

T1 answer sheet

2a. $(3, 10, 7)$

b. $\sqrt{2^2 + 6^2 + 3^2} = \sqrt{49} = 7.$

c. $\frac{1}{7}\vec{a} = (\frac{2}{7}, \frac{6}{7}, \frac{3}{7}).$

d. $2*1 + 6*4 + 3*4 = 38.$

e. $\begin{pmatrix} 2 \\ 6 \\ 3 \end{pmatrix} \times \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 6*2 - 3*3 \\ 3*2 - 2*2 \\ 2*3 - 6*2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix}$

f. any non-zero scaled version of $\vec{b}$, e.g. $-2\vec{b} = (-2, -8, -8).$

3a. $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y = b_x a_x + b_y a_y.$

b. same procedure as a.

c. $\begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} \times \begin{pmatrix} 2 \\ 6 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \\ 6 \end{pmatrix};$ compare to 2e to see that $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}.$

4. The dotproduct of a vector with itself is the square of it's magnitude.

5a. The angle is 90 degrees (the vectors are perpendicular).

b. The angle is greater than 90 degrees.

c. The angle is smaller than 90 degrees.

6. $-1 \begin{pmatrix} a \\ b \\ c \end{pmatrix} \times \begin{pmatrix} d \\ e \\ f \end{pmatrix} = -1 \begin{pmatrix} b*f - c*e \\ c*d - a*f \\ a*e - b*d \end{pmatrix} = \begin{pmatrix} c*e - b*f \\ a*f - c*d \\ b*d - a*e \end{pmatrix} = \begin{pmatrix} e*c - f*b \\ f*a - d*c \\ d*b - e*a \end{pmatrix} = \begin{pmatrix} d \\ e \\ f \end{pmatrix} \times \begin{pmatrix} a \\ b \\ c \end{pmatrix}.$

7. Direction: $1/\|\vec{a}\|$ is a scalar value; hence it does not affect the direction of $\vec{a}.$

Magnitude: we have to prove that the length of the normalized vector $\vec{n} = \vec{a}/\|\vec{a}\|$ is 1.

$$\|\vec{n}\| = \sqrt{\vec{n} \cdot \vec{n}} = \sqrt{\frac{\vec{n}}{\|\vec{n}\|} \cdot \frac{\vec{n}}{\|\vec{n}\|}} = \frac{1}{\|\vec{n}\|} \sqrt{\vec{n} \cdot \vec{n}} = \frac{1}{\|\vec{n}\|} \|\vec{n}\| = 1.$$

8a. $x_{screen} = (x_{world} + 2)/(2 + 2)*1024$

$$y_{screen} = (y_{world} + 1.5)/(1.5 + 1.5)*768$$

d. $x_{screen} = (x_{world} - x_1)/(x_2 - x_1)*W$

$$y_{screen} = (y_{world} - y_1)/(y_2 - y_1)*H$$

e. Width: $(x_2 - x_1)/W$; height: $(y_2 - y_1)/H.$

f. Replace y by $x*(H/W).$

g. $x_{world} = (x_{screen}/W + x_1) * (x_2 - x_1)$

$$y_{world} = (y_{screen}/H + y_1) * (y_2 - y_1)$$