T3 answer sheet

1a. $A + A = 2A = \begin{pmatrix} 2 & 6 \\ 4 & 4 \\ 6 & 2 \end{pmatrix}$.

b. No; $n_A \neq m_B$.

c. $A^T = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$.

d. $A\,A^T = \begin{pmatrix} 10 & 8 & 6 \\ 8 & 8 & 8 \\ 6 & 8 & 10 \end{pmatrix}$.

2a. $|A|$=17

b. $|B|$=-17

c. The column vectors in A are defined counter-clockwise; in B they are defined clockwise.

3. This is the determinant of matrix $A = \begin{pmatrix} 1 & 7 \\ 5 & -1 \end{pmatrix}$, which is -36.

4a. det $\begin{pmatrix} -2 & 2 & 3 \\ -1 & 1 & 3 \\ 2 & 0 & -1 \end{pmatrix} = -2\begin{vmatrix} 1 & 3 \\ 0 & -1 \end{vmatrix} * -1^2 + 2\begin{vmatrix} -1 & 3 \\ 2 & -1 \end{vmatrix} * -1^3 + 3\begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix} * -1^4$

$= -2(1 * -1 + 3 * 0 * -1) * 1 + 2(-1 * -1 + 3 * 2 * -1) * -1 + 3(-1 * 0 + 1 * 2 * -1) * 1$

$= -2 * -1 + 2 * -5 * -1 + 3 * -2 = 6.$

b. $(-2 * 1 * -1 + 2 * 3 * 2 + 3 * -1 * 0) - (3 * 1 * 2 + 2 * -1 * -1 + (-2) * 3 * 0)$

$(2 + 12 + 0) - (6 + 2 - 0) = 6.$

5a. 0

b. Column 3 is column 1 scaled by -2; the three axes do not span a volume but a plane.

6. Here, it is sufficient to provide one case where, if $AB = AC$, $A \neq C$, e.g.:

$A = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}, C = \begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix}$

AB is the null matrix; AC is the null matrix, but $B \neq C$.

7. Write out $A^{\mathsf{T}}A$ in its general form. Now write down what a coefficient in the main diagonal of the resulting matrix would look like. If done correctly, we see that it is the dot product of a column vector of A with itself. Because of orthonormality, this will yield 1.

Now write down the general form of a random coefficient a_{ij} with $i \neq j$. If done correctly, we see that this is the dot product of two non-equal column vectors of the matrix. Since these are perpendicular to each other, the dot product yields 0. Because both the a_{ii} coefficient from the first part, and the a_{ij} coefficient from the second part have been chosen randomly, the proven statements applies for every possible i and i, j-pair (with $i \neq j$). Hence, it follows that the resulting matrix is the identity matrix.

8a. $C = \begin{pmatrix} 7 & -5 & 2 \\ -1 & 5 & -2 \\ -2 & -2 & 2 \end{pmatrix}$

b. $\tilde{A} = C^T = \begin{pmatrix} 7 & -1 & -2 \\ -5 & 5 & -2 \\ 2 & -2 & 2 \end{pmatrix}$

c. |A|=6

d. $A^{-1} = \frac{C^T}{|A|} = \begin{pmatrix} 7/6 & -1/6 & -2/6 \\ -5/6 & 5/6 & -2/6 \\ 2/6 & -2/6 & 2/6 \end{pmatrix}.$

9a. $A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

b. $A^{-1} = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}$

c. $A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

d. This matrix cannot be inverted; it's determinant is 0.

10. $A = \begin{pmatrix} cos\emptyset & -\frac{1}{4}sin\emptyset \\ 4sin\emptyset & cos\emptyset \end{pmatrix}$ Thanks Jorrit & Joost!

(note: no need to memorize rotation matrices for the exam)

11a. See slides.

b.

c. No; the normals need normalization.

12. $A = \begin{pmatrix} 1 & 0 & x_t \\ 0 & 1 & y_t \\ 0 & 0 & 1 \end{pmatrix}$