Tutorial 3 - Matrices

Basics

Exercise 1.

Given: a 3×2 matrix $A = \begin{pmatrix} 1 & 3 \\ 2 & 2 \\ 3 & 1 \end{pmatrix}$.

a) Calculate the sum of A with itself.
b) Can you multiply A with itself? If so, do this; otherwise explain why not.
c) Determine the transpose of A^T of A.
d) Multiply A with A^T.

Exercise 2.

Given: matrix $A = \begin{pmatrix} 3 & 1 \\ 1 & 6 \end{pmatrix}$ and $= \begin{pmatrix} 1 & 3 \\ 6 & 1 \end{pmatrix}$.

a) Calculate the determinant of A.
b) Calculate the determinant of B.
c) What is the geometric interpretation of these two results?

Exercise 3.

Given: vector $\vec{a} = (1,5)$ and $\vec{b} = (7,-1)$.

Calculate the area of the parallelogram defined by the two vectors.

Exercise 4.

Given: matrix $A = \begin{pmatrix} -2 & 2 & 3 \\ -1 & 1 & 3 \\ 2 & 0 & -1 \end{pmatrix}$.

a) Calculate the determinant of A using Laplace's expansion.
b) Verify your answer using the Rule of Sarrus.

Matrix characteristics

Exercise 5.

Given: matrix $= \begin{pmatrix} 1 & 4 & -2 \\ 3 & 2 & -6 \\ -2 & 5 & 4 \end{pmatrix}$.

a) Calculate the determinant of matrix A using the Rule of Sarrus.
b) Explain the result for a) geometrically.

Exercise 6.

Given:

- $n_A \times m_A$ matrix A;
- $m_A \times m_B$ matrix B;
- $m_A \times m_C$ matrix C.

Prove, that if $AB = AC$, it does not necessarily follow that $B = C$ (even if A is not the null matrix).

Exercise 7.

A matrix is called orthogonal if all column vectors are mutually perpendicular. A matrix is called orthonormal if it is orthogonal and all column vectors have unit length. Show that the inverse of an $n \times n$ orthonormal matrix is its transpose. Hint: $A^{-1}A = I$, so you can solve this by proving that $A^{T}A = I$.

Matrix inversion

Exercise 8.

Given: matrix $= \begin{pmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 0 & 2 & 5 \end{pmatrix}$.

a) Calculate the cofactor matrix C.
b) Calculate the adjoint matrix $\tilde{A} = C^T$.
c) Calculate the determinant for A.
d) Calculate the inverse A^{-1} for A using the results for a), b) and c).

Coordinate systems

Exercise 9.

a) Write down a matrix that scales 2D vectors by a factor of 2.
b) Determine the inverse of this matrix.
c) Write down the matrix that projects 3D vectors on the $x = 0$ plane.
d) If possible, determine the inverse of this matrix. Otherwise, explain why this is not possible.

Exercise 10.

Determine the matrix that rotates vectors along an ellipse centered around the origin, with a height of 4 and a width of 1.

Exercise 11.

Normals must be transformed using $(A^{-1})^T$.

a) Explain why (the solution was given in the lecture, but try to do this yourself).
b) Verify using the example of shearing and a few normals that the transform is correct.
c) Will be proposed transform maintain the magnitude of normals after transformation?

Exercise 12.

Determine a matrix that translates points in $\mathbb{R}^2$ by x_t, y_t.

The End

(for now)