Tutorial 3 - Matrices

Basics

Exercise 1.

Given: a 3×2 matrix $A = \begin{pmatrix} 4 & 1 \\ 2 & 1 \\ 3 & 2 \end{pmatrix}$.

a) Calculate the sum of A with itself.
b) Can you multiply A with itself? If so, do this; otherwise explain why not.
c) Determine the transpose of A^T of A.
d) Multiply A with A^T.

Exercise 2.

Given: matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ and $= \begin{pmatrix} 1 & 2 \\ -3 & -6 \end{pmatrix}$.

a) Calculate the determinant of A.
b) Calculate the determinant of B.
c) What is the geometric interpretation of these two results?

Exercise 3.

Given: vector $\vec{a} = (1,3)$ and $\vec{b} = (5,1)$.

Calculate the area of the parallelogram defined by the two vectors.

Exercise 4.

Given: matrix $A = \begin{pmatrix} -2 & 2 & 3 \\ -1 & 1 & 3 \\ 2 & 0 & -1 \end{pmatrix}$.

a) Calculate the determinant of A using Laplace's expansion.
b) Verify your answer using the Rule of Sarrus.

Matrix characteristics

Exercise 5.

Given: matrix $= \begin{pmatrix} 1 & 4 & -2 \\ 3 & 2 & -6 \\ -2 & 5 & 4 \end{pmatrix}$.

a) Calculate the determinant of matrix A.
b) Explain the result for a) geometrically.

Exercise 6.

Given:

- $n_A \times m_A$ matrix A;
- $m_A \times m_B$ matrix B;
- $m_A \times m_C$ matrix C.

Prove, that if $AB = AC$, it does not necessarily follow that $B = C$ (even if A is not the null matrix).

Exercise 7.

A matrix is called orthogonal if all column vectors are mutually perpendicular. A matrix is called orthonormal if it is orthogonal and all column vectors have unit length. Show that the inverse of an $n \times n$ orthonormal matrix is its transpose. Hint: $A^{-1}A = I$, so you can solve this by proving that $A^{T}A = I$.

Matrix inversion

Exercise 8.

Given: matrix $= \begin{pmatrix} 1 & 5 & 2 \\ -1 & 1 & 4 \\ 0 & 2 & -1 \end{pmatrix}$.

a) Calculate the cofactor matrix C.
b) Calculate the adjoint matrix $\tilde{A} = C^{T}$.
c) Calculate the determinant for A.
d) Calculate the inverse A^{-1} for A using the results for a), b) and c).

Coordinate systems

<u>Exercise 9</u>.

a) Write down a matrix that scales 2D vectors by a factor of 2.
b) Determine the inverse of this matrix.
c) Write down the matrix that projects 3D vectors on the $x = 0$ plane.
d) If possible, determine the inverse of this matrix. Otherwise, explain why this is not possible.

<u>Exercise 10</u>.

Determine the matrix that rotates vectors along an ellipse centered around the origin, with a height of 4 and a width of 1.

<u>Exercise 11</u>.

Determine the matrix for reflection in the line $2x - y = 0$ in $\mathbb{R}^2$.

Hint: split this transformation into simpler ones, and combine the result using matrix multiplication.

<u>Exercise 12</u>.

a) Determine a transformation matrix that rotates by 45° clockwise.
b) Transform the vectors $\binom{1}{0}$, $\binom{0}{1}$ and $\binom{1}{1}$, both to practice matrix-vector multiplication, and to verify the matrix you constructed for a).
c) Multiply the matrix with itself and show that this is a rotation by 90°.

The End

(for now)