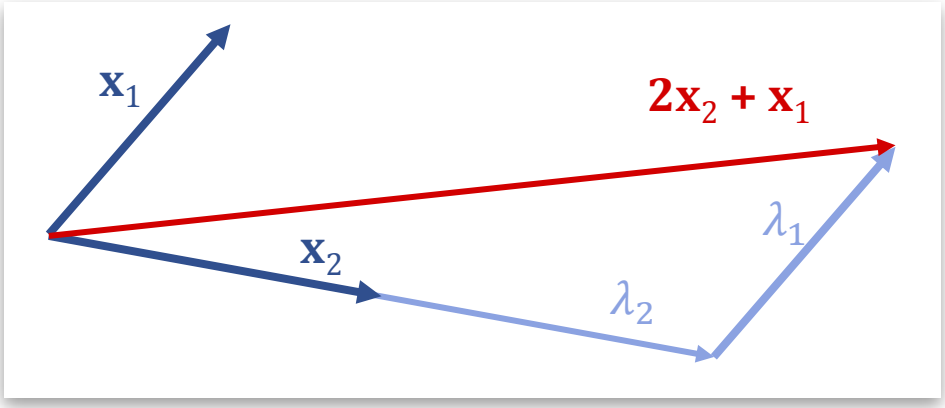


$$\mathbf{y} = \sum_{i=1}^{n} \lambda_i \mathbf{x}^{(i)}$$

Linear Combinations:
This is basically all you can do.

Algebraically

Linear Mappings

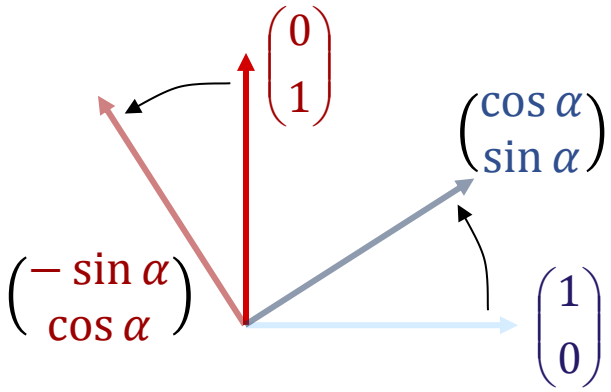


Linear Combinations

$$\mathbf{y} = \sum_{i=1}^{n} \lambda_i \mathbf{x}_i$$

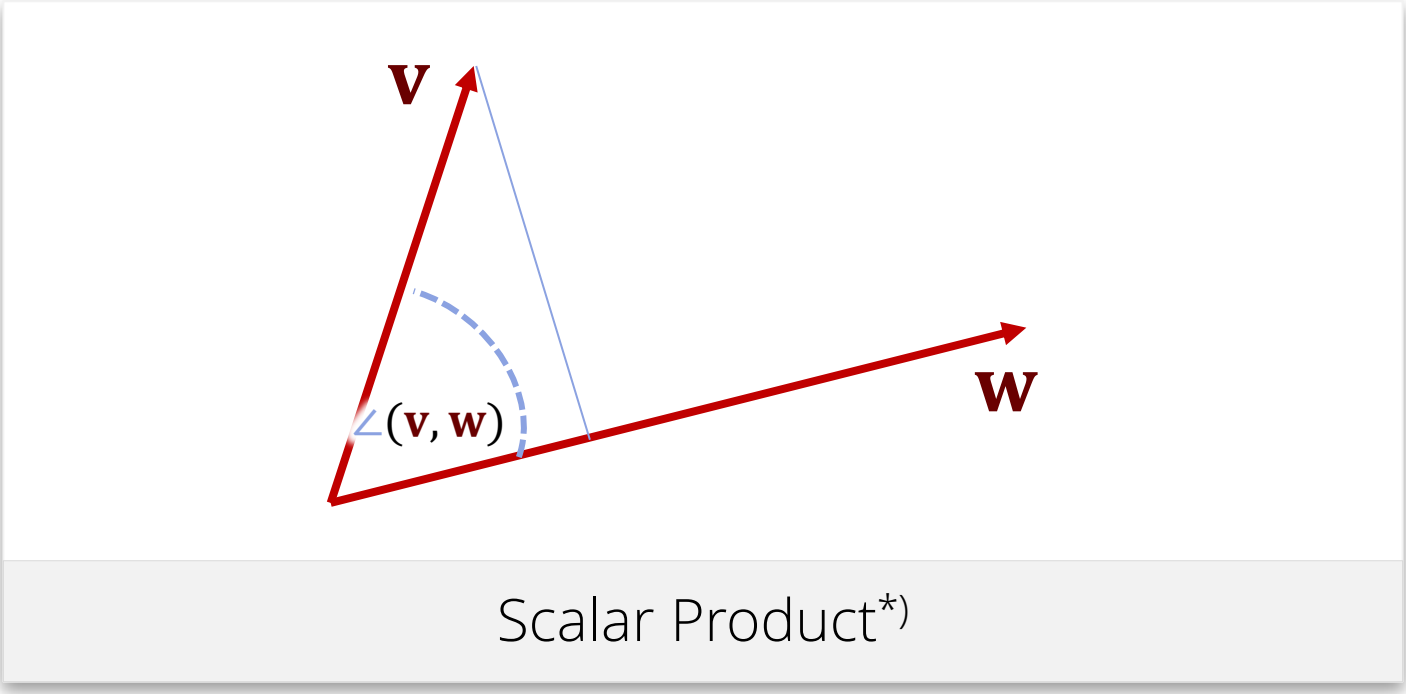
Algebra

Construction: Image of basis vectors



$$\mathbf{M}_{rot} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

Scalar Product



Scalar Product*)

$$\mathbf{v} \cdot \mathbf{w} = \|\mathbf{v}\| \cdot \|\mathbf{w}\| \cdot \cos \angle(\mathbf{v}, \mathbf{w})$$

Comprises: length, projection, angles

*) also known as *inner product* or *dot-product*

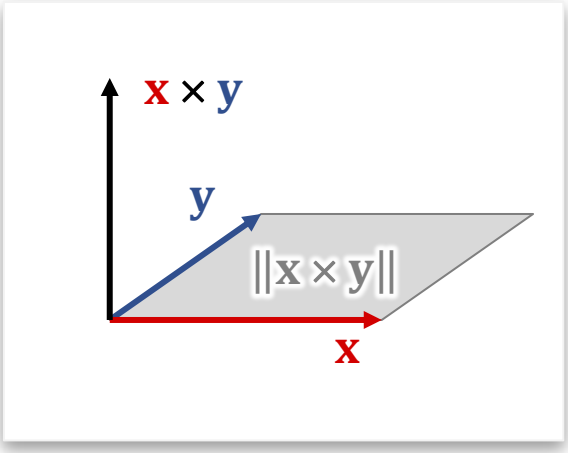
Cross Product

Cross-Product: Only in 3D

- $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$
- $\mathbf{x} \times \mathbf{y} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \times \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} := \begin{pmatrix} x_2 y_3 - x_3 y_2 \\ x_3 y_1 - x_1 y_3 \\ x_1 y_2 - x_2 y_1 \end{pmatrix}$

Geometrically

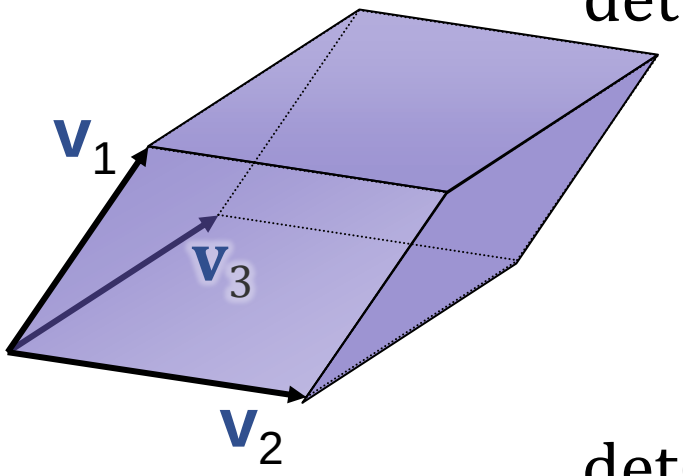
- $\mathbf{x} \times \mathbf{y}$ orthogonal to $\mathbf{x}, \mathbf{y}$
- Right-handed system $(\mathbf{x}, \mathbf{y}, \mathbf{x} \times \mathbf{y})$
- $\|\mathbf{x} \times \mathbf{y}\| = \|\mathbf{x}\| \cdot \|\mathbf{y}\| \cdot \sin\angle(\mathbf{x}, \mathbf{y})$



Determinants

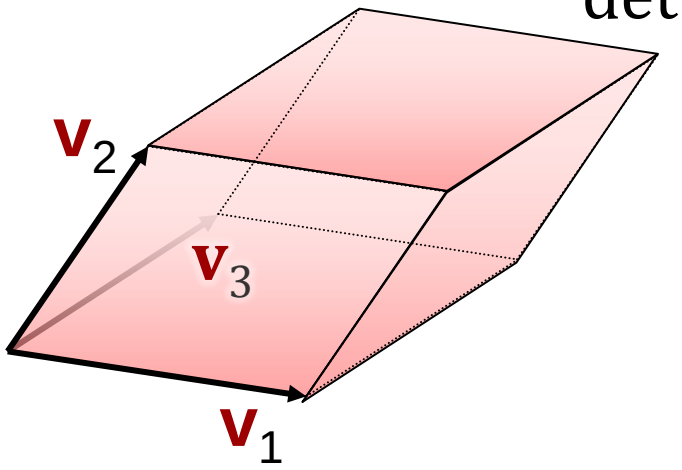
$$\mathbf{M} = \begin{pmatrix} | & | & | \\ \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \\ | & | & | \end{pmatrix}$$

$\det(\mathbf{M}) > 0$



$$\mathbf{M}' = \begin{pmatrix} | & | & | \\ \mathbf{v}_2 & \mathbf{v}_1 & \mathbf{v}_3 \\ | & | & | \end{pmatrix}$$

$\det(\mathbf{M}') < 0$



Signed volume of parallelepiped

Homogeneous Coordinates

Increase dimension $\mathbb{R}^d \rightarrow \mathbb{R}^{d+1}$

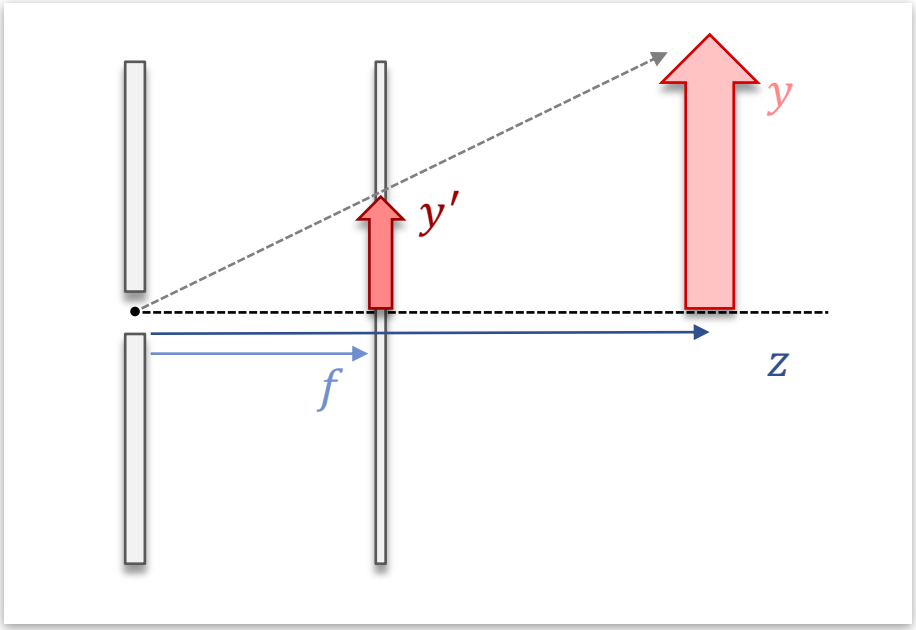
- Last entry = 1 (or normalize by division)

$$\mathbf{M}' \cdot \mathbf{x} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & t_1 \\ m_{21} & m_{22} & m_{23} & t_2 \\ m_{31} & m_{32} & m_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \ddots & & \iddots & | \\ & \mathbf{M} & & \mathbf{t} \\ \iddots & & \ddots & | \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} | \\ \mathbf{x} \\ | \\ 1 \end{pmatrix} = \begin{pmatrix} | \\ \mathbf{Mx} + \mathbf{t} \\ | \\ 1 \end{pmatrix}$$

Perspective

Pinhole Camera



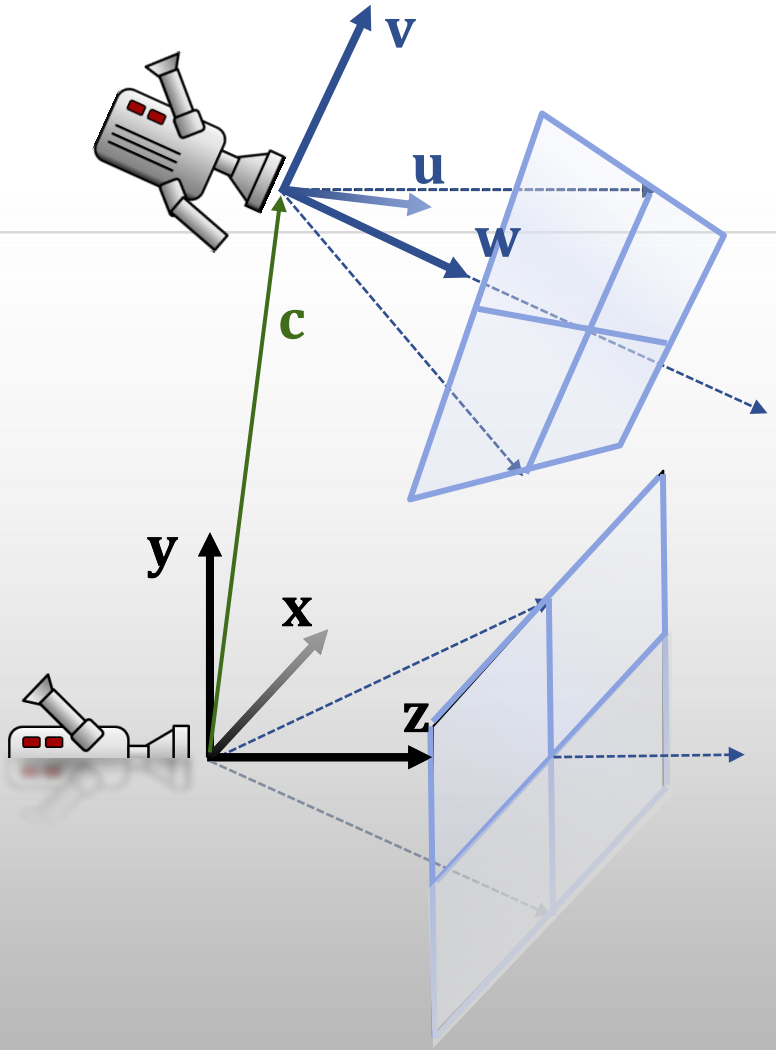
$$x' = f\frac{x}{z}$$

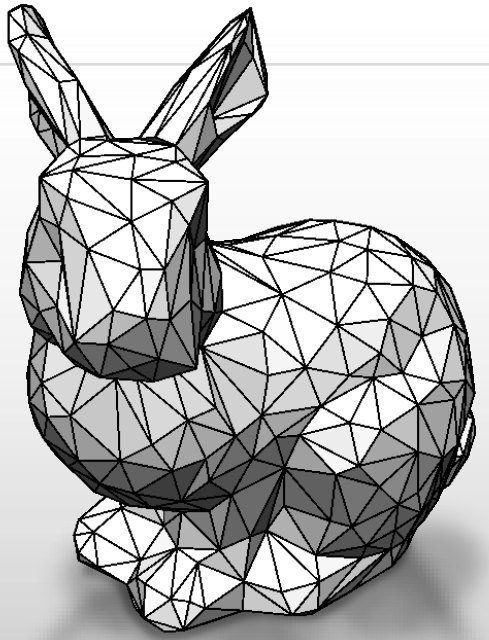
$$y' = f\frac{y}{z}$$

Undetermined degree of freedom

- Focal length vs. image size
- Source of a lot of confusion!

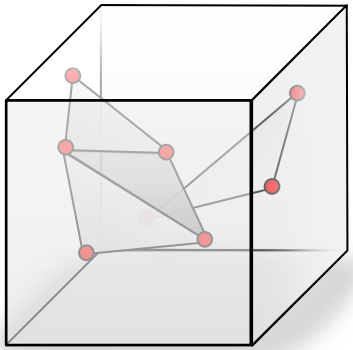
General Camera



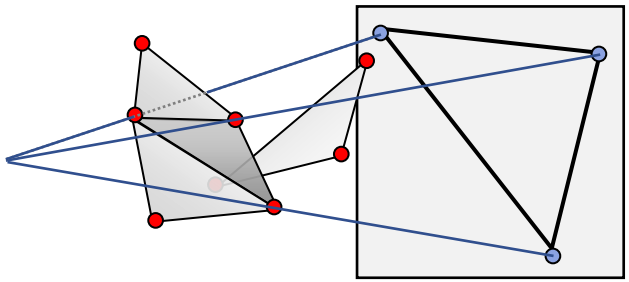


Rasterization

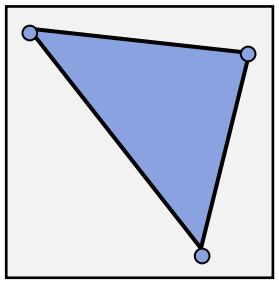
z-Buffer



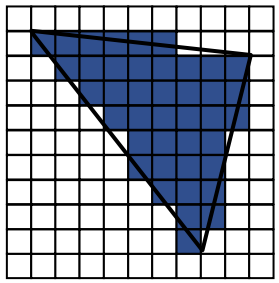
3D Scene



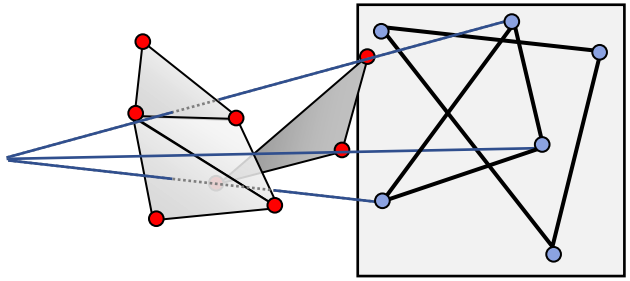
Projection

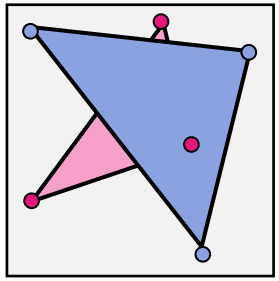


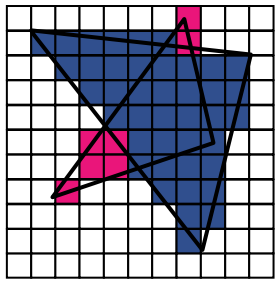
Visibility



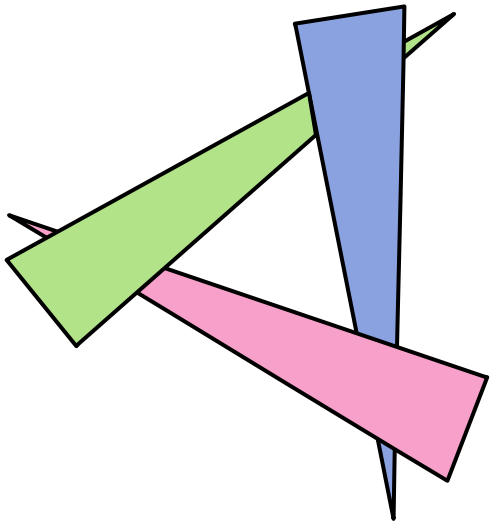
Rasterization

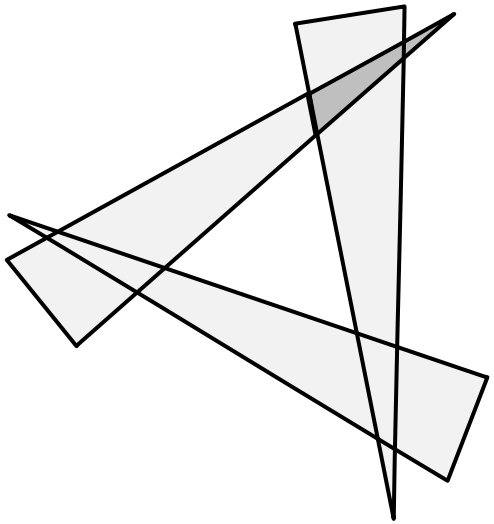






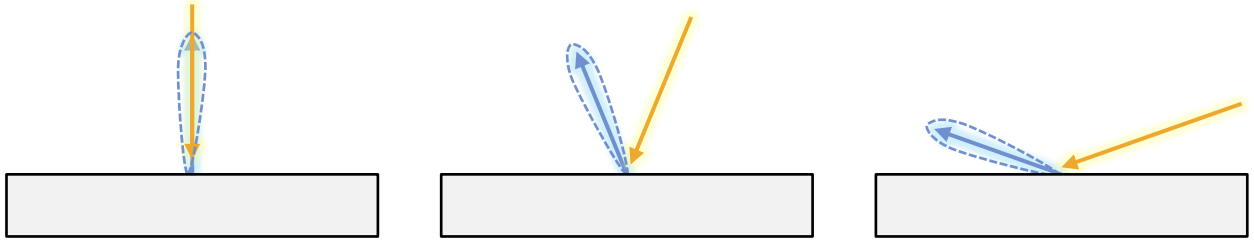
Painters Algorithm



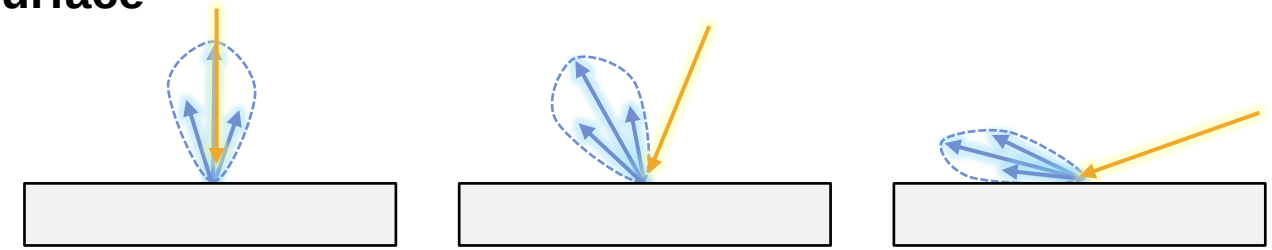


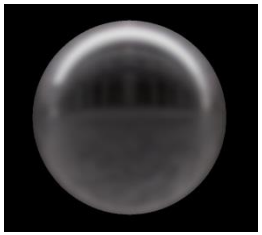
Local Illumination

mirror

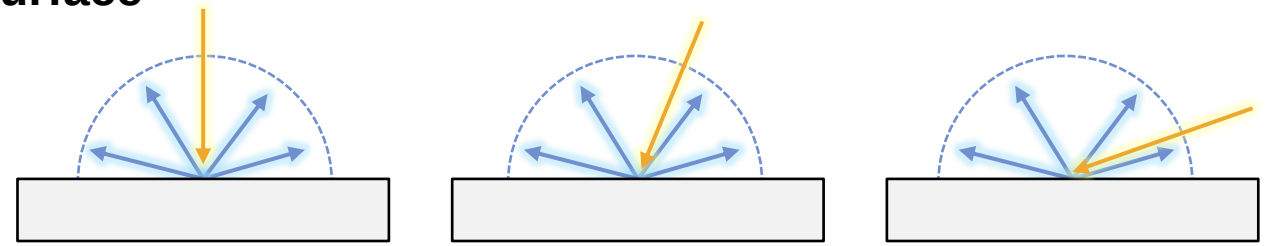


glossy surface

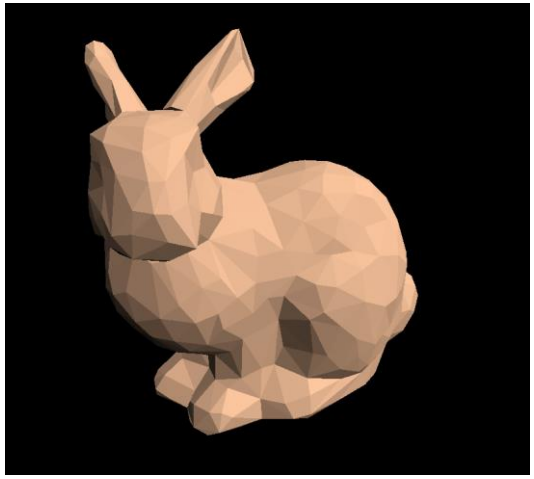




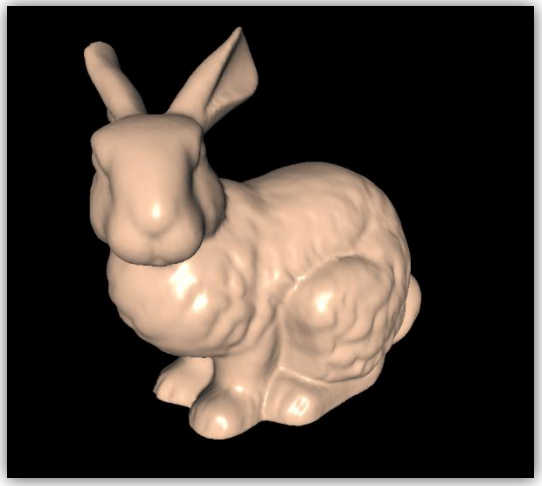
diffuse surface



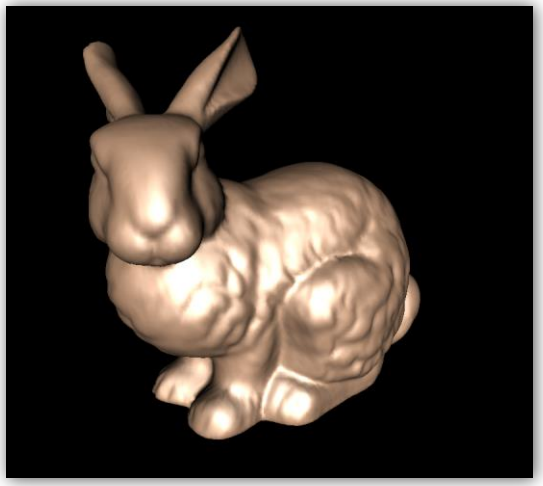
The Glorious Bunnies



Lambert's Bunny + Ambient



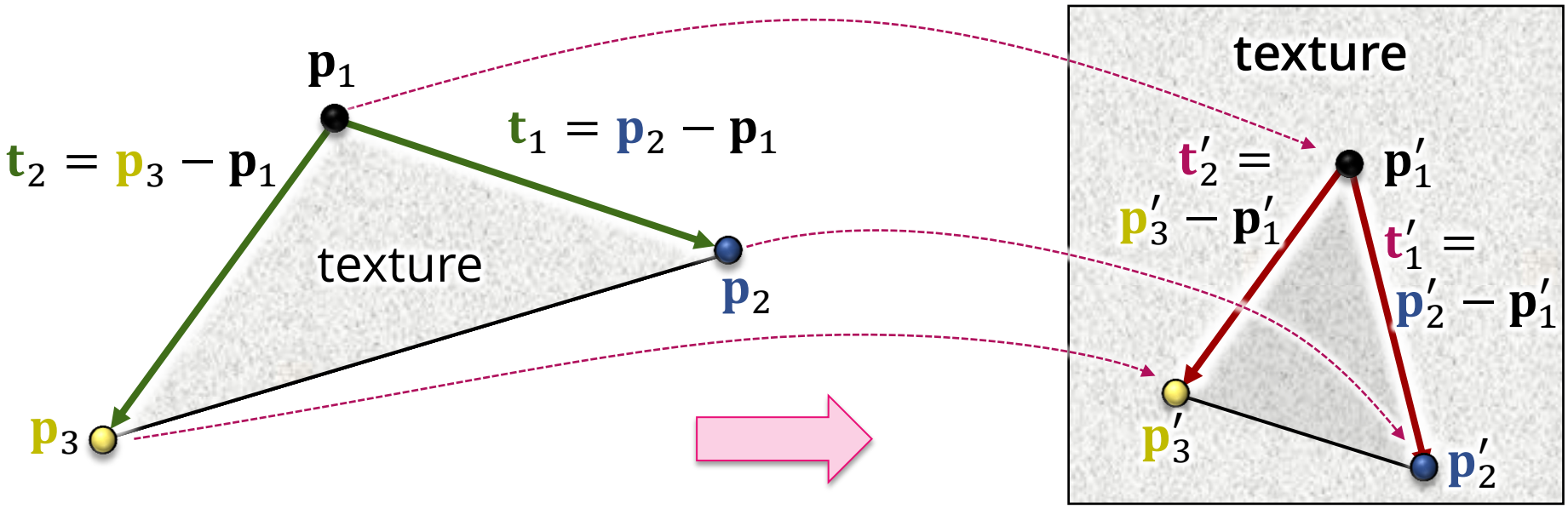
Phong Bunny



Cook-Torrance Bunny

Advanced Rasterization

Texture Mapping



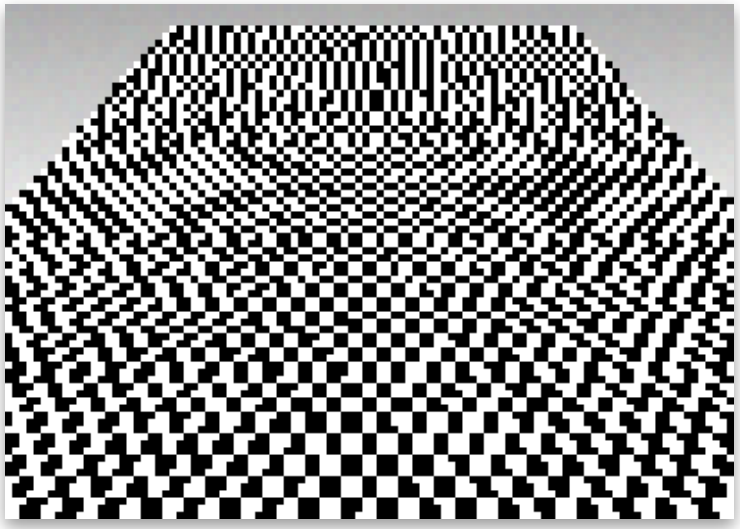
Affine Map

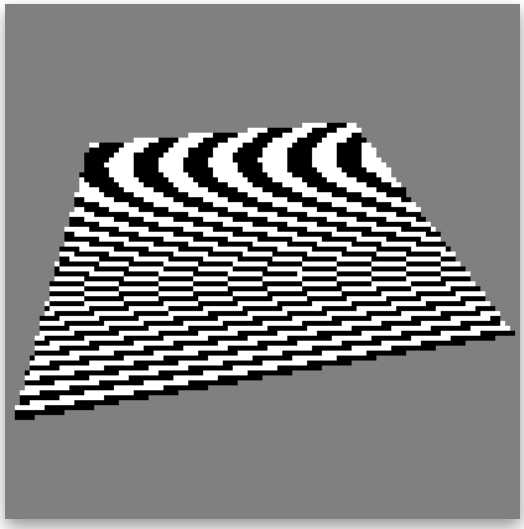
- Map coordinate system $\{\mathbf{p}_1, (\mathbf{t}_1', \mathbf{t}_2')\}$ to $\{\mathbf{p}_2, (\boldsymbol{t}_1, \boldsymbol{t}_2)\}$

$$\mathbf{x} \rightarrow \mathbf{p}_1' + \begin{pmatrix} | & | \\ \mathbf{t}_1' & \mathbf{t}_2' \\ | & | \end{pmatrix} \cdot \begin{pmatrix} | & | \\ \boldsymbol{t}_1 & \boldsymbol{t}_2 \\ | & | \end{pmatrix}^{-1} (\mathbf{x} - \mathbf{p}_1)$$

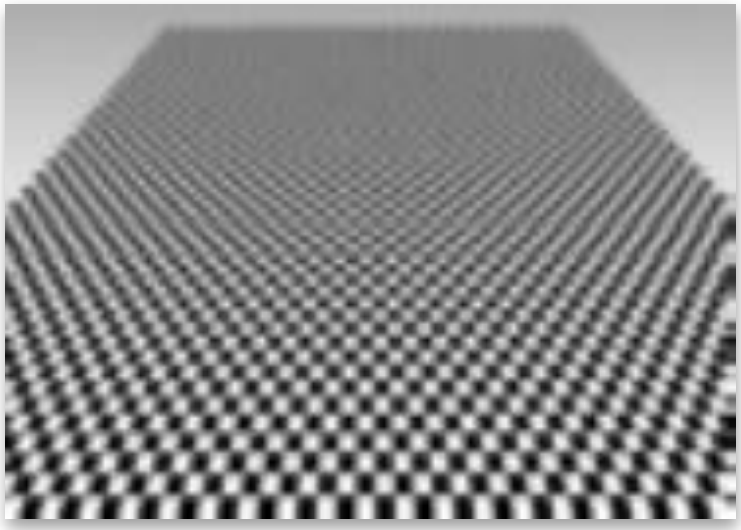
Aliasing

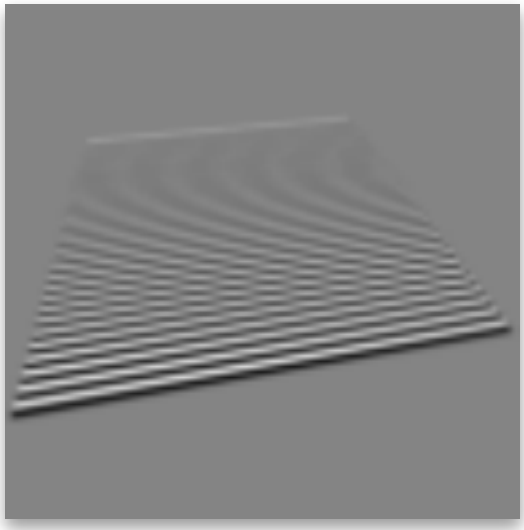
simple sampling



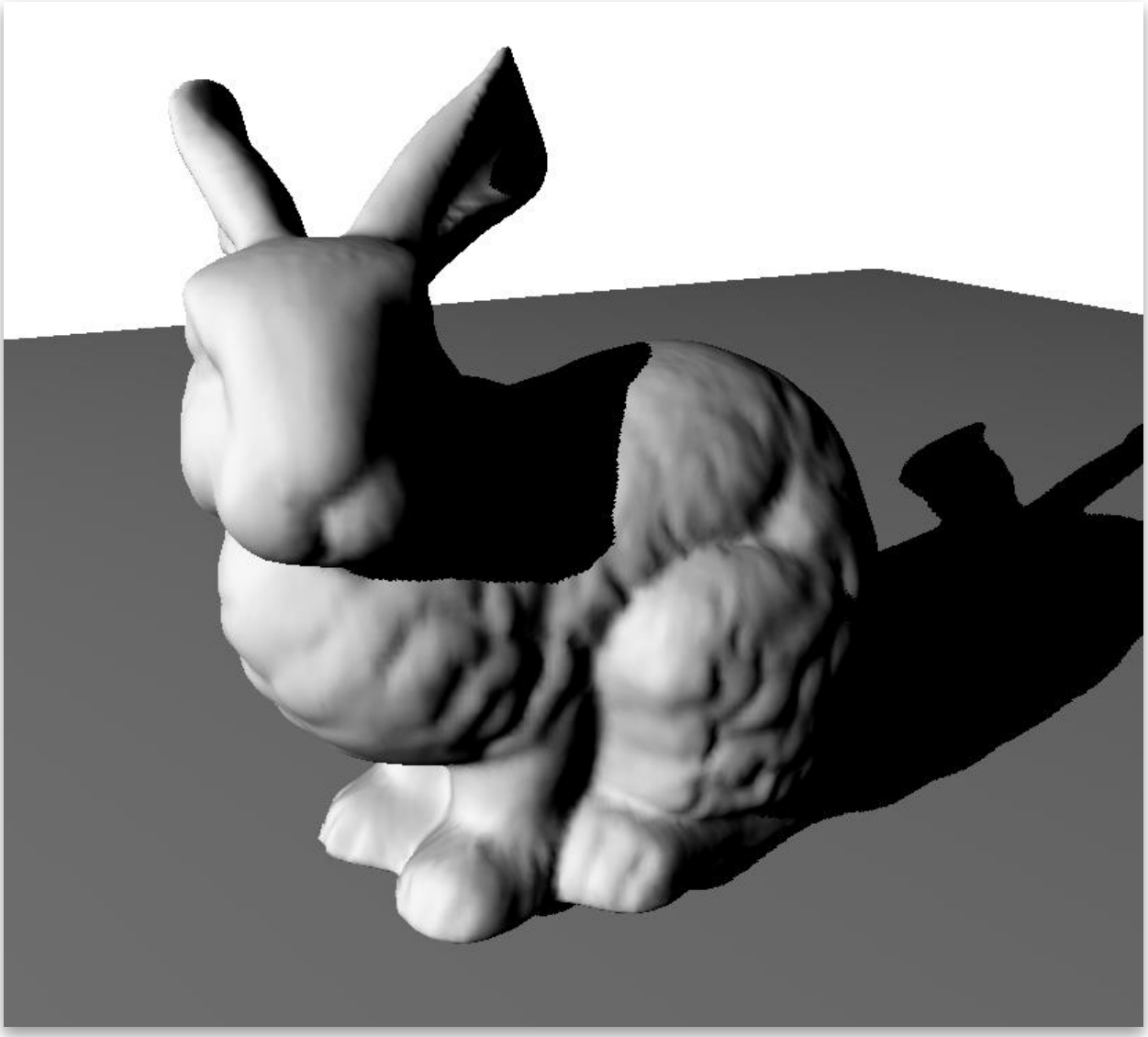


anti-aliasing (Gaussian)

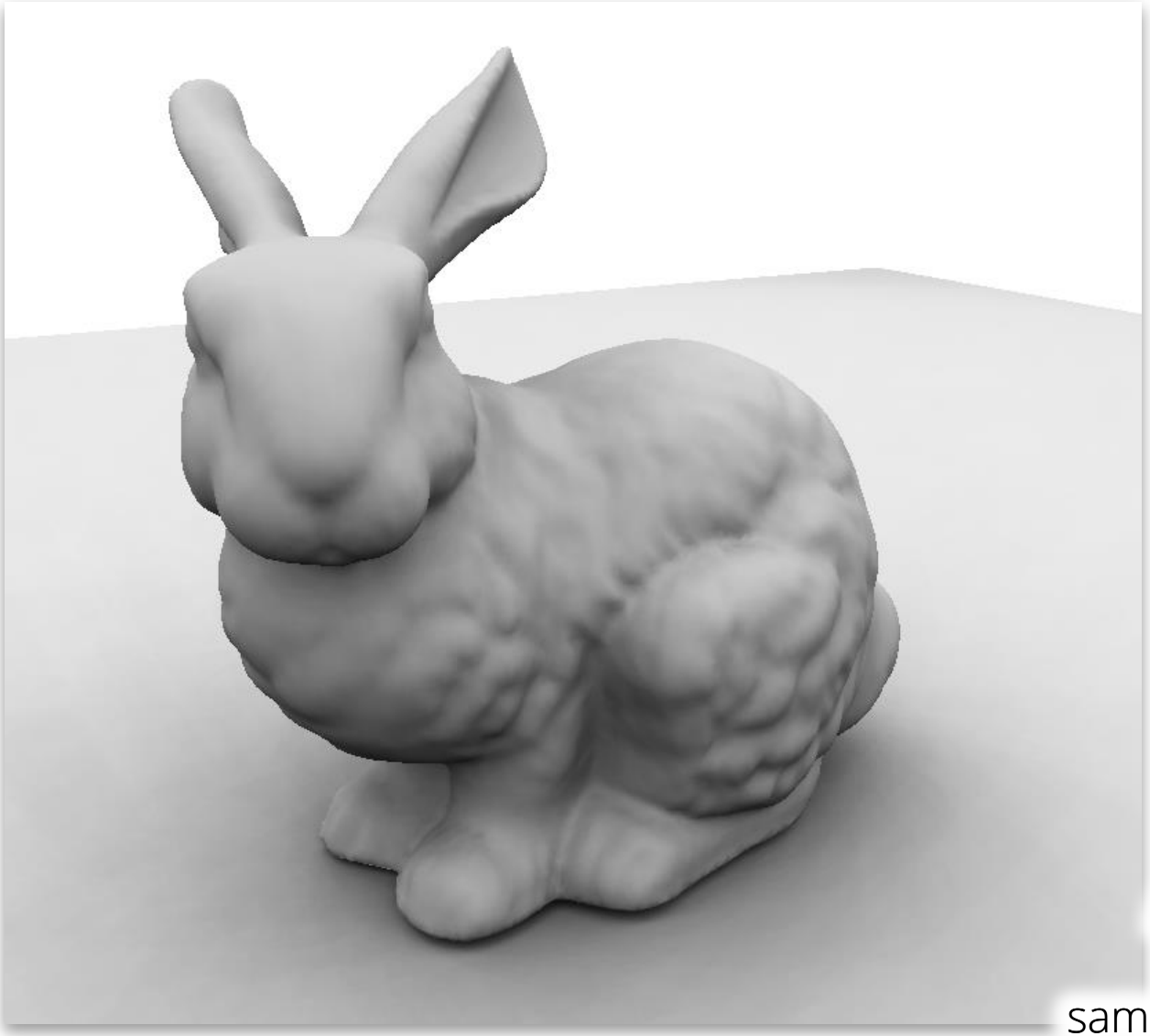


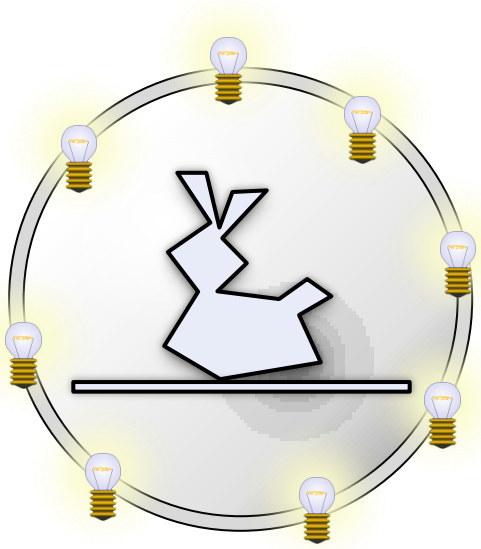


Shadow Map



Ambient Occlusion





Average of 2560 Images
light sources randomly sampled on enclosing sphere

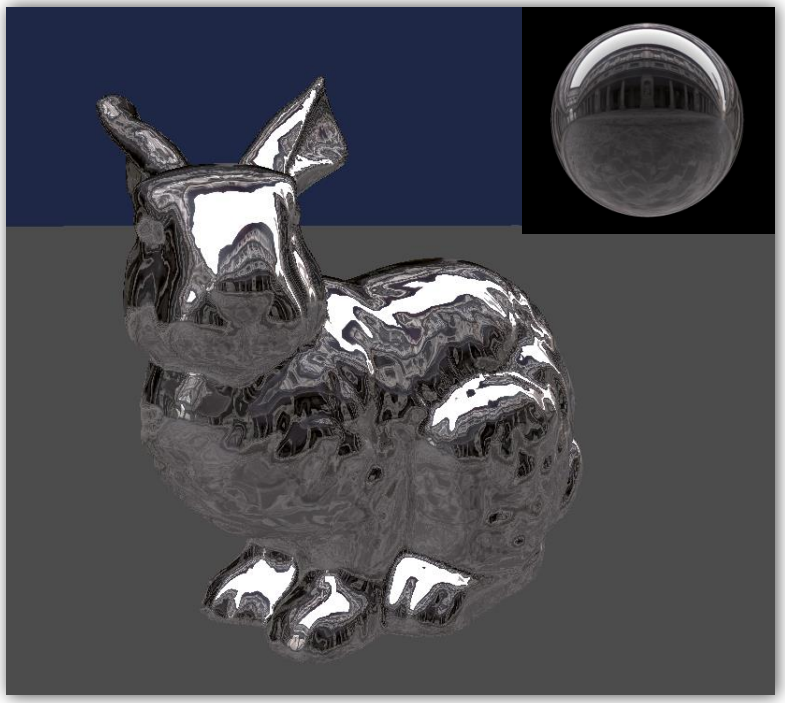
Environment Maps

Approximate Reflections

- Store panoramic image (“360°”) of environment
- Use for reflection

Approximation

- Far away environment
- Single bounce
- No occlusion in path
- Refraction less accurate (single bounce?)

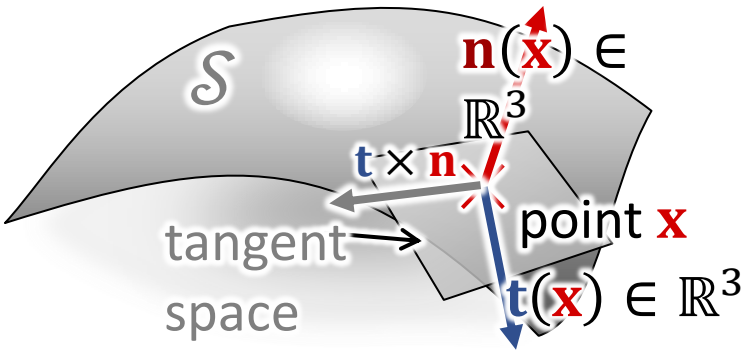


Reflective Bunny
(Environment Mapping)

Bump-Maps / Normal Maps

Normal Maps

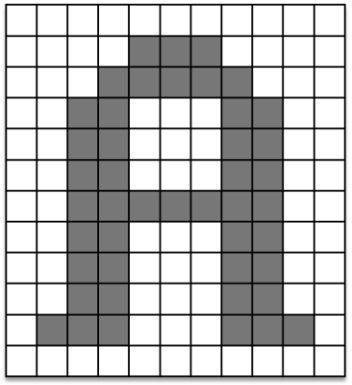
- Store normal in texture
- Map to tangent coordinate frame
- Need normals **n**
  and tangent field **t**
- Then: coordinate transform



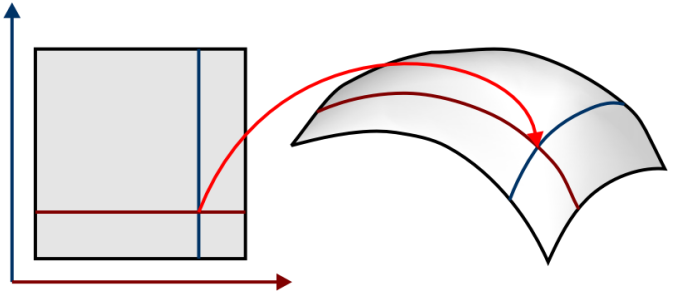
Modeling

Modeling Zoo

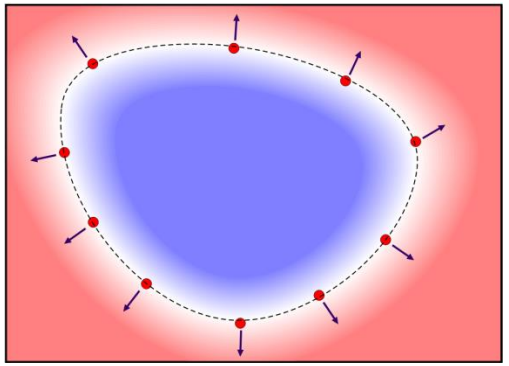
Space Discretization



Parametric Models

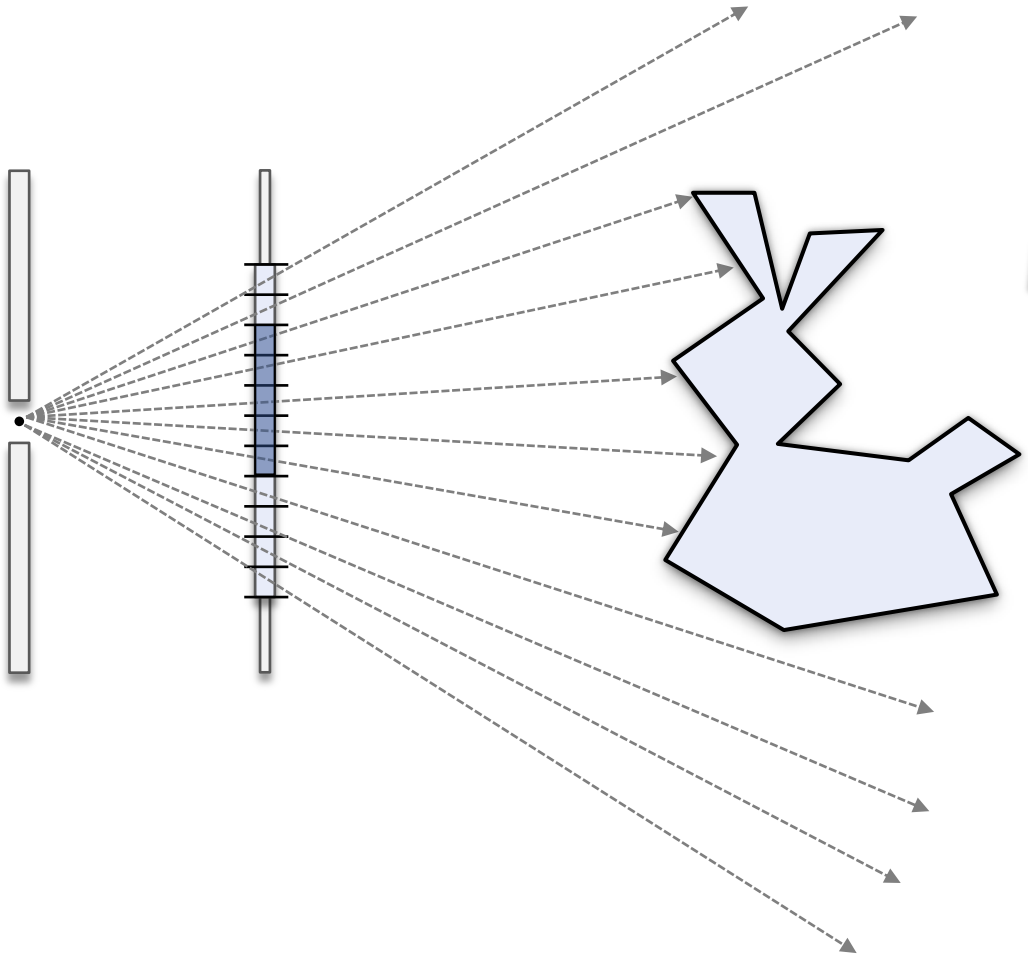


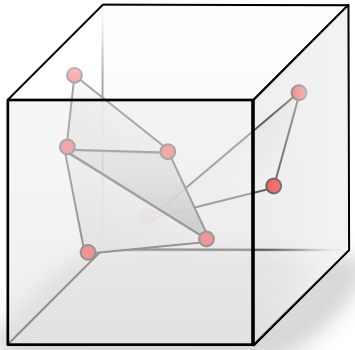
Implicit Models



Raytracing

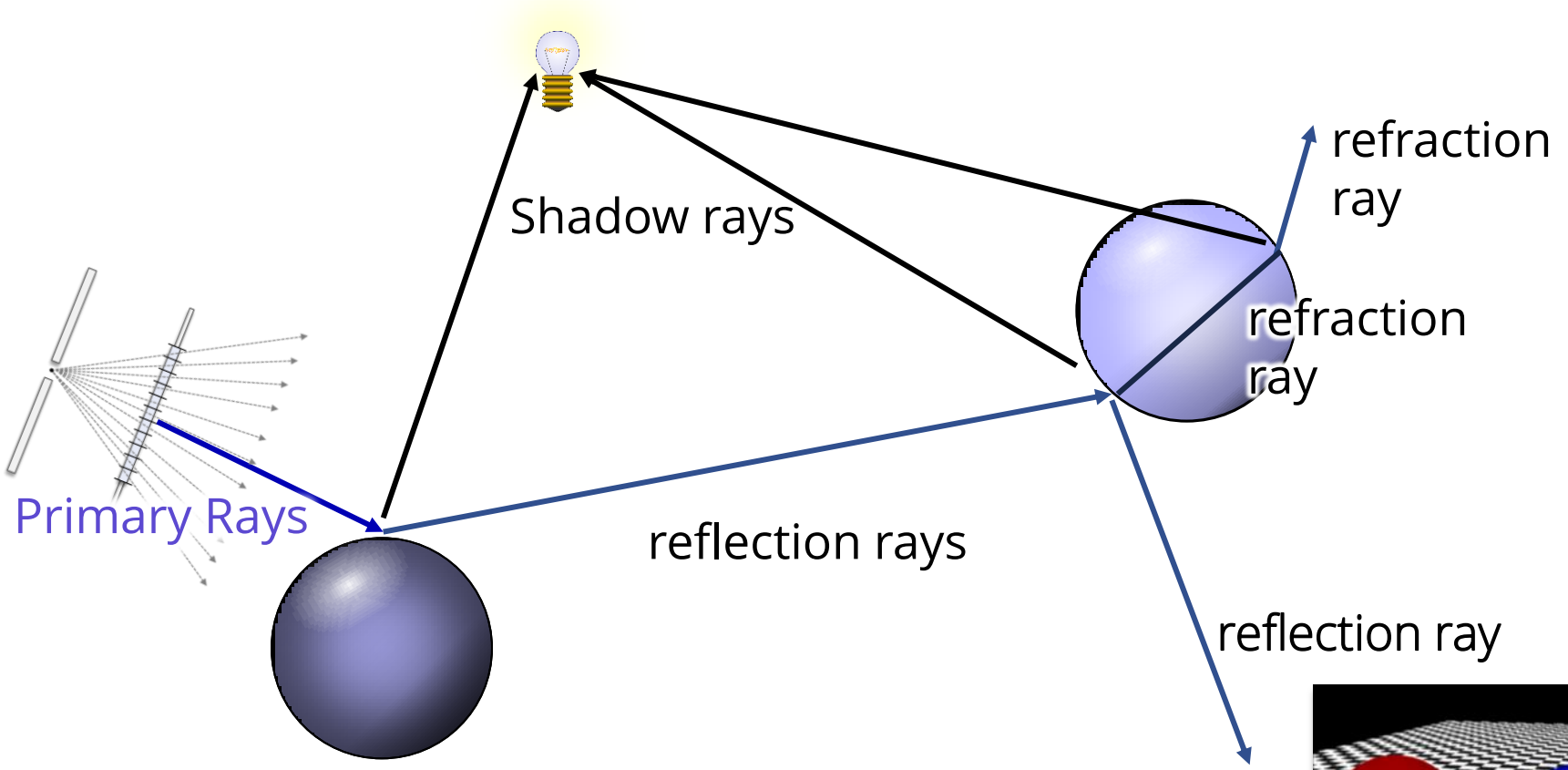
Raytracing

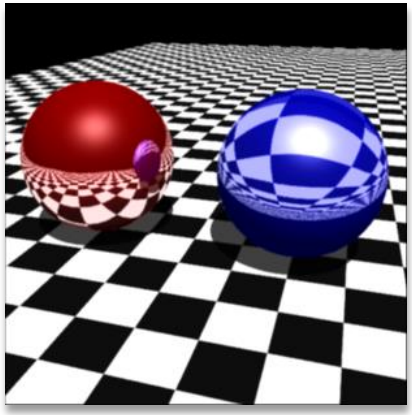




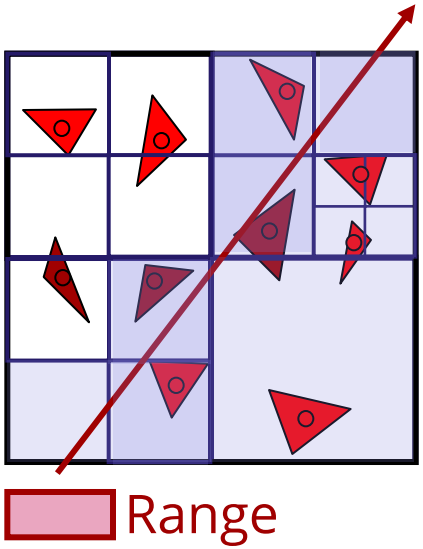
3D Scene

Recursive Raytracing





Raytracing

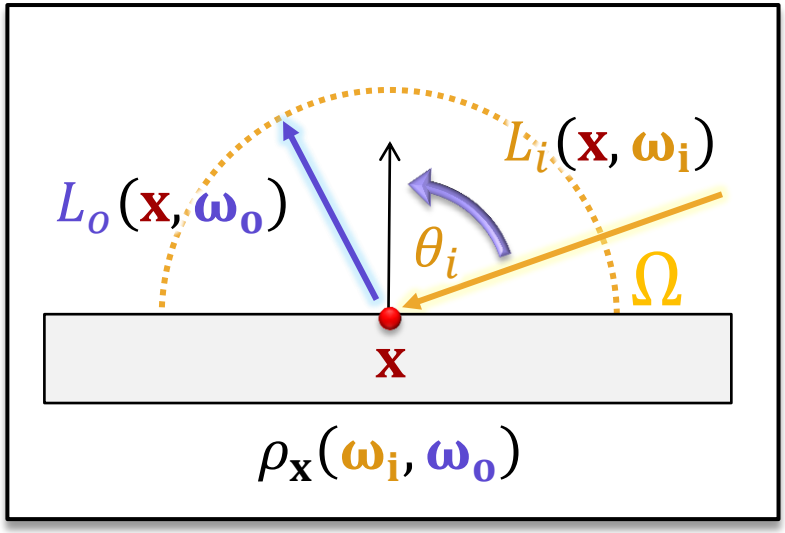


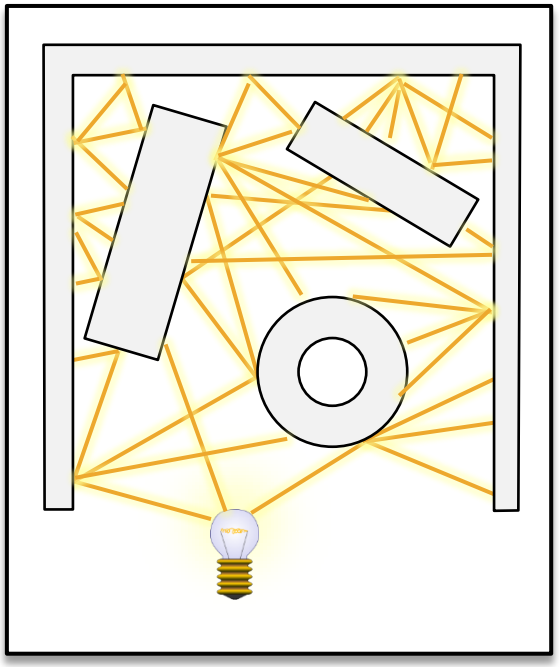
Raytracing: special case

- Ray is the range
- Early ray termination
  - Sorted recursion (child closer to the camera: first)
  - Stop after hit

Global Illumination

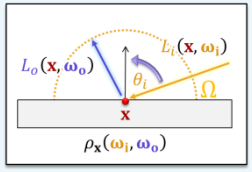
Rendering Equation





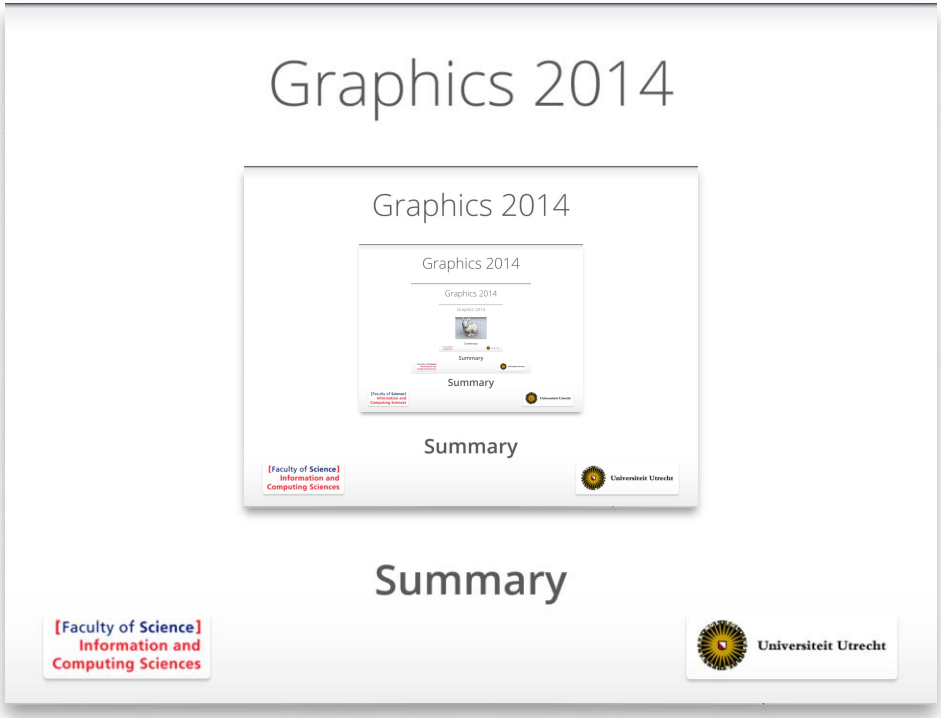
Rendering Equation

$$L_o(\mathbf{x}, \omega_o) = \underbrace{E_o(\mathbf{x}, \omega_o)}_{\text{emission}} + \underbrace{\int_{\omega_i \in \Omega} \left[L_i(\mathbf{x}, \omega_i) \cdot \rho_{\mathbf{x}}(\omega_i, \omega_o) \cdot \cos \theta_i \right] d\omega_i}_{\text{reflection}}$$



Summary

Graphics 2014



Summary

[Faculty of Science] Information and Computing Sciences

Universiteit Utrecht