# Tutorial 2 - Geometry

## Basic geometric entities

### Exercise 1.

Given: a line in $\mathbb{R}^2$ through two points (-2,0) and (4,1).

a) What is the slope-intersect form of this line?
b) Verify your answer for a) for the two given points.
c) What is the implicit form of the line?
d) Determine two normals for the line, with different directions.
e) Determine the general implicit representation for the line, i.e. in the form $Ax + By + C = 0$.
f) What is the relation between the distance of the line to the origin, 'C' in the general representation, and the magnitude of the normal?

### Exercise 2.

Let $l$ be a line in $\mathbb{R}^2$ through the origin, and let $\vec{n}$ be a normal vector for $l$.

a) Show geometrically that all points $p$ that lie on the line satisfy $\vec{n} \cdot \vec{p} = 0$.

Now, let $l$ be a line in $\mathbb{R}^2$ that does not go through the origin, and let $\vec{n}$ be a normal vector for $l$.

b) Show geometrically that all points $p$ that lie on the line satisfy $\vec{n} \cdot \vec{p} - \vec{n} \cdot \overrightarrow{p'} = 0$, where $p'$ is an arbitrary point on the line.

Note: "geometrically" here means that you can solve this question by drawing an image and using it to explain the solution.)

### Exercise 3.

Given: a line in $\mathbb{R}^3$ through two points, (-2,0,1) and (4, 1, 5).

a) What is the length of this line?
b) What is the *parametric* representation of this line?
c) Determine two normals for the line, with different directions.
d) How many normals of unit length exist for this line, and how are they organized?

## Exercise 4.

Given: a line in $\mathbb{R}^4$ through two points, (-2,0,1,1) and (4, 1, 5, -1).

a) Determine two normals for the line, with different direction.
b) How many normals of unit length exist for this line, and how are they organized?

## Exercise 5.

In the lecture, we discussed three different kinds of representations of a line: the parametric representation, the implicit representation, and the slope-intercept representation. Write down the general form of these equations and explain how we can do a conversion from one form to another. Take an example and transfer it into the other representations.

## Exercise 6.

Let $l$ be a line in $\mathbb{R}^2$ with implicit equation $f(x, y) = ax + by + c = 0$. For points that do not lie on the line, we have $f(x, y) \neq 0$. We say that the line $l$ splits $\mathbb{R}^2$ in two half-spaces: the positive half-space $l^+$, containing the points in $\mathbb{R}^2$ that lie on the side of $l$ to which the normal vector $(a, b)$ points, and the negative half-space $l^-$, containing the remaining points not on $l$. Show that the name *positive half-space* is appropriate by proving that $f(x_p, y_p) > 0$ for any point $p = (x_p, y_p)$ that lies in the halfspace to which the normal vector $(a, b)$ points.

## Exercise 7.

Given: a circle in $\mathbb{R}^2$ with radius $r = 3$, and center $c = (3,5)$.

a) What is the implicit representation of the circle?
b) What is the parametric representation of the circle?

## Spheres and planes

### Exercise 8.

Given: a sphere in $\mathbb{R}^3$ with radius $r = 3.14$ and center $c = (3,5,1)$.

a) What is the parametric representation of the sphere?
b) Using the parametric representation, calculate two opposing points on the sphere.
c) What is the distance between the points calculated in b) ?

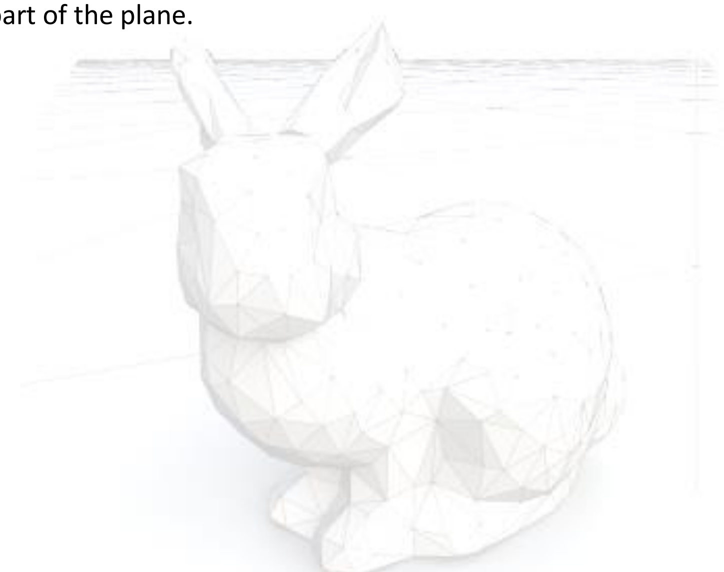
### Exercise 9.

Given: a plane in $\mathbb{R}^3$, for which the following is true: the x-coordinate of all points on the surface is 1.

a) Determine two linearly independent vectors in this plane.
b) Determine the normal of the plane.
c) Calculate the distance of point $p = (5,1,1)$ to the plane.
d) How many planes exist that are perpendicular to the plane, and have a distance $d = 0$ to the origin?

### Exercise 10.

a) Give a parametric equation for the plane $V$ in $\mathbb{R}^3$ through the points $p_1 = (0,7,6)$, $p_2 = (8,0,8)$, and $p_3 = (12,10,0)$.
b) Give an implicit equation for the plane from the previous sub-problem.
c) Verify that the tree points are indeed part of the plane.
d) Now give a normal vector to the plane.



## Exercise 11.

It's generally not possible to create an implicit representation of "1-dimensionalish" curves (e.g. lines, circles, spirals) in 3D. But with the parametric representation we can. For example, we can create a spiral using the $sin$ and $cos$ functions. Another (rather simple, but very useful) example is a line in $\mathbb{R}^3$.

a) Construct the parametric equation of a line $l_1$ in $\mathbb{R}^3$ that goes through the points $p_1 = (0,7,6)$ and $p_2 = (8,0,8)$.
b) Now create the parametric equation of a line $l_2$ in $\mathbb{R}^3$ that goes through $p_1 = (0,7,6)$ and $p_3 = (12,10,0)$.
c) What is the intersection point of both lines? You can either calculate it, or write it down directly. In the latter case, you have to argue why this is an intersection point, and also why it is the only one.

## Exercise 12.

Given: a sphere in $\mathbb{R}^3$, with center $c = (3,3,3)$ and a point on the surface of the sphere, $p = (2,5,1)$.

a) Determine the implicit representation of the sphere.
b) Calculate the distance of the sphere to point $q = (4,9,-1)$ (note: this is not the distance to the centre of the sphere).
c) Define a plane that has $p$ as the only intersection point with the sphere (i.e., the tangent plane of the sphere at position $p$).
d) Define two perpendicular unit vectors in this plane, which are both also perpendicular to the normal of the sphere in $p$.

The vectors found in exercise 12, together with the normal of the sphere in $p$ span up an orthonormal basis, which we call *tangent space*. We operate in this space e.g. for doing normal mapping.

# The End

*(for now)*