T2 answer sheet

1a. dx = 4 – (-2) = 6, dy = 1 – 0 = 1 ➔ a = 1/6
c = 0 + 2 * 1/6 = 1/3
➔ y = 1/6x + 1/3.

c. A = -1, B = 6
C = -$\vec{N} \cdot P$ = -((-1,6) · (4,1)) = -1*(-4+6) = -2
-1x + 6y - 2 = 0.

d. (-1,6) and (1,-6) (and all scaled versions of these)

e. See c

f. 'C' is the distance of the line to the normal, times the magnitude of the normal. In this case, the magnitude of the normal is sqrt(1*1+6*6)=sqrt(37); the line is thus at a distance of 2/sqrt(37) from the origin (~0.329).

2a. Draw the line and the normal $\vec{n}$. A vector $\vec{p}$ from the origin to an arbitrary point p on the line is parallel to the line, and thus perpendicular to the normal, hence $\vec{n} \cdot \vec{p} = 0$.

b. Draw a line (not through the origin). Pick two points on the line, p and p' ($p \neq p'$). The vector $p - p'$ is perpendicular to $\vec{n}$, and thus $\vec{n}.(p - p') = 0$. Rewriting this guives $\vec{n}.p - \vec{n}.p' = 0$.

3a. dx = 4-(-2)=6, dy = 1-0 = 1, dz = 5-1 = 4;
length = sqrt(dx * dx + dy * dy + dz * dz) = sqrt(53) = ~7.28.

b. p(t) = (-2,0,1) + t * (6, 1, 4).

c. e.g. -1, 6, 0 (swapping x/-y, zeroing z) and -4, 0, 6 (swapping x/-z, zeroing y).

d. Unlimited, in a disc of radius 1 around the line.
Note: vectors do not have a position, so a disc or cylinder 'around the line' is a bit misleading.

4a. dx = 6, dy = 1, dz = 4, dw = -2.
Normals: e.g. (-1,6,0,0) (swapping x/-y, zeroing y and z) and (0,0,2,4) (swapping z/-w, zeroing x and y).

b. Unlimited, in a sphere of radius 1 around the 4D line.

5. Slope-intersect to implicit: shuffle to isolate 0;
Implicit to slope-intersect: shuffle to isolate y;
Implicit to parametric: determine two points on the line: after normalization, one point is the origin plus the normal times C; the second point is the first point plus (B,-A).

6. Consider l and a point p that lies in the halfplane to which (a,b) points. Let p_0 be the perpendicular projection of p onto l. We can write: $p = p_0 + \lambda(a, b)$, with $\lambda > 0$. The distance $f(p_x, p_y)$ of p to l is then $f(p_0) + \lambda\sqrt{a^2 + b^2}$. Since $f(p_0) = 0$ (it is on the line), $\lambda > 0$, and the square root must be greater than zero, we have that $f\left(p_x, p_y\right) > 0$.

7a. $(x - 3)^2 + (y - 5)^2 - 9 = 0$.

b. $p(ø) = \begin{pmatrix} 3 + 3\cos ø \\ 5 + 3\sin ø \end{pmatrix}$

8a. $p(a,b) = \begin{pmatrix} 3 + 3.14\cos a \sin b \\ 5 + 3.14 \sin a \sin b \\ 1 + 3.14 \cos b \end{pmatrix}$

b. using the result from a, use e.g. a = 0, b = 0 and a = 0, b = π.

9a. E.g., (0,1,0) and (0,0,1).

b. (1,0,0).

c. 4.

d. An infinite number: all planes parallel to the x-axis (such as y=0 and z = 0).

10a. $f(u,v) = p_1 + u(p_2 - p_1) + v(p_3 - p_1)$.

b-d. Solve this by calculating a vector from p_1 to p_2, and one from p_1 to p_3; determine the cross product between these vectors to find the normal of the plane. This yields A,B and C for the general implicit plane equation. Determine D by filling in p_1.

11a,b. $f(t) = p_1 + t(p_2 - p_1)$.

c. (0,7,6); the lines are linearly independent, but share a point, which must therefore be the only intersection point.

12a. $(x-3)^2 + (y-3)^2 + (z-3)^2 - r^2 = 0$

$r = ||p - c|| = sqrt(1 + 4 + 4) = 3.$

b. $d = ||q - c|| - r$

c. The normal of the plane is the vector $p - c$; use p itself to find D.

d. Swap e.g. x/-y and y/-z to obtain two tangent vectors.