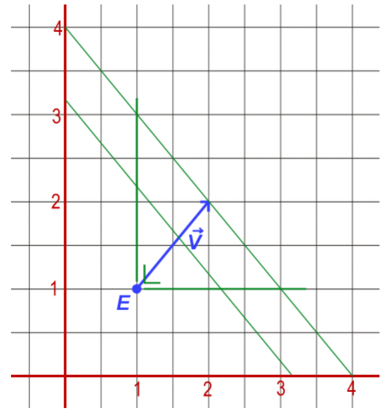
**PART 1 – Mathematics – For 35 Points**

1. Given: an eye position $E = (1,1,1)$, a view vector $\vec{V} = (1,0,1)$, an up vector $\overrightarrow{up} = (0,1,0)$, a FOV of 90 degrees and a screen resolution of 512x512 pixels. *(for 5x5 points)*

    a. For the purpose of ray tracing, calculate the four corners of a virtual screen plane at a distance 1 from $E$, perpendicular to $\vec{V}$ and $\overrightarrow{up}$, taking into account the specified FOV.




The geometry for this question is shown on the right. Note that V is not normalized, yet the requirement is that the plane is at a distance of 1 from E, and must thus lie on the closest green line. It can now be easily seen that for the left side of the screen, $x = 1$ and $z = 1 + \sqrt{2}$; on the right, $z = 1$ and $x = 1 + \sqrt{2}$. Since the screen plane is 2 wide, it must also be 2 high, and therefore the y-coordinates are 0 and 2.

    b. Determine the normalized direction of ray $r$, which originates from $E$ and extends through pixel (0,256).

Pixel 0,256 is on the far left side of the plane, halfway y=0 and y=2. Without further calculations its position must be $(1,1,1 + \sqrt{2})$. A vector from E to this point is $(0,0,\sqrt{2})$; normalized this is simply (0,0,1).

    c. Determine the 3D coordinate of the intersection of $r$ with the plane: $-X - Z = 10$.

Normal of the plane: (-1,0,-1) (with d=-10). Use the equation from the slides (or derive it if you forgot it): $t = (O \cdot \vec{N} + d)/(\vec{D} \cdot \vec{N})$; $t = (-2 - 10)/-1 = -12$; $I = O + t\vec{D} = (1,1,-11)$. Verify in the plane equation: $-1 + 11 = 10$. A valid answer for this question is either the calculated point $I$, or 'no intersection', because $t$ is negative. Calculating the intersection correctly with a different ray is also OK (in case you got 1b wrong).

    d. Calculate the intersection of $r$ with the sphere $||p - C|| = \sqrt{20}$, where $C = (-1,1,-1)$.

Using the formulas from the slides: a=1; b=4; c=-12 ➔ $t = \frac{-4\pm\sqrt{4^2+48}}{2}$; $t = 2$ or $-6$. Using t=2, we get $I = (1,1,3)$. Calculating the intersection correctly with a different ray is also OK (in case you got 1b wrong), but probably more difficult.

    e. Point $P = (\sqrt{10} - 1, 1, \sqrt{10} - 1)$ is a point on the sphere $||p - C|| = \sqrt{20}$. Calculate the normalized shading normal for $P$ on the <u>inside</u> of the sphere.

Take a vector from P to C: $(1 - (\sqrt{10} - 1), 0, 1 - (\sqrt{10} - 1))$ and normalize it. Since the x and y component are the same, the normalized vector must be $(\frac{-1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}})$.

2. Given: an eye position $E = (-2,0,1)$, a view vector $\vec{V} = (2,1,4)$ and an up vector $\overrightarrow{up} = (0,1,0)$. Construct the orthonormal view ('look-at') matrix. *(for 5 points)*

In our matrix, $z$ will be the normalized version of $\vec{V}$, and $x$ will be the vector perpendicular to $\vec{V}$ and $\overrightarrow{up}$. Finally, $y$ is the vector perpendicular to $x$ and $z$. Normalization can be postponed until you have the three vectors, to avoid having to work with unpleasant numbers. This process yields:

$z = \vec{V};\ x = \overrightarrow{up} \times \vec{V} = (4{,}0,-2);\ y = z \times x = (-2{,}20,-4)$.
Normalizing these vectors: divide $x$ by $\sqrt{20}$, $y$ by $\sqrt{220}$ and $z$ by $\sqrt{21}$.
Construct the final matrix as a 4x4 matrix using x, y and z. The translation is $(x \cdot -E, y \cdot -E, z \cdot -E)$.
This process is described in Tutorial 4 q. 6, and in the book.

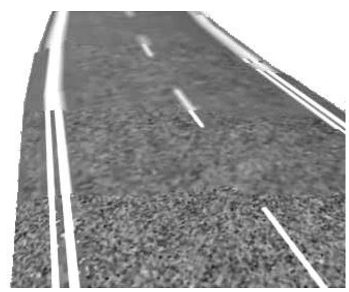
3. Given: a matrix with the column vectors $x = (\frac{3}{5}, 0, \frac{-4}{5})$, $y = (\frac{7}{25}, \frac{24}{25}, 0)$ and $z = \left(\frac{4}{5}, 0, \frac{3}{5}\right)$. The column vectors are unit vectors, but the matrix is not orthonormal. Make this matrix ortho-normal without changing the view direction (i.e., do not change the z-vector). *(for 5 points)*

This question is easy when you realize x and z are perpendicular: 0.6*0.8+(-0.8)*0.6=0. The y-axis must be the problem; without further calculations it can simply be set to (0,1,0) as this vector will be perpendicular to x and z.

**PART 2 – Graphics Theory – For 65 + 10 Points**

4. *Observe the three images below, and fill in the missing parts.* *(for 10 points)*

The left image shows blocky textures due to ……oversampling….. . In the center image, this is not as noticeable thanks to the use of …bilinear interpolation…. . In the image on the right, distant road sections would have suffered from …undersampling.. , but clearly ……MIP-mapping…. was used to combat this. The horizontal bands would have been reduced if the renderer would also have supported …trilinear filtering…. .

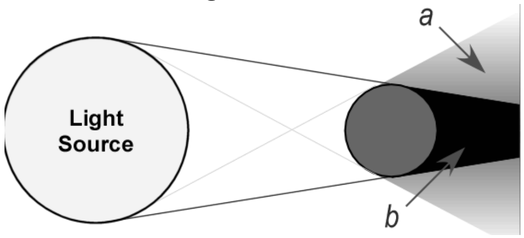



5. Culling schemes that mark some objects as visible when they are not (but not the other way round) are called: (for 5 points)

………conservative…………

6. *Mark each correct option. There may be more than one correct option. (for 5 points)*
When using guard bands, the following polygons are <u>not</u> rasterized:
   a. Polygons outside the visible screen area <u>and</u> the guard band
   b. Polygons outside the visible screen area, but (partially) inside the guard band
   c. Polygons partially inside the visible screen area, and partially in the guard band
   d. Polygons completely inside the visible screen area.

7. Observe this image, and label areas *a* and *b*. *(for 5 points)*




Area 'a' is the ...Penumbra.....................

Area 'b' is the ...Umbra.........................

8. Name three methods we can use to suggest bright lights on a regular monitor. *(for 10 points)*

Method 1: ...Lens flares.............................................................

Method 2: ...Bloom / glow........................................................

Method 3: ...Exposure control / tone mapping.....................

9. Which of the following is <u>not</u> a spatial subdivision scheme? Mark one option. *(for 5 points)*

a) A nested grid; b) a kD-tree; c) a BVH using spheres; d) a quad-tree.

10. Glass causes two types of scattering. Name both. *(for 5 points)*

First type: ...Reflection.........................

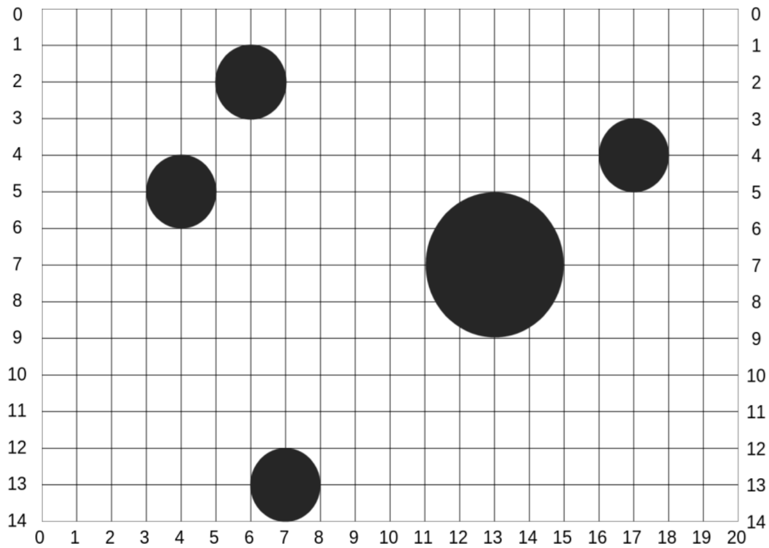
Second type: ...Refraction / transmission..

*11.* What is the difference between Distribution Ray Tracing (Cook) and Path Tracing (Kajiya)? Mark one option. *(for 5 points)*

a. Distribution Ray Tracing cannot render depth of field, Path Tracing can.
b. Distribution Ray Tracing uses a ray tree, Path Tracing doesn't.
c. Distribution Ray Tracing is based on ray optics, Path Tracing isn't.

12. Given: a scene with 5 discs, as shown below. Determine the best split using the Surface Area Heuristic. Consider <u>only</u> vertical splits, assume integer coordinates only, and assume that objects on a split plane are assigned to the left side. Write down the best split plane position, the cost before splitting and the cost after splitting. *(for 15 points)*
*Note: since this is a 2D scene, use circumference as you would normally use surface area.*

Splitting at x=6, we get:
left box = 4 by 5, circumf. = 18, N=2; right box = 12 by 11, circumf. = 46, N=3; cost: 2x18 + 46x3 = 174.
Splitting at x=7, we get:
left box = 5 by 13, circumf. = 36, N=3; right box = 6 by 7, circumf. = 26, N=2; cost: 3x36 + 26x2 = 160.
Splitting at x=13, we get:
left box = 12 by 13, circumf. = 50, N=4; right box = 2 by 2, circumf. = 8, N=1; cost: 50x4 + 8x1 = 208.
Splitting at 7 is optimal.
The cost of not splitting: box = 15x13, circumf. = 56, cost = 280.



Cost before splitting:

………280……………………………………

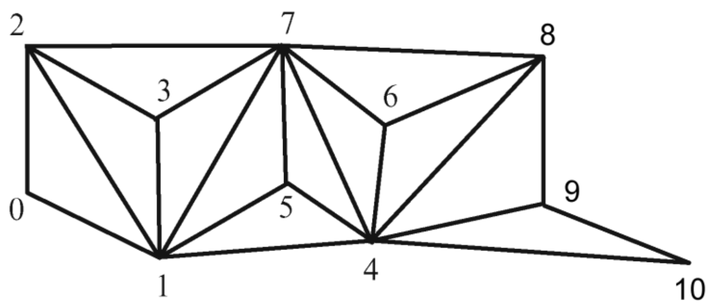
Position of best split plane:

………7……………………………………………

Cost after splitting:

……160………………………………………

13. Observe the mesh in this image, with vertices 0..10. The mesh can be represented by a single triangle strip. Write down the layout of the strip. *(for 10* ***BONUS*** *points)*




Note: this is an unfold cube. ☺

Answer:

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Many of you managed to solve this one, which I made a bonus question because I thought it was evil. The correct solution is: 0 1 2 3 7 1 5 4 7 6 8 4 9 10. Note that each triangle must use the last two vertices in the strip.