Tutorial 1 - Basic maths / math recap / vectors

Introduction

The tutorials are designed to give you a way to practice the material discussed in the lectures. You can use the tutorial sessions that are arranged for you in April through June to get assistance on these tutorials. For the academic year 2014/215, assistance will be provided by TAs Forough Madehkhaksar, Coert van Gemeren and Anna Aljanaki, and student assistants Tigran Gasparian, Jordi Vermeulen, Casper Schouls, Sander Vanheste and Jan Posthoorn.

The exercises in these tutorials are representative for the exams (one halfway the block, one at the end).

Note that the answers to the exercises are not always directly available from the slides: it may (and will) require some tinkering to apply concepts. Feel free to ask for help doing this during the tutorial sessions, or on the forum:

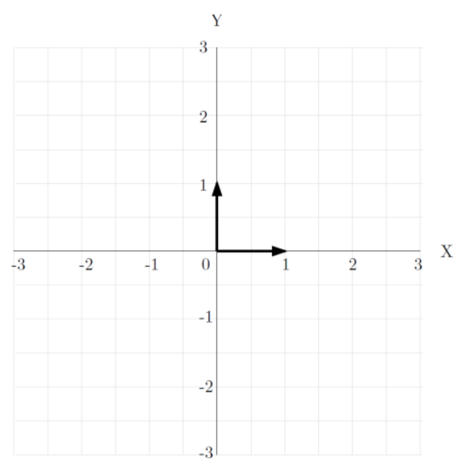
http://infogr2015.freeforums.org

These tutorials are partially derived from materials by Michael Wand and Wolfgang Hürst.

Basic vectors

Exercise 1.

Given: three vectors in $\mathbb{R}^2$: $a = \begin{pmatrix}1\\1\end{pmatrix}, b = \begin{pmatrix}2\\2\end{pmatrix}, c = \begin{pmatrix}1\\0\end{pmatrix}$.



a) Draw the vectors on a sheet of paper (using a grid such as the one on the right).
b) Compute the sum of a and b. Draw the result.
c) Scale vector a by the factors $1, 2, -1$ and $\sqrt{2}$. Draw the results.
d) Determine $b - c$ graphically, using the rule from the lecture: first, join the starting points of b and c, then draw an arrow from the tip c of to the tip of b, which gives you the result.
e) Compute and draw $2(a + c)$ and $2a + 2c$. Visualize how this gives the same result (distributive rule).
f) Build a linear combination v of the vectors a, b, c, i.e. $v = \lambda_1 a + \lambda_2 b + \lambda_3 c$.

Remark: This first assignment is only meant to familiarize yourself with geometric vectors. There is no big insight here. If you already took vector algebra in high-school, this should be very easy.

Exercise 2.

Given: three vectors $\vec{a} = (2,6,3)$, $\vec{b} = (1,4,4)$ and $\vec{c} = (2,3,2)$ in $\mathbb{R}^3$.

a) What is the sum of vectors $\vec{a}$ and $\vec{b}$?
b) What is the length of vector $\vec{a}$?
c) Calculate <u>unit</u> vector $\vec{u}$, which has the same direction as vector $\vec{a}$.
d) Calculate the dot product of $\vec{a}$ and $\vec{b}$.
e) Calculate the cross product of $\vec{a}$ and $\vec{c}$.
f) Create a vector $\vec{p}$ parallel to $\vec{b}$.

Exercise 3.

Commutative and associative properties of vector operations:

a) Show that the scalar product of two vectors is commutative, i.e. for any two random vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$.
b) Show that vector addition is associative, i.e. for any three random vectors $\vec{a}, \vec{b}$ and $\vec{c}$, $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$.
c) Show (using one concrete example) that the cross product is *not* commutative, i.e. $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$.

Exercise 4.

What is the relation between the magnitude of a vector and the dot product of the vector with itself?

Exercise 5.

Given: two vectors $\vec{a}$ and $\vec{b}$.

a) If $\vec{a} \cdot \vec{b} = 0$, what do we know about the angle between $\vec{a}$ and $\vec{b}$?
b) If $\vec{a} \cdot \vec{b} < 0$, what do we know about the angle between $\vec{a}$ and $\vec{b}$?
c) If $\vec{a} \cdot \vec{b} > 0$, what do we know about the angle between $\vec{a}$ and $\vec{b}$?

Exercise 6.

Show that for two random vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$.

Exercise 7.

Prove that if $\vec{a}$ is a random vector in $\mathbb{R}^2$, $\vec{a}/\|\vec{a}\|$ is a unit vector that points in the same direction as $\vec{a}$.

Coordinate systems

Graphics are drawn on a raster, which can have an arbitrary size. Common screen resolutions are 1600x1200, 1920x1080, 2560x1920. Especially on mobile devices, resolutions can differ greatly, which is why it is important to decouple screen coordinates from world coordinates.

Exercise 8.

Given:

- a screen with a resolution of 1024×768 pixels and pixel x-coordinates $0..1023$ (left to right) and y-coordinates $0..767$ (top to bottom);
- a rectangle in $\mathbb{R}^2$, with corners $(-2,1.5)$ (top left corner), $(2, 1.5)$ (top right corner), $(2, -1.5)$ (bottom right corner) and $(-2, -1.5)$ (bottom left corner).

a) Calculate x_{screen}, y_{screen} based on parameters x_{world} and y_{world}.
b) Verify your solution for the four corners, e.g. $(2, -1.5)$ should yield $(1023,767)$.
c) Verify your solution for the world origin $(0,0)$.
d) Generalize your solution for a screen of $W \times H$ pixels, and a world rectangle of $x_1..x_2, y_1..y_2$.
e) For a screen of $W \times H$ pixels and a world rectangle of $x_1..x_2, y_1..y_2$, what is the size of a pixel in world coordinates?
f) Modify your solution for d) so that it only requires $x_1..x_2$; use the screen aspect ratio to determine $y_1..y_2$.

For the general solution of d), calculate the inverse operation, i.e. determine x_{world} and y_{world} based on parameters x_{screen} and y_{screen}.

The End

(for now)