Part 1: Vectors (20pts out of 100pts)

Five multiple choice questions, for 1 point each:
clearly mark the correct answer. Note: for these questions, there is only one correct answer.

1. The two vectors (-2,1) and (1,2) are…

   a) …linearly dependent of each other
   b) …forming an orthonormal basis
   c) …perpendicular to each other
   d) …pointing in the opposite direction of each other
   e) none of the above.

2. The scalar product (aka dot product) of two perpendicular vectors is

   a) …0
   b) …1
   c) …2 PI
   d) …-2 PI
   e) none of the above.

3. If s is a scalar value and $\vec{v}, \vec{w}$ are two vectors in $\mathbb{R}^3$, then the result of $s + (\vec{v} \times \vec{w}) \cdot (\overrightarrow{v \times w})$ is…

   a) …a vector in $\mathbb{R}^3$
   b) …a scalar
   c) …undefined
   d) …a 3 by 3 matrix.

4. If the angle between two vectors (both having a non-zero magnitude) is greater than $90°$ and smaller than $270°$, then the scalar product (dot product) of these vectors is…

   a) …positive
   b) …negative
   c) …undefined
   d) …positive when the angle is smaller than $180°$, negative when the angle is greater than $180°$

5. If the scalar product (dot product) of two unit vectors is zero, they are…

   a) …linearly dependent
   b) …forming an orthonormal basis
   c) …pointing in the same direction
   d) …at an angle of 180 degrees to each other.

Four open questions, for 2 points each:
provide the correct answer. There is no need to detail your solution.

6. What is the Euclidean length $||\vec{v}||$ of vector $\vec{v} = (0,3,4,0)$?

Answer: $||\vec{v}||$ = $\sqrt{0^2 + 3^2 + 4^2 + 0^2} = \sqrt{25} = 5$

7. What is the scalar product (or dot product) $\vec{v} \cdot \vec{w}$ of the two vectors $\vec{v} = (0,5,-2)$ and $\vec{w} = (3,1,-2)$?

Answer: $\vec{v} \cdot \vec{w}$ = (0*3)+(5*1)+(-2 * -2) = 9
Note: if you interpreted the vectors as (0.5 , -2) and (3.1 , -2), you scored full points for 5.55.

8. What is the cross product $\vec{v} \times \vec{w}$ of the two vectors $\vec{v} = (0,5,-2)$ and $\vec{w} = (3,1,-2)$?

Answer: $\vec{v} \times \vec{w}$ = (-8, -6, -15)

9. Given vector $\vec{u} = (2,2)$, create a unit vector $\vec{u}'$ that points in the same direction as $\vec{u}$.

Answer: $\vec{u}\ /\ length(\vec{u}) = \vec{u}/\sqrt{8} = (2/\sqrt{8}, 2/\sqrt{8})$. *(and all further reductions of this)*

10. Prove that $\lambda(-y, x)$ is a normal vector of (x, y) for all $\lambda \neq 0$.

If $\lambda(-y, x)$ is a normal vector of (x, y), then $\lambda(-y, x) \cdot (x, y) = 0$.
$\lambda(-y, x) \cdot (x, y) = (\lambda * -y * x) + (\lambda * x * y) = 0$. QED

Stating that the normal of a vector (x,y) is (-y,x) is repeating the question, and not a proof.

End of Part 1: Vectors.

Part 2: Basic Geometric Entities (20pts out of 100pts)

Five multiple choice questions, for 1 point each:
clearly mark the correct answer. Note: for these questions, there is only one correct answer.

1. If $y = ax + c$ denotes the *slope-intersect* form of a line in 2D, then c gives us…

   a) …the slope of the line
   b) …the fraction of the slope in the x-direction
   c) …the fraction of the slope in the y-direction
   d) …the intersection of the line with the x-axis
   e) …the intersection of the line with the y-axis

2. If $2x - y + 5 = 0$ denotes the *implicit* representation of a line in 2D, then the vector $(2, -1)$ is…

   a) …a point on the line
   b) …a vector parallel to the line
   c) …a vector perpendicular to the line
   d) none of the above.

3. If $p(t) = (1,1) + t(-2,1)$ denotes the *parametric* equation of a line in 2D, then $(1,1)$ is…

   a) …a point on the line
   b) …a vector parallel to the line
   c) …a vector perpendicular to the line
   d) none of the above.

4. The equation $2x + y + z = 0$ represents…

   a) …the implicit representation of a line in 3D
   b) …the implicit representation of a plane in 3D
   c) …the implicit representation of a line or a plane in 3D
   d) none of the above.

5. In $\mathbb{R}^3$, the equation $(x-3)^2 + (y-3)^2 + (z-3)^2 - 9 = 0$ represents…

   a) …the implicit representation of a sphere with radius 9 and center $(3,3,3)$
   b) …the implicit representation of a sphere with radius 3 and center $(\mathbf{3}, \mathbf{3}, \mathbf{3})$
   c) …the implicit representation of a sphere with radius 9 and center $(-3, -3, -3)$
   d) …the implicit representation of a sphere with radius 3 and center $(-3, -3, -3)$.

Three open questions, for 5 points each:
provide the correct answer. Where appropriate, write down intermediate calculations. You may receive points for partially correct answers.

6. Write down the *parametric equation* of a plane in 3D that goes through three points $p_0 = (1,1,1)$, $p_1 = (2,3,1)$ and $p_2 = (0,0,3)$.

e.g. p(u,v) = p0 + u (p1 - p0) + v (p2 - p0)

Any other pair of vectors using p0, p1 and p2 is of course correct as well.
Any other base vector (from p0, p1 and p2) is also correct.

7. Determine a normal vector for the plane described in question 6.

e.g. (p1-p0) X (p2-p0);

(4,-2,1) or (-4,2,-1).

Not correct: a normal of (p1-p0); this is just a normal of a line, not of the plane.

8. Write down the implicit representation for the plane described in question 6.

Format: Ax+By+Cz+D=0;
using normal (4,-2,1): fill in p0 to determined D:
D=1*4-2*1+1*1=3.

⇨ 4x – 2y + z – 3 = 0.

You can verify this by filling in p1:

4*2-2*3+1*1-3 = 0.

If you used a wrong normal from question 7 in a correct way, you scored full points here.

End of Part 2: Basic Geometric Entities

Part 3: Matrices and Determinants (20pts out of 100pts)

Three open questions:
provide the correct answer. Where appropriate, write down intermediate calculations. You may receive points for partially correct answers.

1. Calculate the result of the following matrix multiplication: *(for 5 points)*

$$\begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 4 & 1 \end{pmatrix} \quad \begin{pmatrix} 3*3+0*4 & 3*2+0*-1 \\ -1*3+2*4 & -1*2+2*-1 \\ 4*3+1*4 & 4*2+1*-1 \end{pmatrix}$$

$$=\begin{pmatrix} 9 & 6 \\ 5 & 4 \\ 16 & 7 \end{pmatrix}$$

2. Compute the determinant of matrix $A = \begin{pmatrix} 1 & 4 & 2 \\ 0 & 1 & 2 \\ -2 & -1 & 1 \end{pmatrix}$ *(for 5 points)*

$\det(M) = a_{11} \cdot a_{22} \cdot a_{33} + a_{12} \cdot a_{23} \cdot a_{31} + a_{13} \cdot a_{21} \cdot a_{32}$
$- a_{31} \cdot a_{22} \cdot a_{13} - a_{32} \cdot a_{23} \cdot a_{11} - a_{33} \cdot a_{21} \cdot a_{12}$

Apply Sarrus to get -9.

3. Calculate the cofactor of a_{23}^c of matrix A from question 2 (Note: just this single cofactor; no need to do the full Laplace's expansion). *(for 10 points)*

$a_{23} = 2$

$$a_{23}^c = (2\begin{vmatrix} 1 & 4 \\ -2 & -1 \end{vmatrix}) - 1^{(2+3)} = -2\begin{vmatrix} 1 & 4 \\ -2 & -1 \end{vmatrix}$$

$$= -2(1*(-1) - 4*-2))$$

$$= -2*(-1+8) = -14.$$

Many people forgot the -2. In that case, you still got 8 points for the answer 7.
<u>CORRECTION</u>: The -2 is needed in Laplace, but that was not the question here. Two free points on this question.

End of Part 3: Matrices

Part 4: Transformations (15pts out of 100pts)

Three open questions:
provide a brief, correct answer.

1. Given the following matrix for linear transformations in 3D, with $a, b \neq 0$:

$$A = \begin{pmatrix} 1 & b & c \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

What kind of transformation do we get if we apply matrix A to a vector in 3D? *(for 2 points)*

Answer: Shearing or skewing.

2. Write down matrix A', which is the inverse of A from question 1. *(for 6 points)*

$$\begin{pmatrix} 1 & -b & -c \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Note: many people made this far too difficult. The inverse reverses the transform; we therefore simply shear in the opposite direction.

3. Write down a matrix for non-uniform scaling <u>with respect to the point (1,1)</u> by a factor 2 in the x-direction, and a factor 4 in the y-direction in 2D. *(for 7 points)*

Solution: use three matrices; the first one shifts point (1,1) to the origin; the second one applies the specified scale; the third shifts back to (1,1). So we get:

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Doing some matrix multiplications then yields

$$\begin{pmatrix} 2 & 0 & 1 \\ 0 & 4 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

The 2x2 matrix with 2,4 on the diagonal scored 2 points.
A slight error in the above calculation scored 8 points (e.g. doing the matrix multiplication in the wrong order).

End of Part 4: Transformations

Part 5: Graphics Theory (25pts out of 100pts)

Five multiple choice questions, for 1 point each:
clearly mark the correct answer. Note: for these questions, there is only one correct answer.

1. When using 16-bit per color, we typically use 5 bits for red and blue, but 6 for green. Why?

   a) To compensate for the lack of alpha
   b) To better align the bits in memory
   c) Because the human eye is more sensitive to green than to red and blue
   d) Because display hardware can display green with more intensity levels

2. In the context of texture mapping, 'oversampling' refers to:
   Free point on this question due to error in some grading templates.

   a) Reading from several textures for a single fragment
   b) Reading several pixels from the same texture for a single fragment
   c) Writing to several fragments using the same texture pixel
   d) None of the above

3. Per-vertex lighting…

   a) …involves linear interpolation and renormalization of the normal over a triangle
   b) …involves linear interpolation of illumination over a triangle
   c) …increases the cost of lighting calculations compared to per-fragment lighting
   d) …increases the accuracy of lighting calculations compared to per-fragment lighting

4. 'Overdraw' in the context of a rasterizer refers to…

   a) …not clearing the display between subsequent frames
   b) …rendering the image at a higher resolution for anti-aliasing
   c) …rendering outside the screen boundaries
   d) None of the above *It refers to rendering multiple times to the same screen pixel.*

5. 'Z-fighting' happens when…

   a) …the renderer tries to render objects behind the camera
   b) …an object is visible through a closer translucent surface
   c) …two overlapping polygons produce fragments with (almost) the same depth
   d) None of the above

Three open questions:
provide the correct answer. Where appropriate, write down intermediate calculations. You may receive points for partially correct answers.

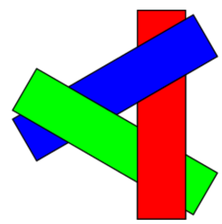
1. Describe (in 50 words or less) how a BSP allows us to render a polygonal scene in back-to-front order. *(for 5 points)*

The BSP recursively splits the world in 2 half spaces. The polygons are sorted back-to-front by traversing the tree, while comparing the camera against the split plane, and taking the 'far' side first, then the 'near' side.

Keywords: half spaces, recursion, camera (or viewpoint).
Not correct: "the tree leafs are further away than the root".
Also note that the BSP is not built during rendering.

2. Provide a case where the Painter's algorithm fails to properly sort polygons. Explain briefly (50 words or less) why this case is problematic. *(for 5 points)*

Anything that looked or sounded like this:



Some people had really clever alternatives, which were also accepted (if correct). Example: intersecting polygons.

3. Given three vertices in screen space: $p_0 = (-5,30)$, $p_1 = (15,10)$ and $p_2 = (5,50)$ and a screen with a resolution of 640×480 pixels. Use Sutherland-Hodgeman to clip the triangle against the left side of the screen. Show the steps involved, and write down the order and the coordinates of the vertices of the resulting polygon. *(for 10 points)*

The clipped coordinates are: (0,25) and (0,40).
More important is the process:

p0 to p1: coming in, so: emit clipped coord and p1.
p1 to p2: staying in, so: emit p2.
p2 to p0: going out, so: emit clipped coord.
Result: clipped coord, p1, p2, clipped coord.

Correct process but wrong clipped cords yielded 8 points. Correct clipped coord but bogus process yielded 2 points.

End of Part 5: Graphics Theory - ***End of exam.***