Graphics (INFOGR 2015-2016) – Final Exam

Thursday June 30th, 17.00 – 19.30 – EDUC-GAMMA

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- Fill in your name and student ID at the top of this page, and write it on every additional paper you want to turn in.
- Answer the questions in the designated areas on these exam sheets. For the math questions, only the final answer is needed. If you need more space for a problem, state this in the designated area for the problem and continue on the paper provided by us. On the additional paper, state your name and student ID, and clearly indicate the problem number.
- If a question is unclear to you, write down how you interpret the question, then answer it.

PART 1 – THEORY

[redacted]

1. Complete the following text with appropriate terminology from the slides:

"When rendering large scenes, we try to determine the set of visible polygons. On a coarse level, we do this by ……culling…… objects against the view ……volume / frustum…… using their bounding volumes. These tests are often ……conservative…… ; this means that sometimes an object is rendered when it is in fact not visible. On the lowest level, we use ……backface…… - ……culling…… to discard individual triangles based on camera position."

[redacted]

2. Complete the following text with appropriate terminology from the slides:

"Renderers use ……HDR…… internally to represent colors that can be brighter than the pixels on a monitor. To simulate bright pixels, we simulate the response of eyes to bright lights by adjusting ……exposure…… . We can also use ……lens flares / glow…… to simulate the effects of light scattering inside an eye or a camera. Finally, we apply ……gamma…… ……correction…… to compensate for the fact that the response of older monitors to energy values is not linear."

[redacted] 3. Which of the following statements is correct? (circle one option, no explanation required)

i. The construction of the BSP and the drawing order are both independent from the camera position.
ii. Only the construction of the BSP is independent from the camera position.
iii. Only the drawing order is independent from the camera position.
iv. The construction of the BSP and the drawing order are both dependent from the camera position.

[redacted] 4. Assume s is a scalar value, and $\vec{v}$ and $\vec{w}$ are two vectors in $\mathbb{R}^3$. "$\times$" denotes the cross product of two vectors and "$\cdot$" denotes the scalar product (or inner product or dot product). For a, b, c and d, circle the correct option (i,ii or iii).

a) The result of $(\vec{v} \times \vec{w}) \cdot (\vec{v} \times \vec{w})$ is: (i) a scalar value, (ii) a vector in $\mathbb{R}^3$, or (iii) undefined.
b) The result of $(s \times \vec{w}) \cdot (s \times \vec{w})$ is: (i) a scalar value, (ii) a vector in $\mathbb{R}^3$, or (iii) undefined.
c) The result of $s\vec{v} + \vec{v} \times \vec{w}$ is: (i) a scalar value, (ii) a vector in $\mathbb{R}^3$, or (iii) undefined.
d) The result of $s\vec{w} + \vec{v} \cdot \vec{w}$ is: (i) a scalar value, (ii) a vector in $\mathbb{R}^3$, or (iii) undefined.

PART 2 – MATHEMATICS

[redacted] 5. Matrix M scales vectors and positions in $\mathbb{R}^3$ by a factor 2 in the x-, y- and z-axis, and translates positions by T_x, T_y, T_z.

a) Write down matrix M.

$$M = \begin{pmatrix} 2 & 0 & 0 & T_x \\ 0 & 2 & 0 & T_y \\ 0 & 0 & 2 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

b) Write down the equation that transforms a position using M.

$$\begin{pmatrix} \\ \\ \\ \end{pmatrix} = M \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

c) Write down the equation that transforms a vector using M.

$$\begin{pmatrix} \\ \\ \\ \end{pmatrix} = M \begin{pmatrix} x \\ y \\ z \\ 0 \end{pmatrix}$$

[redacted] 6. Assume a 2×2 matrix A where the two column vectors are parallel to each other. Prove that the determinant $det\ A$ of this matrix is zero.

The determinant of a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is: $det\ A = ad - bc$.

The column vectors are parallel ➔ there must be a λ such that $\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \lambda c \\ \lambda d \end{pmatrix}$.

We thus get $t\ A = \lambda cd - \lambda dc = 0$. QED.

[redacted] 7. Given: matrix $A = \begin{pmatrix} 3 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 0 & 3 \\ 1 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix}$.

a) Calculate the product AB using matrix multiplication.

$$AB = \begin{pmatrix} 3 & 3 \\ 3 & 2 \end{pmatrix}$$

b) Calculate the sum $A + B^T$.

$$A + B^T = \begin{pmatrix} 4 & 1 & 1 \\ 2 & 3 & 3 \end{pmatrix}$$

c) Calculate the determinant of matrix C.

$\det C =$ ....-7...........

[redacted] 8. Given:

- a triangle with screen coordinates $v_0 = (-2,1)$, $v_1 = (1,-1)$ and $v_2 = (4,4)$
- a screen with a resolution of 512x384 pixels.

a) Calculate the intersections of the triangle with the left side of the screen. Note: only fill out $I_2 \ldots I_4$ if they exist, leave blank if you believe they don't.

intersection $I_1 = \left(0, -\frac{1}{3} \right)$ intersection $I_2 = \left(0,\ 2 \right)$

intersection $I_3 = (\quad)$ intersection $I_4 = (\quad)$

b) Use Sutherland-Hodgeman to clip the triangle against the left side of the screen. Write down, for each edge, which vertices are emitted (use $v_0, v_1, v_2, I_1 \ldots I_4$ rather than full coordinates).

Note: I_1 and I_2 swapped (only!) if swapped in a) as well.

edge v_0, v_1: ....I_1, v_1.... edge v_1, v_2: ....v_2.... edge v_2, v_0: ....I_2....

c) Calculate the intersections of the n-gon obtained in b) with the top of the screen.

intersection $I_1 = (\quad)$ intersection $I_2 = (\quad)$

intersection $I_3 = \left(0, 0 \right)$ intersection $I_4 = \left(1\frac{3}{5}, 0 \right)$

d) Use Sutherland-Hodgeman to clip the n-gon against the top of the screen. <u>Use the notation of 8b</u>.

Assuming they clip I_1, v_1, v_2, I_2 we now have a small notation problem.
Let's say we continued numbering, so I_1 becomes I_3 and I_2 becomes I_4:

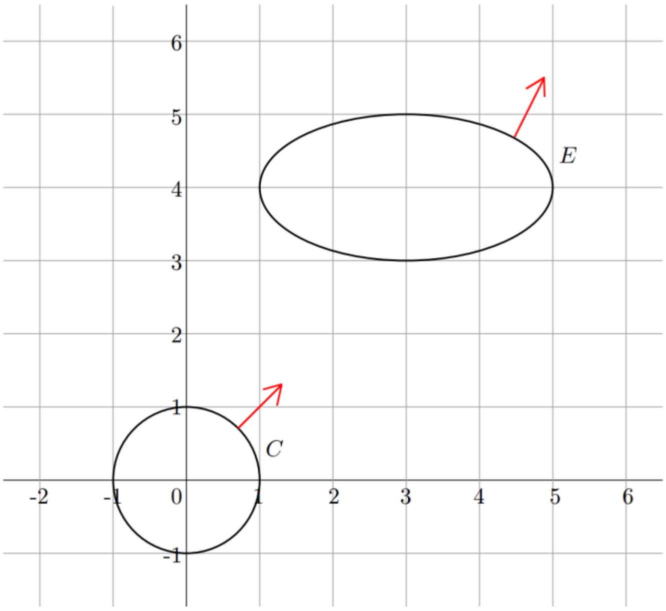
edge I_1, v_1: $\boxed{\text{nothing}}$ edge v_1, v_2: $\boxed{I_3 \text{ and } v_2}$ edge v_2, I_2: $\boxed{I_2}$ edge I_2, I_1: $\boxed{I_4}$

[redacted] 9. In $\mathbb{R}^2$, the matrix $\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$ defines a counterclockwise rotation about the angle α about the origin. Write down the 3x3 transformation matrix A for a similar rotation about the x-axis in $\mathbb{R}^3$.

Rotation around the x-axis does not change the x-values. For the y- and z-values: it is basically a rotation in the y-z-plane. Hence, we can just take the 2d rotation matrix for the y- and z-coordinates and fill in some values for the x-coordinates that don't change it.

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{pmatrix}$$

[redacted] 10. Consider the situation in the picture on the right.



a) Write down matrix M which transforms circle C into ellipse E.

$$M = \begin{pmatrix} \mathbf{2} & \mathbf{0} & \mathbf{3} \\ \mathbf{0} & \mathbf{1} & \mathbf{4} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

$M = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}.$

Verify for (-1,0) and (0,1): $M\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}$; $M\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix}$ ➔ All OK.

b) As discussed in the slides (and evident in the image), the normal of circle C at position $\left(\frac{1}{2}\sqrt{2}, \frac{1}{2}\sqrt{2}\right)$ is not properly transformed by matrix M. Determine matrix M_N that we can use to correctly transform normals. Hint: the ingredients for this are easily obtained when you think about it.

$$M_N = \begin{pmatrix} \mathbf{0.5} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} \quad \textbf{or:} \quad \begin{pmatrix} \mathbf{0.5} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

Note: translation is irrelevant here; any value will do.

Normals are correctly transformed using matrix $M_N = (M^{-1})^T$.
The inverse does the opposite of the original transform, so we determine that

$M^{-1} = \begin{pmatrix} 0.5 & 0 \\ 0 & 1 \end{pmatrix}$. Note that the translation is irrelevant. This is a diagonal matrix, so its transpose is the same.

Verify for $\left(\frac{1}{2}\sqrt{2}, \frac{1}{2}\sqrt{2}\right)$: $M_N \begin{pmatrix} 0.5\sqrt{2} \\ 0.5\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0.25\sqrt{2} \\ 0.5\sqrt{2} \end{pmatrix}$, which passes visual inspection in the picture. This still needs to be normalized.

Each question scores up to 10 points. ***Good luck!***