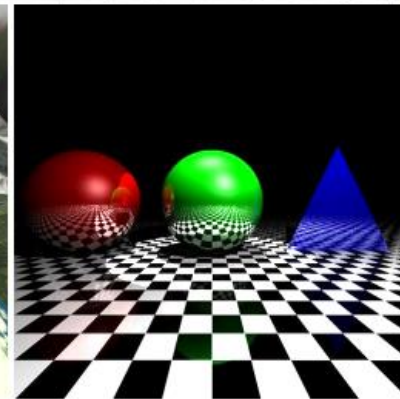
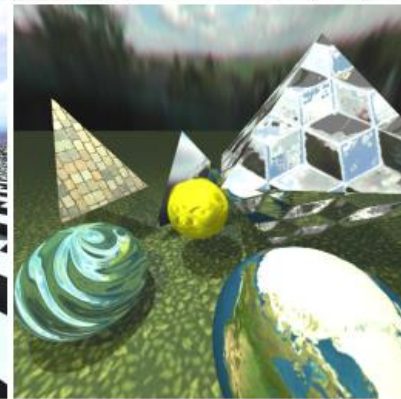
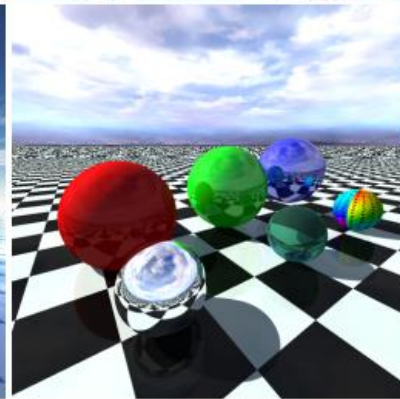
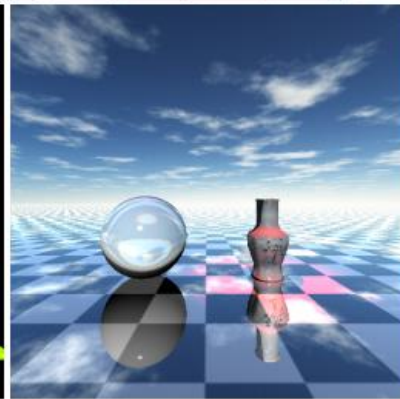
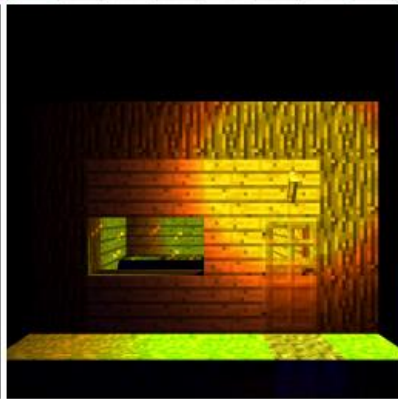
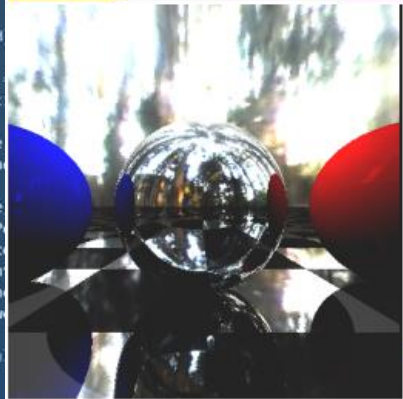
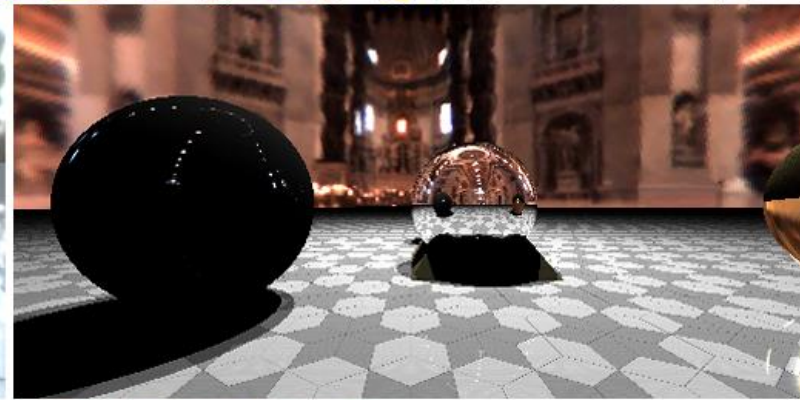
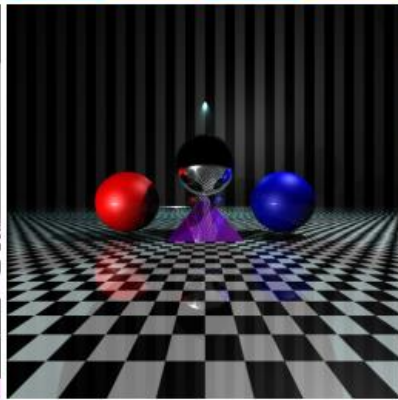
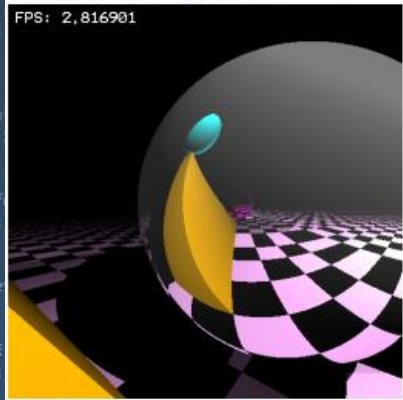
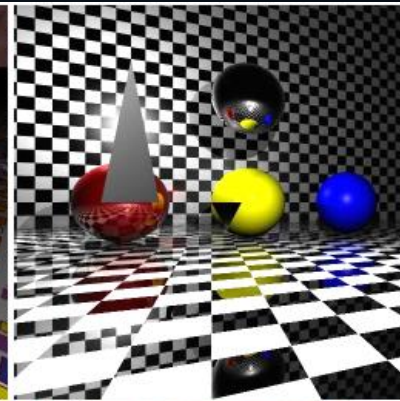
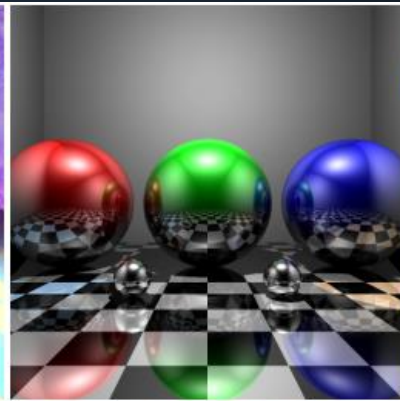


# INFOGR – Computer Graphics

J. Bikker - April-July 2016 - Lecture 11: “Visibility”

# Welcome!





```
FPS: 2,816901  
MAXDEPTH  
survive  
estimat  
if;  
-radiance  
e.x + ra  
w = true  
at brdfP  
at3 fact  
at weigh  
at cosTh  
E * ((w  
andom wa  
ive)
```

```
at3 brdf = SampleDiffuse( diffuse, N, r1, r2, RR, spot  
survive;  
pdf;  
n = E * brdf * (dot( N, R ) / pdf);  
sion = true;
```





## Smallest Ray Tracers:

### Executable

Valeri Erling  
Roderick Spaans

- 5692598 & 5683777: RTMini\_minimal.exe – 2803 bytes
- 5741858: ASM\_CPU\_Min\_Exe – 994 bytes

### Source

Marijn Suijten

Ivo Gabe de Wolff  
Lars Folkersma

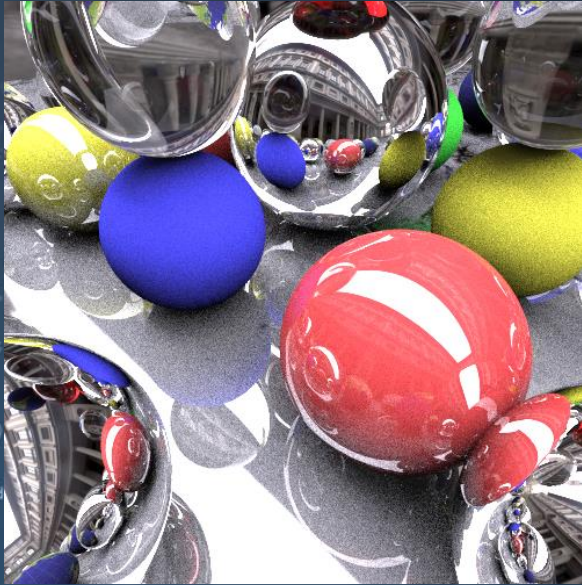
- 4279433 & 5543800: Haskell ray tracer; 2280 characters.
- 5741858: C# ray tracer, 1235 characters.

Marijn Suijten

```
using V=System.Numerics.Vector3;using static System.Math; using f=System.Single;using System.Drawing;class
S{public V P,C=V.One;public int T;public f r,R;public S(V p,f a,f b){P=p;R=a*a;r=b;}public void I(R r){V L
=P-r.0;f a=V.Dot(L,r.D),d=V.Dot(L,L)-a*a;if(a>0&&d<R){f t=a-(f)Sqrt(R-d);if(t>0&&t<r.i){r.i=t;r.N=
V.Normalize(r.0+t*r.D-P);r.p=this;}}}}class R{public V O,D,N;public S p;public f i=99;public R(V o,V d){O=
o+1e-4f*d;D=d;}}public R(V d){D=d;}}class A{V P=V.One;S x=new S(V.UnityY*-500,498,.7f){T=1},y=new S(new V(-1
,0,4),.6f,1),z=new S(new V(1,0,4),.6f,0){C=V.UnitX};void D(R r){x.I(r);y.I(r);z.I(r);}A(){int S=512;Bitmap
b=new Bitmap(S,S);for(int i=0;i<S*S;i++)b.SetPixel(i%S,i/S,R(B(new R(V.Normalize(new V((f)(i%S)/S-.5f,.5f-
(f)(i/S)/S, 1)))))); b.Save("r.bmp");}V B(R r){D(r);V C=V.Zero;if(r.p!=null){V I=r.0+r.i*r.D,c=r.p.T>0?new
V((int)(I.X-9)+(int)(I.Z-9)&1):r.p.C,L=V.Normalize(P-I);f f=r.p.r,d;R a=new R(I,L);D(a);if(a.p==null){if (
(d=V.Dot(L,r.N))>0)C+=c*d*(1-f)/(V.Distance(I,P)/9+1);if((d=V.Dot(r.D,V.Reflect(L,r.N)))>0)C+=new V((f)Pow
(d,9)*f);}C+=f*B(new R(I,V.Reflect(r.D,r.N)))*c;} return C;}Color R(V v)=>Color.FromArgb(S(v.X),S(v.Y), S
(v.Z));int S(f f)=>(int)(f<0?0:f>1?1:Sqrt(f)*255);static void Main(){new A();}}
```



# Fastest Ray Tracer:



```
...
(depth)
inside
nt / n
os2t = 1.0f
D, N );
)
at a = nt
at Tr = 1 -
Tr) R = (D
E * diffuse
= true;
efl + refr)
D, N );
-efl * E * d
= true;
MAXDEPTH)
survive = SurvivalProbability( diffuse
estimation = doing it properly
if;
-radiance = SampleLight( &rand, I, M, N, L, R, S, T, U, V, W, X, Y, Z );
e.x + radiance.y + radiance.z > 0) return radiance;
v = true;
at brdfPdf = EvaluateDiffuse( L, N ) * Recursive( L, N, R, S, T, U, V, W, X, Y, Z );
at3 factor = diffuse * INVPI;
at weight = Mis2( directPdf, brdfPdf );
at cosThetaOut = dot( N, L );
E * ((weight * cosThetaOut) / directPdf) * radiance;
random walk - done properly, closely following death
ive)
;
at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &S, &T, &U, &V, &W, &X, &Y, &Z );
urvive;
pdf;
n = E * brdf * (dot( N, R ) / pdf);
sion = true;
```



# Today's Agenda:

- Depth Sorting
- Clipping
- Visibility

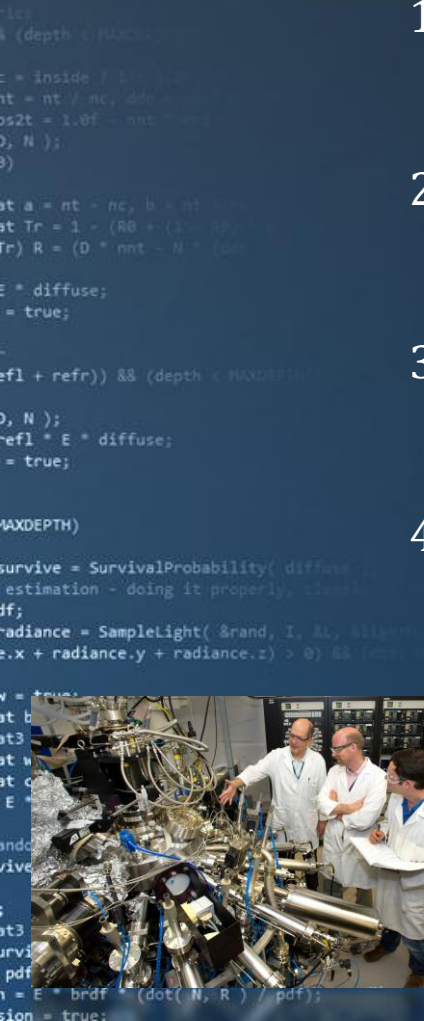




# Depth Sorting

## Rendering – Functional overview

1. Transform:  
translating / rotating meshes
2. Project:  
calculating 2D screen positions
3. Rasterize:  
determining affected pixels
4. Shade:  
calculate color per affected pixel



# Depth Sorting

## 3. Rasterize: *determining affected pixels*

### Questions:

- What is the screen space position of the fragment?
- Is that position actually on-screen?
- Is the fragment the nearest fragment for the affected pixel?

How do we efficiently determine visibility of a pixel?

Animation, culling,  
tessellation, ...

meshes

Transform

vertices

Project

vertices

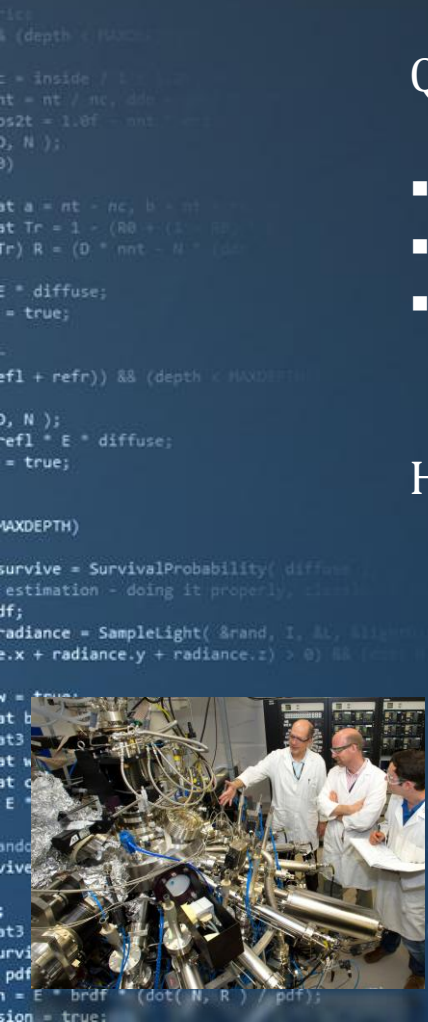
Rasterize

fragment positions

Shade

pixels

Postprocessing





Part of the tree is off-screen

Too far away to draw

Tree requires little detail

City obscured by tree

Torso closer than ground

Tree between ground & sun





# Depth Sorting

## Old-skool depth sorting: Painter’s Algorithm

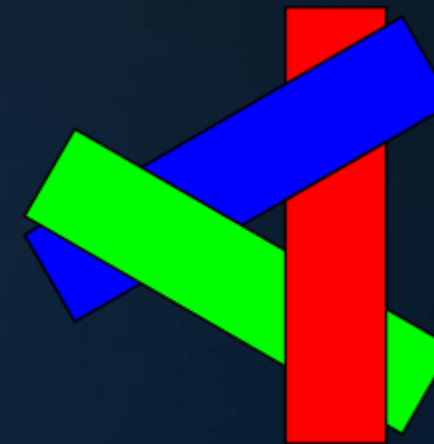
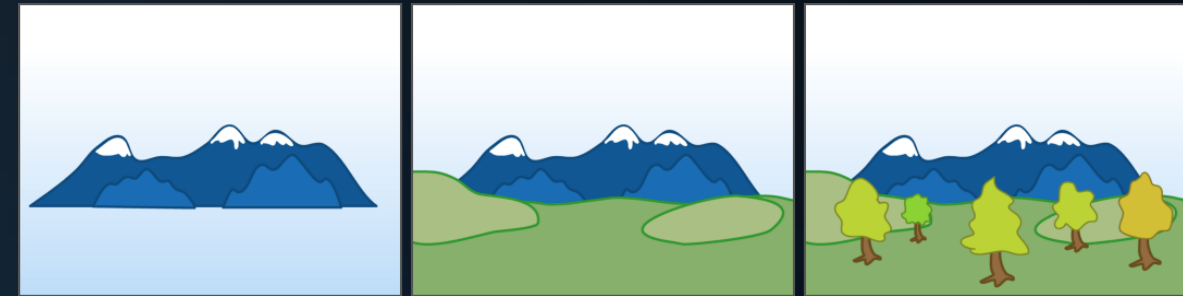
- Sort polygons by depth
- Based on polygon center
- Render depth-first

### Advantage:

- Doesn’t require z-buffer

### Problems:

- Cost of sorting
  - Doesn’t handle all cases
- Overdraw

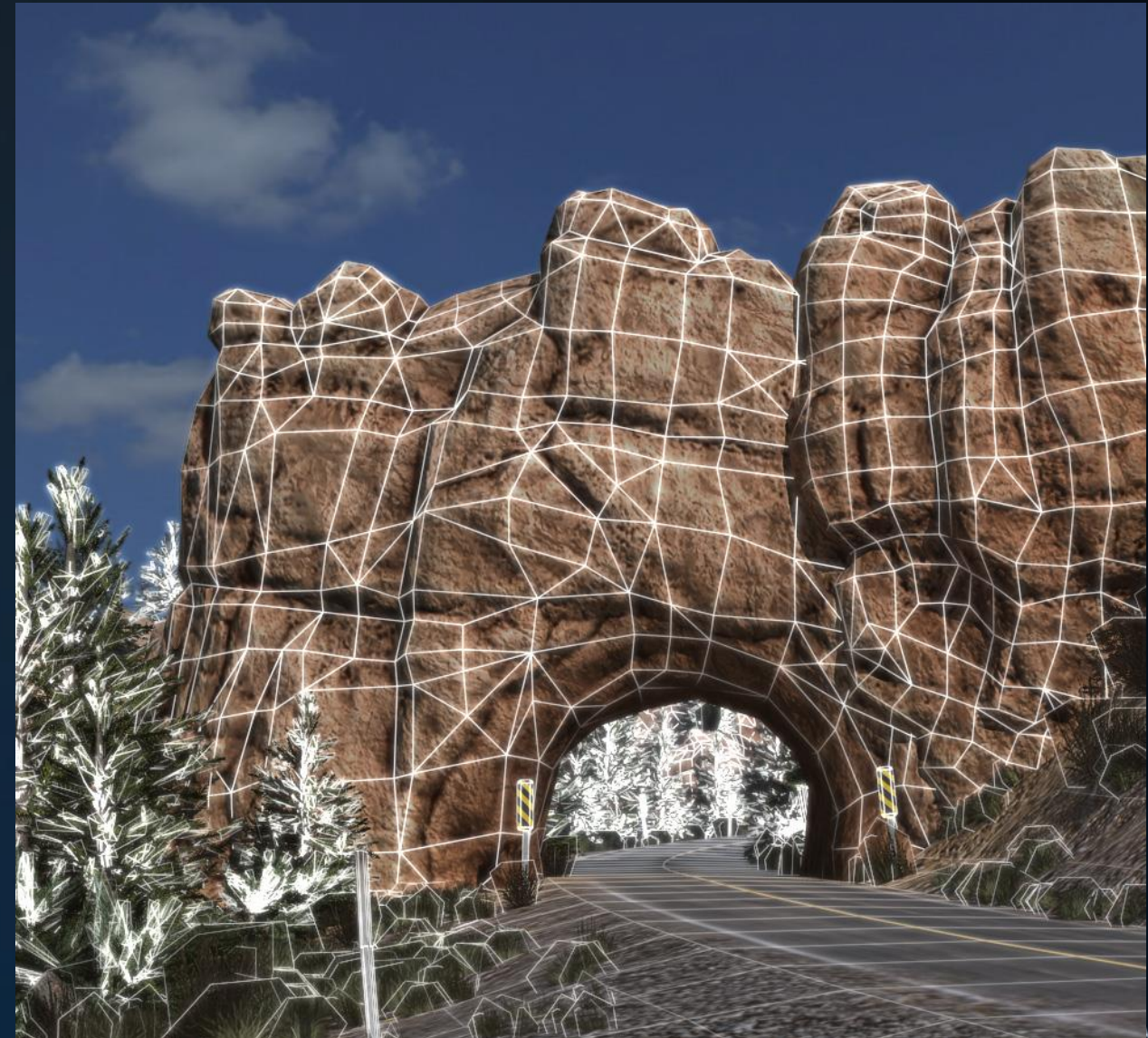


# Depth Sorting

Overdraw:

Inefficiency caused by drawing multiple times to the same pixel.

```
...
    & (depth < MAXDEPTH)
...
    c = inside / nc;
    nt = nt / nc;
    pos2t = 1.0f / nnt;
    D, N );
    )
...
    at a = nt - nc, b = nt;
    at Tr = 1 - (R0 + (1 - R0) * c);
    Tr) R = (D * nnt - N * (a + b * c));
...
    E * diffuse;
    = true;
...
    refl + refr) && (depth < MAXDEPTH)
...
    D, N );
    refl * E * diffuse;
    = true;
...
MAXDEPTH)
...
survive = SurvivalProbability( diffuse;
estimation - doing it properly, check
if;
radiance = SampleLight( @rand, I, M, Alignment
e.x + radiance.y + radiance.z) > 0) && (max
...
y = true;
at b
at3
at w
at c
E *
...
andc
vive
...
at3
surv
pdf
n = E * brdf * (dot( N, R ) / pdf);
sion = true;
```

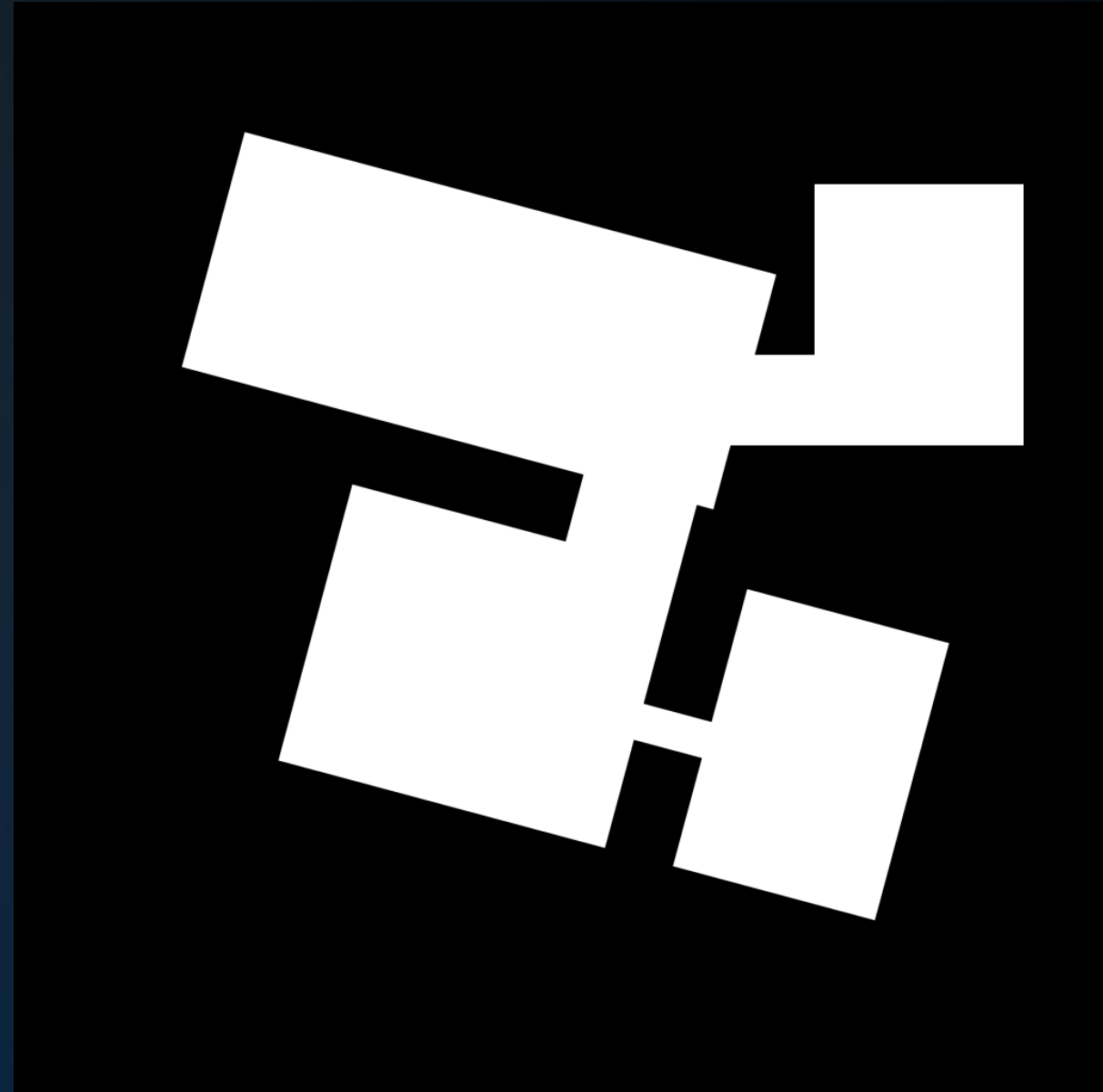




# Depth Sorting

Correct order: BSP

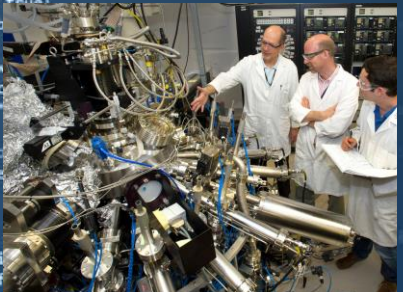
root



```

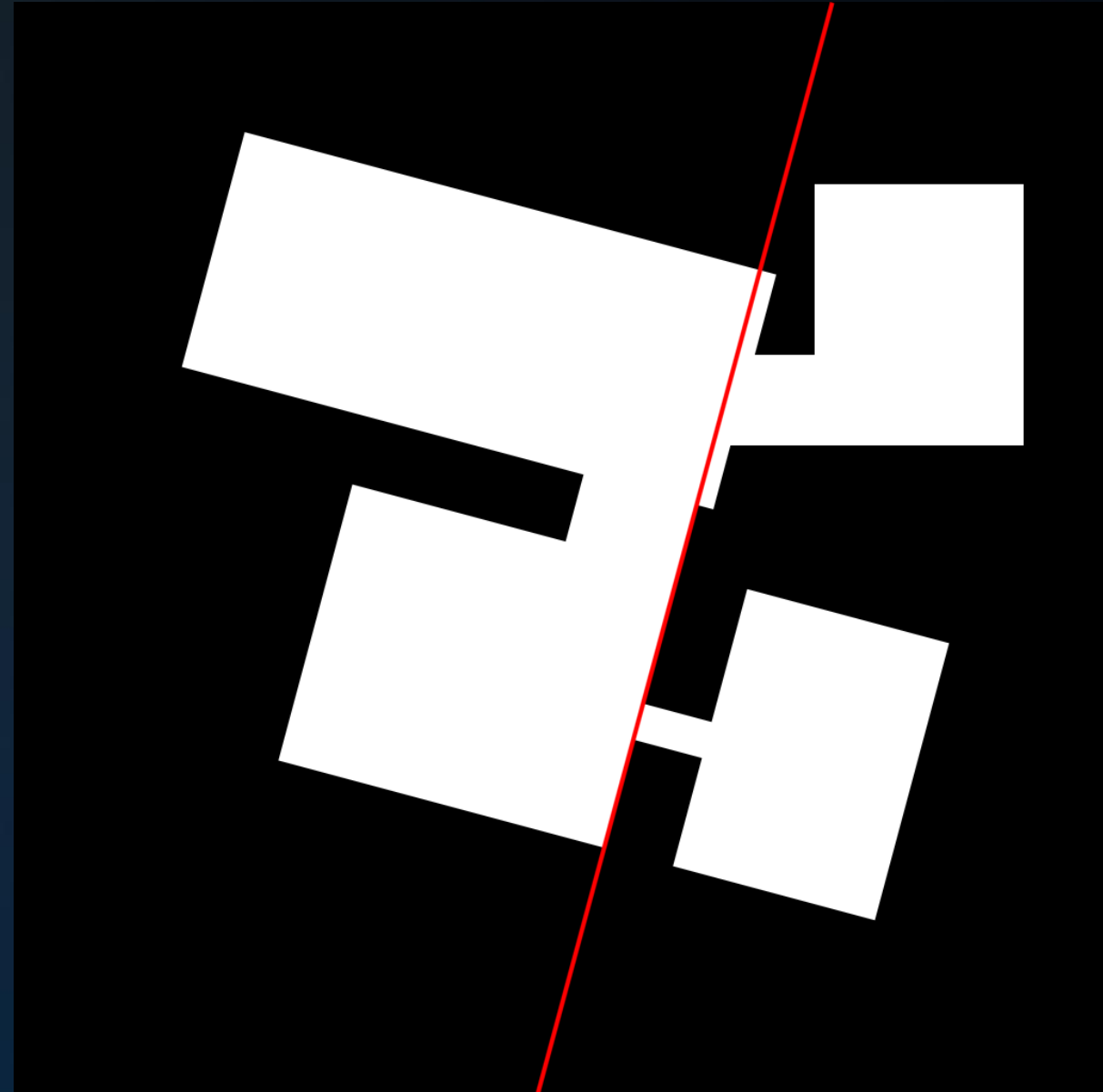
...
    & (depth < MAXDEPTH)
...
    c = inside / 1.0;
    nt = nt / nc; ndd = n * n;
    cos2t = 1.0f - nnt;
    D, N );
    )
...
    at a = nt - nc; b = nt - nc;
    at Tr = 1 - (R0 + (1 - R0) * c);
    Tr) R = (D * nnt - N * (dd
...
    E * diffuse;
    = true;
...
    refl + refr) && (depth < MAXDEPTH)
...
    D, N );
    refl * E * diffuse;
    = true;
...
MAXDEPTH)
...
survive = SurvivalProbability( diffuse;
estimation - doing it properly, class
if;
radiance = SampleLight( &rand, I, M, Alignment
e.x + radiance.y + radiance.z) > 0) && (max
...
y = true;
at b
at3
at w
at c
E *
...
and
yive
...
at3
surv
pdf
n = E * brdf * (dot( N, R ) / pdf);
ion = true;

```



# Depth Sorting

Correct order: BSP



```

...
    & (depth < MAXDEPTH)
...
    c = inside / 1.0;
    nt = nt / nc; nct = nct / nc;
    cos2t = 1.0f - nnt;
    D, N );
...
    at a = nt - nc; b = nt;
    at Tr = 1 - (R0 + (1 - R0) * c);
    Tr) R = (D * nnt - N * (a *
...
    E * diffuse;
    = true;
...
    refl + refr) && (depth < MAXDEPTH)
...
    D, N );
    refl * E * diffuse;
    = true;
...
MAXDEPTH)
...
survive = SurvivalProbability( diffuse;
estimation - doing it properly, check
if;
radiance = SampleLight( &rand, I, M, Alignment
e.x + radiance.y + radiance.z) > 0) && (max
...
y = true;
at b
at3
at w
at c
E *
...
and
yive
...
at3
surv
pdf
n = E * brdf * (dot( N, R ) / pdf);
ision = true;

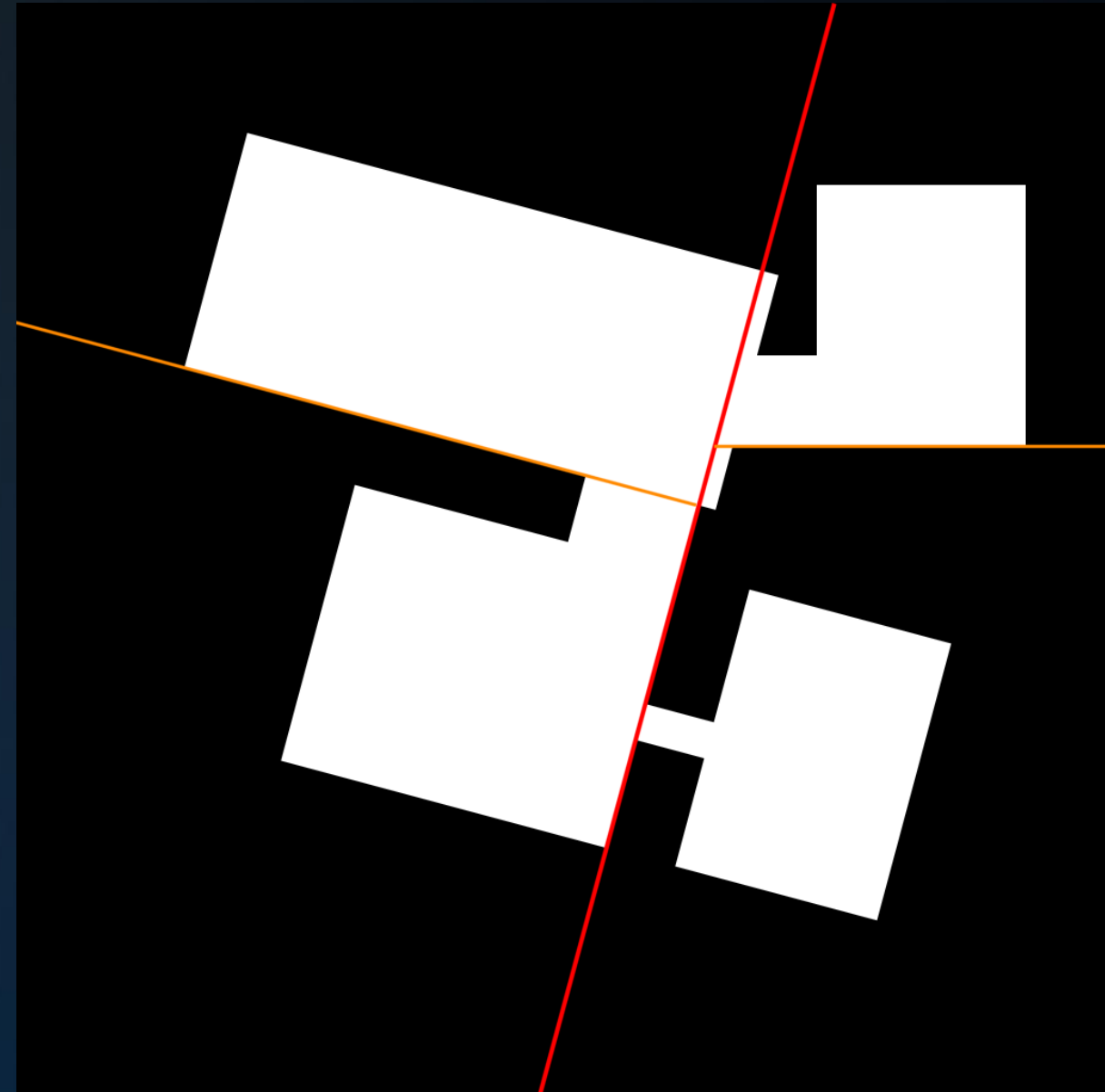
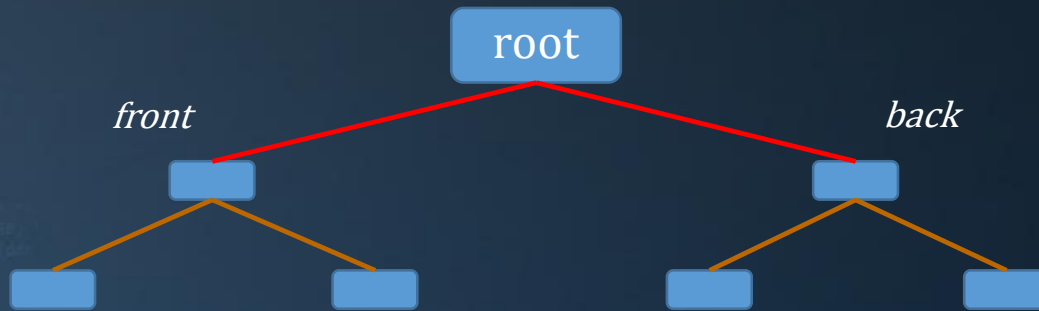
```





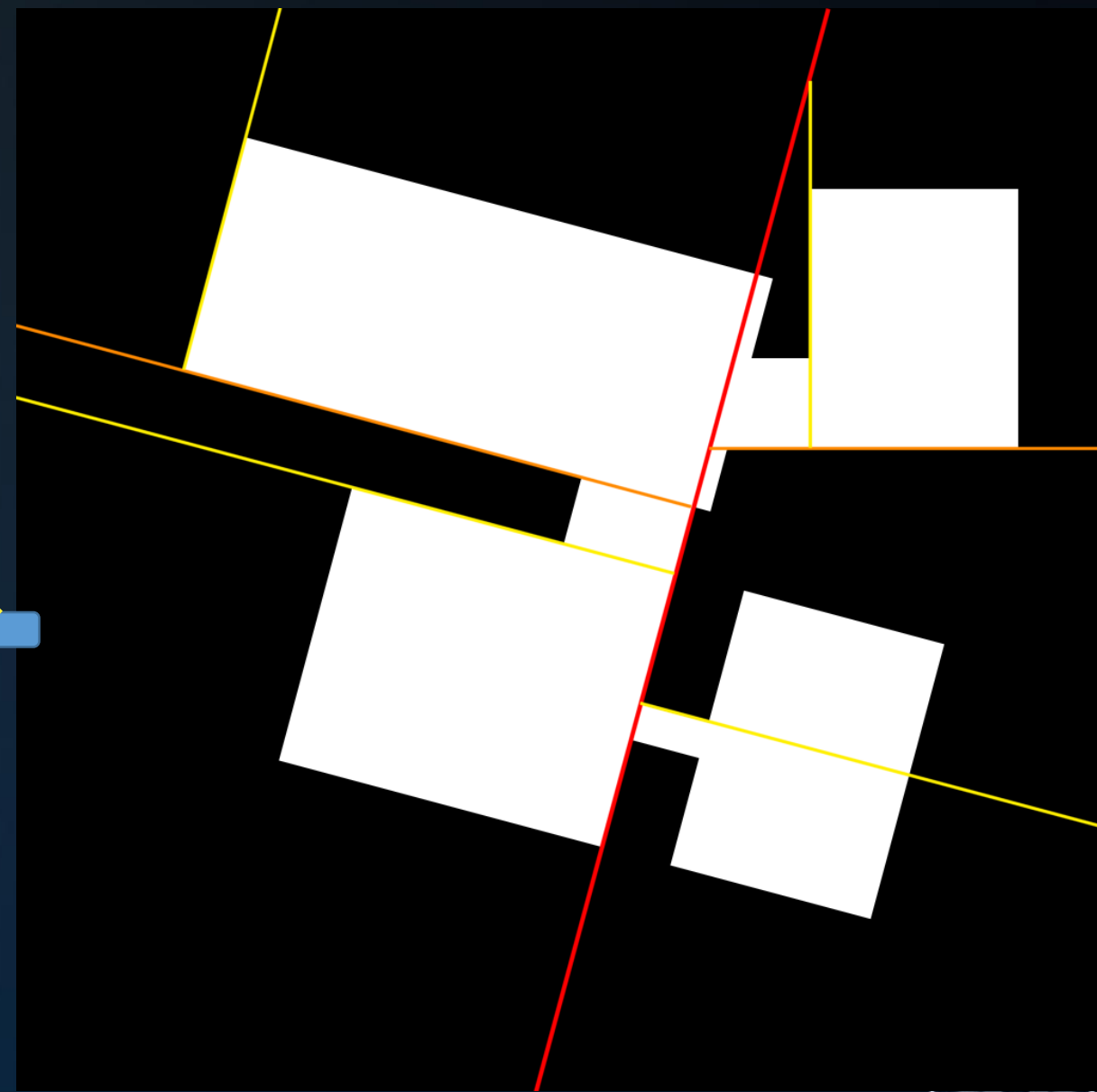
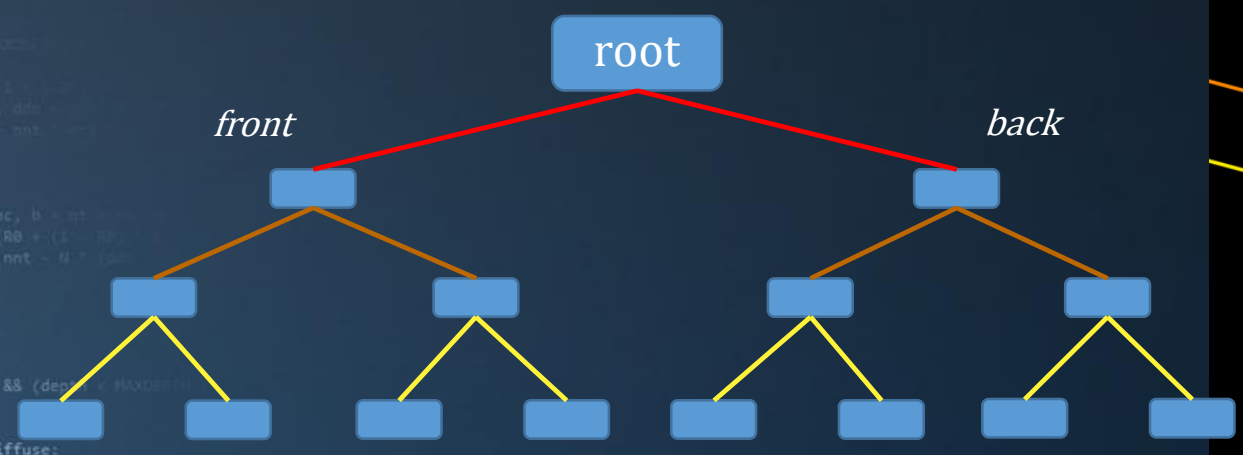
# Depth Sorting

Correct order: BSP



# Depth Sorting

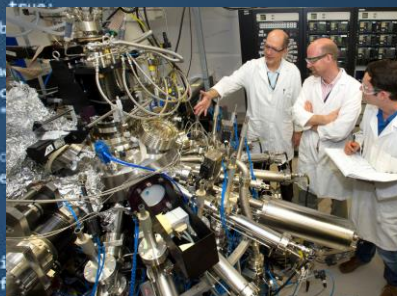
Correct order: BSP



```

...
    & (depth < MAXDEPTH)
...
    c = inside / (1.0f - r);
    nt = nt / nc; nct = nc;
    nt = nt / nc; nct = nc;
    pos2t = 1.0f - nnt;
    D, N );
    )
...
    at a = nt - nc, b = nt;
    at Tr = 1 - (R0 + (1 - R0) * c);
    Tr) R = (D * nnt - N * (1 - nnt));
...
    E * diffuse;
    = true;
...
    refl + refr) && (depth < MAXDEPTH)
...
    D, N );
    -refl * E * diffuse;
    = true;
...
MAXDEPTH)
...
survive = SurvivalProbability( diffuse;
estimation - doing it properly, check
if;
radiance = SampleLight( &rand, I, M, Alignment;
e.x + radiance.y + radiance.z) > 0) && (maxDepth > 0)
...
y = true;
at b
at3
at w
at c
E *
...
and
yive
...
at3
survive
pdf
n = E * brdf * (dot( N, R ) / pdf);
ision = true;

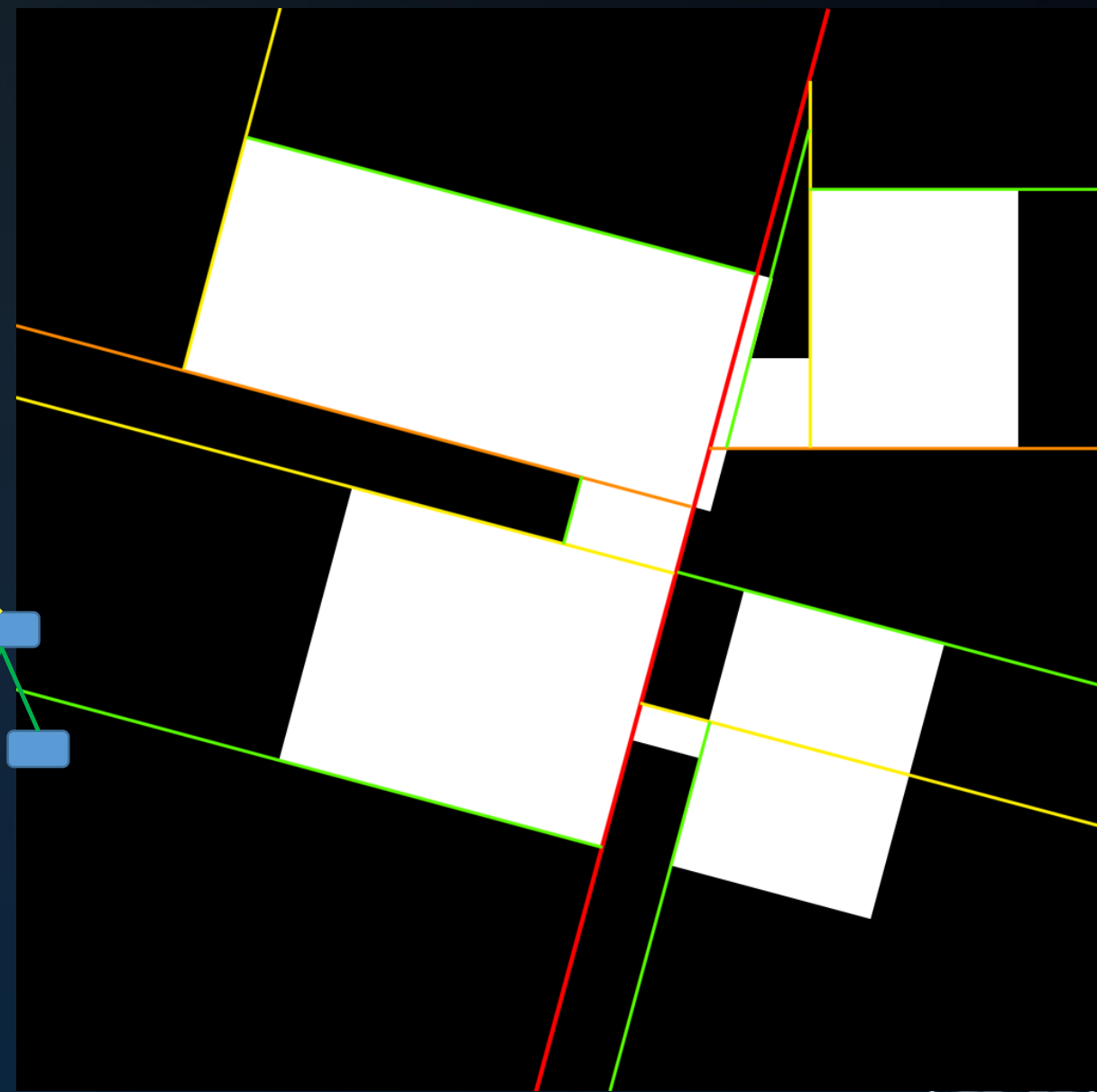
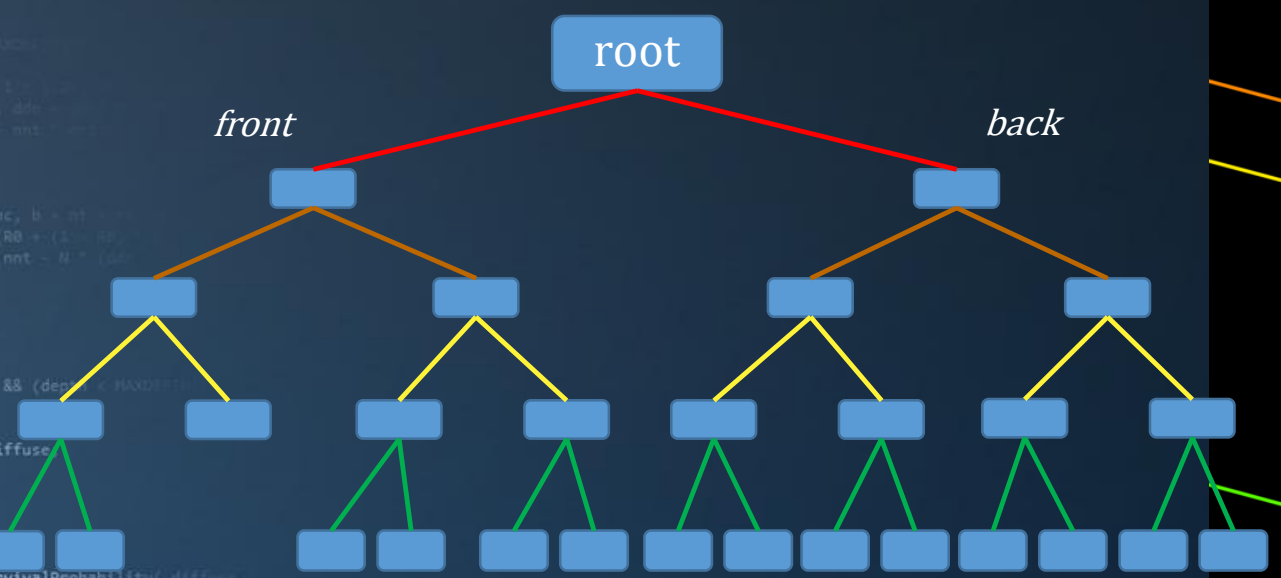
```



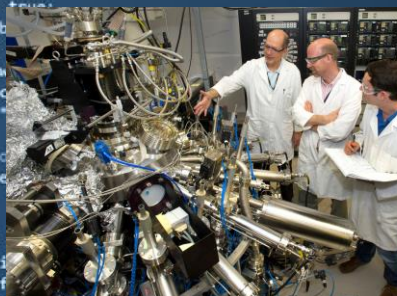


# Depth Sorting

Correct order: BSP

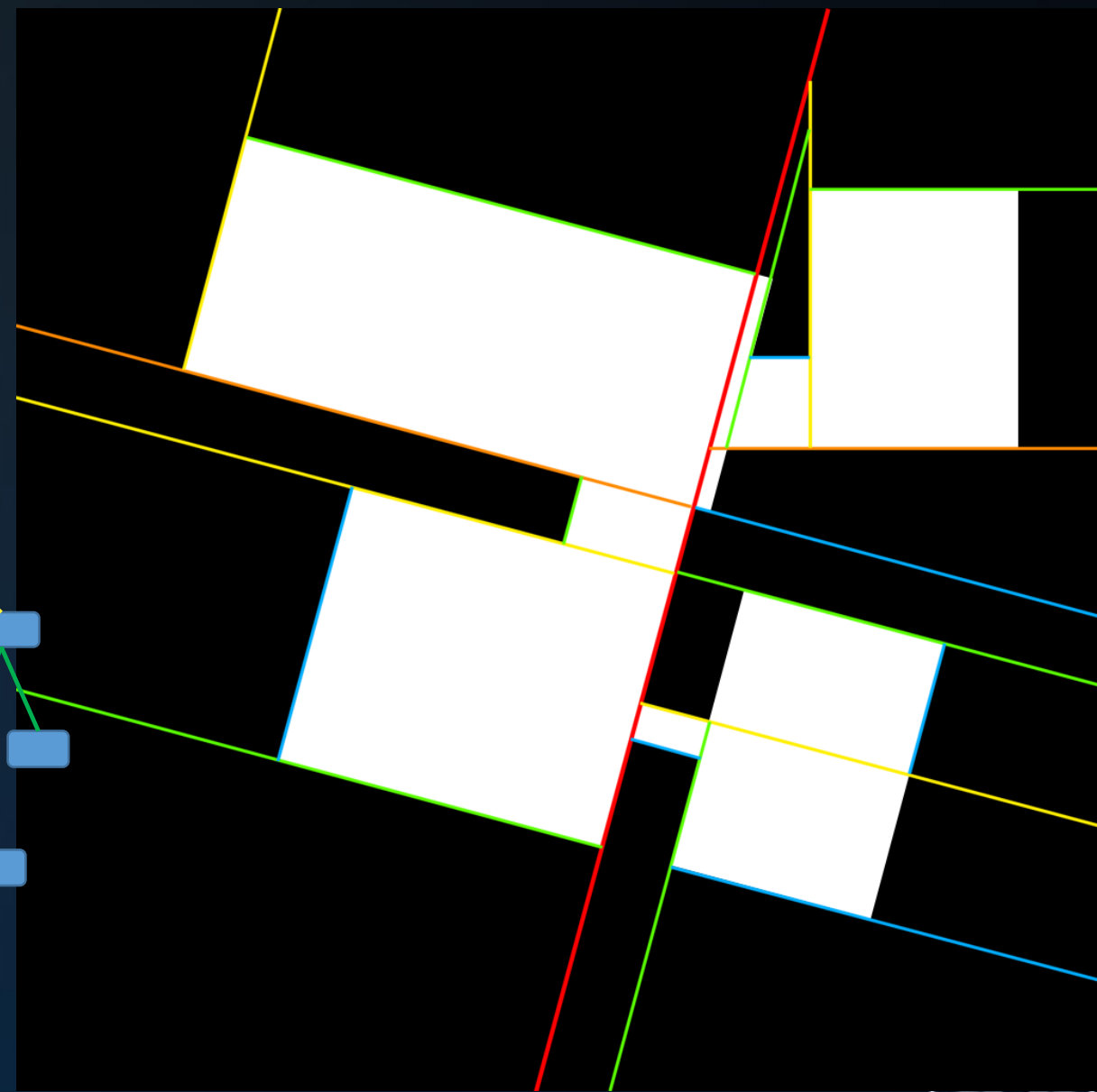
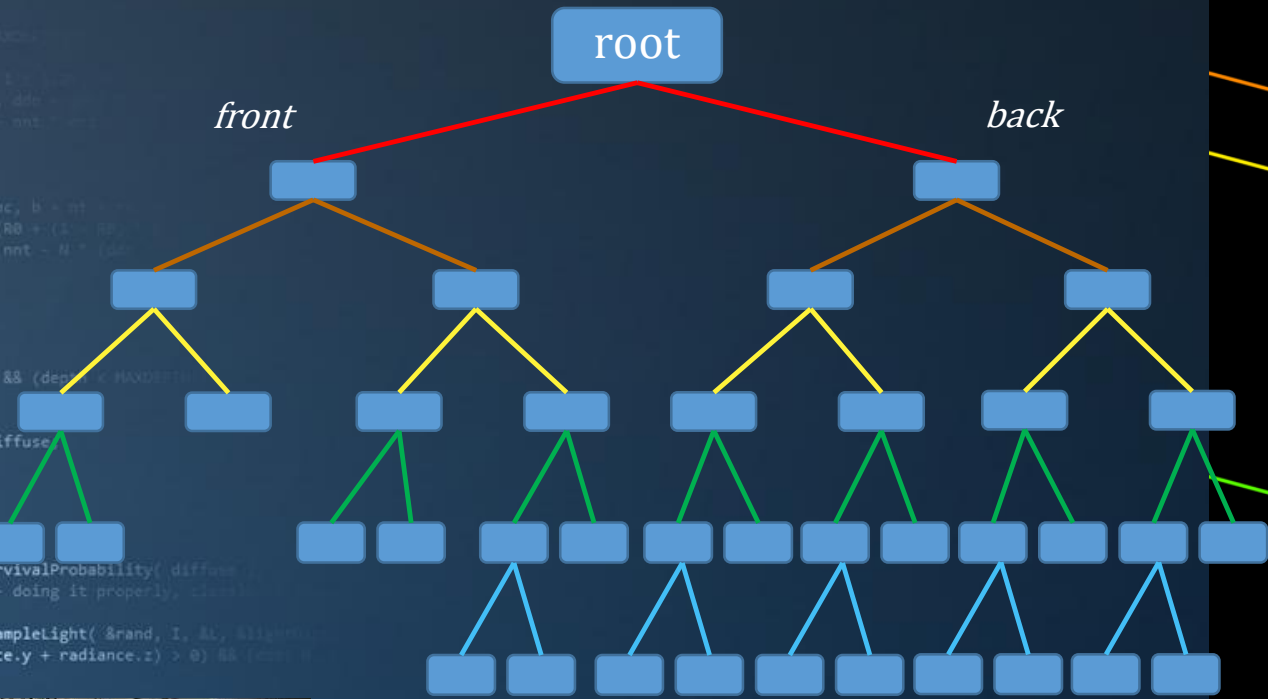


```
...ics  
... & (depth < MAXDEPTH)  
...  
... c = inside / 1.0f;  
... nt = nt / nc; add =  
... os2t = 1.0f - nnt;  
... D, N );  
... )  
...  
... at a = nt - nc, b = nt;  
... at Tr = 1 - (R0 + (1 - R0)  
... Tr) R = (D * nnt - N * (1 -  
...  
... E * diffuse;  
... = true;  
...  
... efl + refr) && (depth < MAXDEPTH)  
...  
... D, N );  
... efl * E * diffuse;  
... = true;  
...  
... MAXDEPTH)  
... survive = SurvivalProbability( diffuse;  
... estimation - doing it properly, check  
... if;  
... radiance = SampleLight( &rand, I, M, Alignment  
... e.x + radiance.y + radiance.z) > 0) && (survive  
...  
... y = true;  
... at b  
... at3  
... at w  
... at c  
... E *  
...  
... and  
... yive  
...  
... at3  
... urvi  
... pdf  
... n = E * brdf * (dot( N, R ) / pdf);  
... sion = true;
```

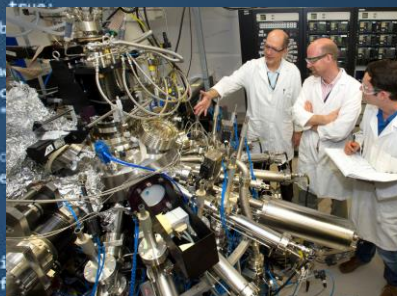


# Depth Sorting

Correct order: BSP



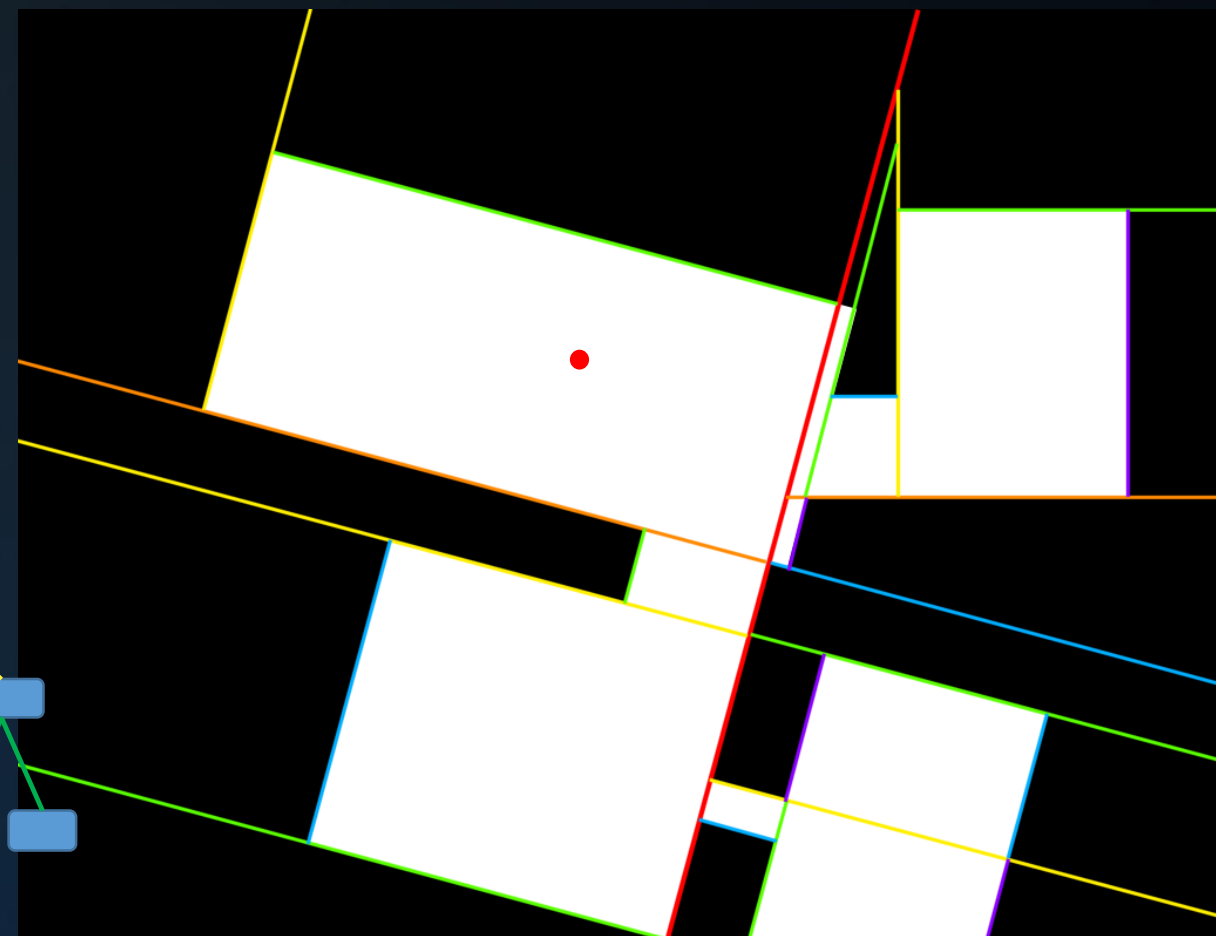
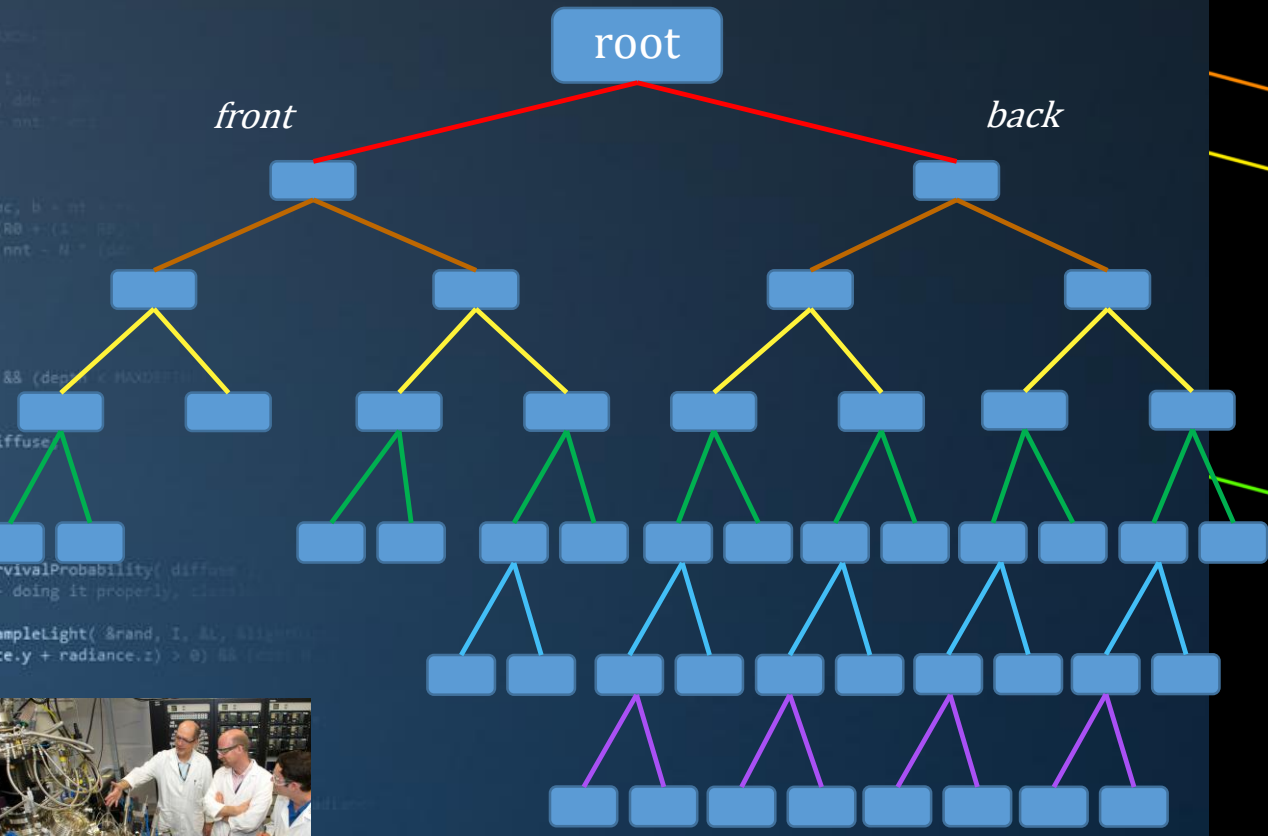
```
...
at (depth < MAXDEPTH)
{
    // inside / outside test
    int nt = nc / nc;
    double pos2t = 1.0f - nnt;
    double D, N;
    // ...
    at a = nt - nc, b = nt;
    at Tr = 1 - (R0 + (1 - R0));
    Tr = (D * nnt - N * ...);
    E * diffuse;
    = true;
    refl + refr) && (depth < MAXDEPTH)
    D, N);
    refl * E * diffuse;
    = true;
    MAXDEPTH)
    survive = SurvivalProbability( diffuse;
    estimation - doing it properly, check);
    if;
    radiance = SampleLight( &rand, I, M, Alignment);
    e.x + radiance.y + radiance.z) > 0) && (depth <
    y = true;
    at b;
    at3;
    at w;
    at c;
    E *
    and;
    yive;
    at3;
    urvi;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
}
```





# Depth Sorting

Correct order: BSP

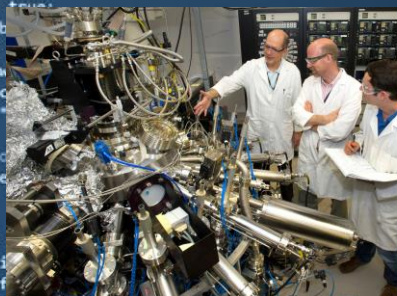


Sorting by BSP traversal:  
 Recursively  
 1. Render far side of plane  
 2. Render near side of plane

```

...
at a = nt - nc, b = nt - nc;
at Tr = 1 - (R0 + (1 - R0) * t);
Tr) R = (D * nnt - N * t);
...
E * diffuse;
= true;
...
efl + refr) && (depth < MAXDEPTH);
...
D, N);
-refl * E * diffuse;
= true;
...
MAXDEPTH);
...
survive = SurvivalProbability( diffuse;
estimation - doing it properly, check);
if;
radiance = SampleLight( @rand, I, Rt, Alignment);
e.x + radiance.y + radiance.z) > 0) && (survive);
...
y = true;
at b;
at3;
at w;
at c;
at e;
...
and;
vive;
...
at3;
urv;
pdf;
n = E * brdf * (dot( N, R ) / pdf);
ision = true;
...

```



# Depth Sorting

Draw order using a BSP:

- Guaranteed to be correct (hard cases result in polygon splits)
- No sorting required, just a tree traversal

But:

- Requires construction of BSP: not suitable for dynamic objects
- Does not eliminate overdraw

```

...
    & (depth < MAXDEPTH)
...
    t = inside / (1.0 - n * nc);
    nt = nt / nc;
    cos2t = 1.0f - nnt;
    D, N );
    )
...
    at a = nt - nc, b = nt;
    at Tr = 1 - (R0 + (1 - R0) * t);
    Tr) R = (D * nnt - N * (1 - nnt));
...
    E * diffuse;
    = true;
...
    refl + refr)) && (depth < MAXDEPTH)
...
    D, N );
    -refl * E * diffuse;
    = true;
...
MAXDEPTH)
...
survive = SurvivalProbability( diffuse;
estimation - doing it properly, check
if;
radiance = SampleLight( @rand, I, M, Alignment;
e.x + radiance.y + radiance.z) > 0) && (survive
...
y = true;
at b
at3
at w
at c
E *
...
and
ive
...
at3
surv
pdf
n = E * brdf * (dot( N, R ) / pdf);
ision = true;

```



# Depth Sorting

## Z-buffer

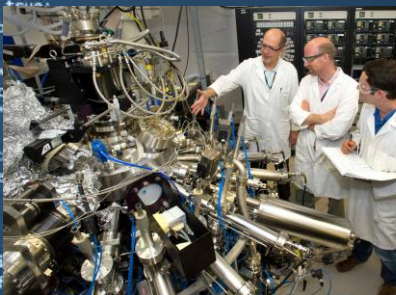
A z-buffer stores, per screen pixel, a depth value.

The depth of each fragment is checked against this value:

- If the fragment is further away, it is discarded
- Otherwise, it is drawn, and the z-buffer is updated.

The z-buffer requires:

- An additional buffer
- Initialization of the buffer to  $Z_{max}$
- Interpolation of  $z$  over the triangle
- A z-buffer read and compare, and possibly a write.







# Depth Sorting

## Z-buffer

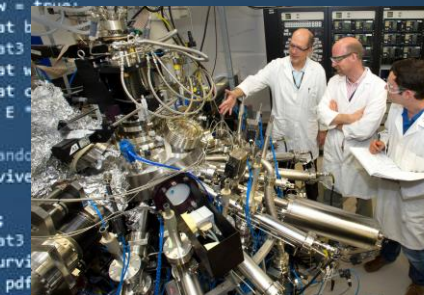
What is the best representation for depth in a z-buffer?

1. Interpolated z (convenient, intuitive);
2.  $1/z$  (or:  $n + f - \frac{fn}{z}$ ) (more accurate nearby);
3.  $(\text{int})((2^{31}-1)/z)$ ;
4.  $(\text{uint})((2^{32}-1)/-z)$ ;
5.  $(\text{uint})((2^{32}-1)/(-z + 1))$ .

Note: we use  $z_{\text{int}} = \frac{(2^{32}-1)}{-z+1}$ :

this way, any  $z < 0$  will be in the range  $z_{\text{adjusted}} = -z_{\text{original}} + 1 = 1.. \infty$ , therefore  $1/z_{\text{adjusted}}$  will be in the range  $0..1$ , and thus the integer value we will store uses the full range of  $0..2^{32} - 1$ .

Here,  $z_{\text{int}} = 0$  represents  $z_{\text{original}} = 0$ , and  $z_{\text{int}} = 2^{32} - 1$  represents  $z_{\text{original}} = -\infty$ .



# Depth Sorting

## Z-buffer optimization

In the ideal case, the nearest fragment for a pixel is drawn first:

- This causes all subsequent fragments for the pixel to be discarded;
- This minimizes the number of writes to the frame buffer and z-buffer.

The ideal case can be approached by using Painter’s to ‘pre-sort’.

```

...
    (depth < MAXDEPTH)
...
    inside / inside;
    nt = nt / nc;
    cos2t = 1.0f - nnt;
    D, N );
    )
...
    at a = nt - nc, b = nt;
    at Tr = 1 - (R0 + (1 - R0) *
    Tr) R = (D * nnt - N * (a
...
    E * diffuse;
    = true;
...
    refl + refr) && (depth < MAXDEPTH)
...
    D, N );
    refl * E * diffuse;
    = true;
...
MAXDEPTH)
...
survive = SurvivalProbability( diffuse;
estimation - doing it properly, check
if;
radiance = SampleLight( @rand, I, M, Alignment
e.x + radiance.y + radiance.z) > 0) && (depth <
...
y = true;
at b
at3
at w
at c
E *
...
and
ive
...
at3
surv
pdf
n = E * brdf * (dot( N, R ) / pdf);
ion = true;

```



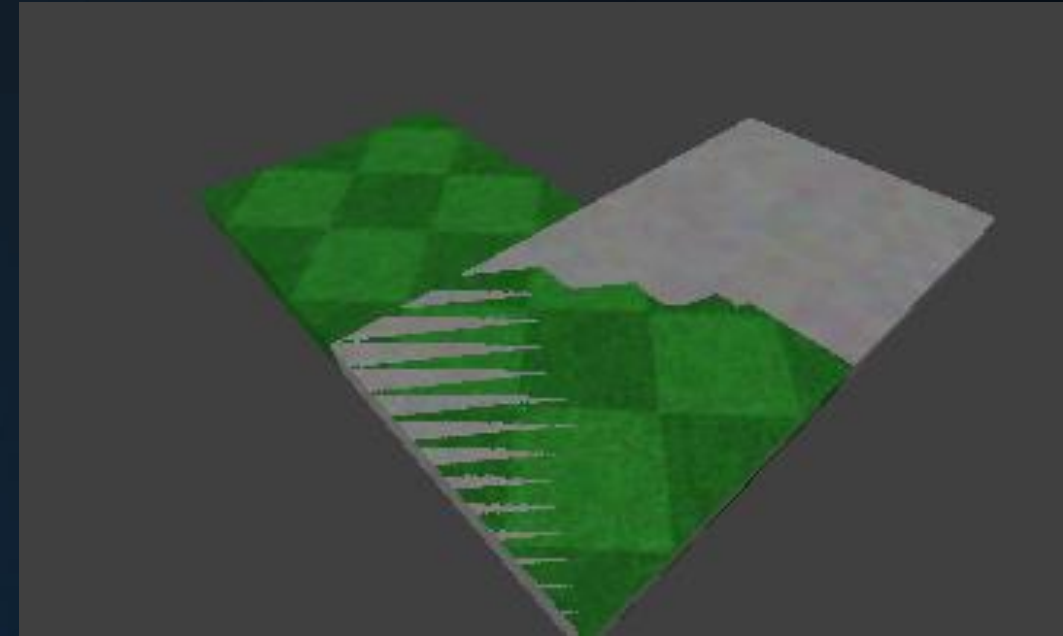


# Depth Sorting

‘Z-fighting’:

Occurs when two polygons have almost identical z-values.

Floating point inaccuracies during interpolation will cause unpleasant patterns in the image.



```
...ics
& (depth < MAXDEPTH)
...
t = inside / (1.0 - outside);
nt = nt / nc;
os2t = 1.0f - nnt;
D, N );
...
)
...
at a = nt - nc, b = nt;
at Tr = 1 - (R0 + (1 - R0)
Tr) R = (D * nnt - N * (a
...
E * diffuse;
= true;
...
efl + refr) && (depth < MAXDEPTH)
...
D, N );
-efl * E * diffuse;
= true;
...
MAXDEPTH)
...
survive = SurvivalProbability( diffuse;
estimation - doing it properly, check
if;
-radiance = SampleLight( @rand, I, M, Alignment
e.x + radiance.y + radiance.z) > 0) && (survive
...
y = true;
at b
st3
st w
it c
it c
E *
...
and
yive
...
st3
surv
pdf
n = E * brdf * (dot( N, R ) / pdf);
ion = true;
```





Part of the tree is off-screen

Stuff that is too far to draw

Tree requires little detail

City obscured by tree

Torso closer than ground

Tree between ground & sun





# Today's Agenda:

- Depth Sorting
- Clipping
- Visibility





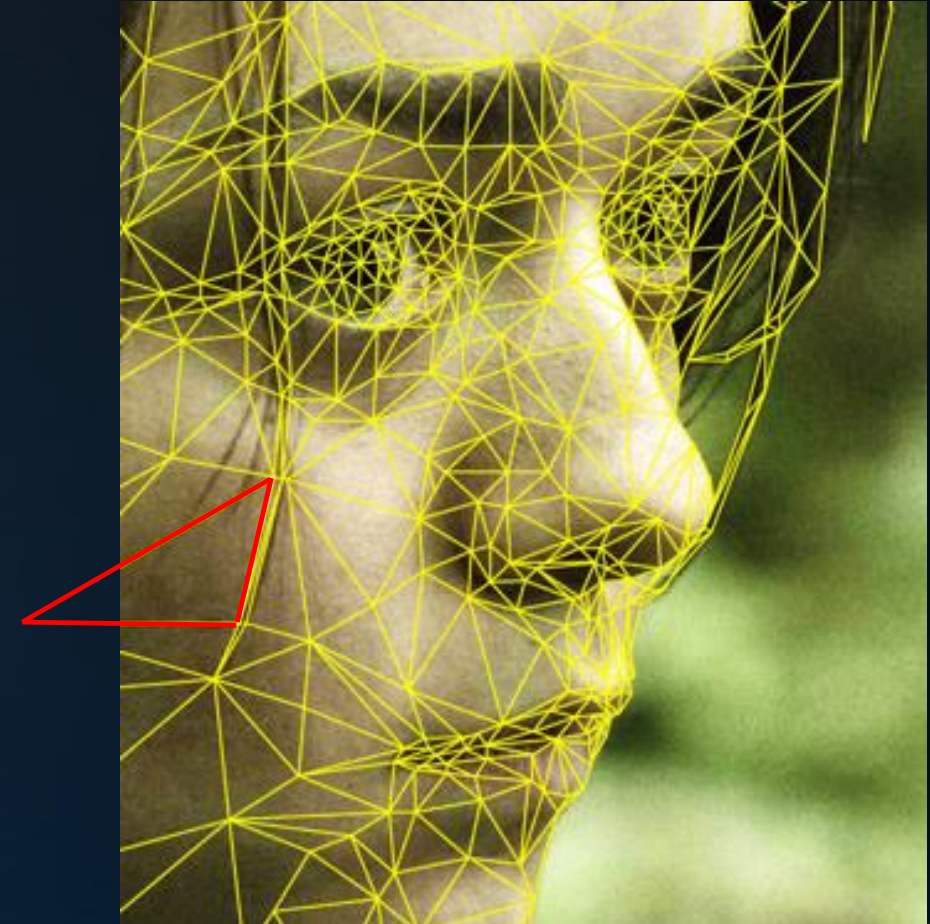
# Clipping

## Clipping

Many triangles are partially off-screen. This is handled by *clipping* them.

Sutherland-Hodgeman clipping:

Clip triangle against 1 plane at a time;  
Emit n-gon (0, 3 or 4 vertices).



# Clipping

## Sutherland-Hodgeman

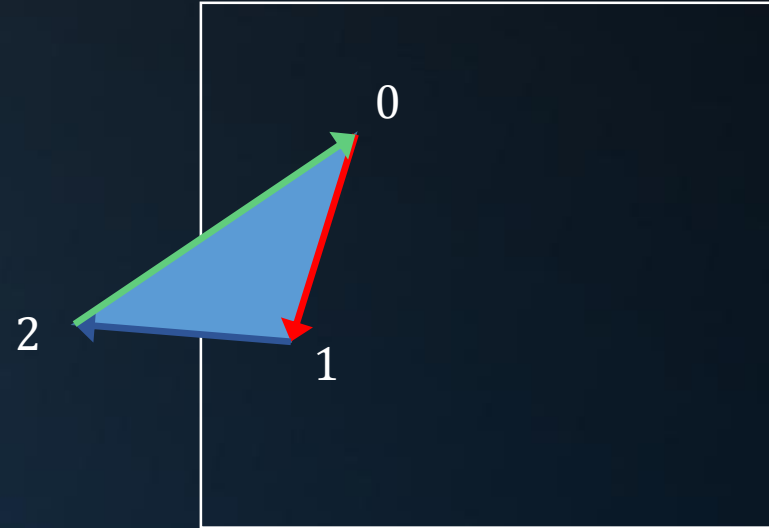
Input: list of vertices

Algorithm:

Per edge with vertices  $v_0$  and  $v_1$ :

- If  $v_0$  and  $v_1$  are ‘in’, emit  $v_1$
- If  $v_0$  is ‘in’, but  $v_1$  is ‘out’, emit C
- If  $v_0$  is ‘out’, but  $v_1$  is ‘in’, emit C and  $v_1$

where C is the intersection point of the edge and the plane.



Output: list of vertices,  
defining a convex n-gon.

in	out
Vertex 0	Vertex 1
Vertex 1	Intersection 1
Vertex 2	Intersection 2
	Vertex 0



# Clipping

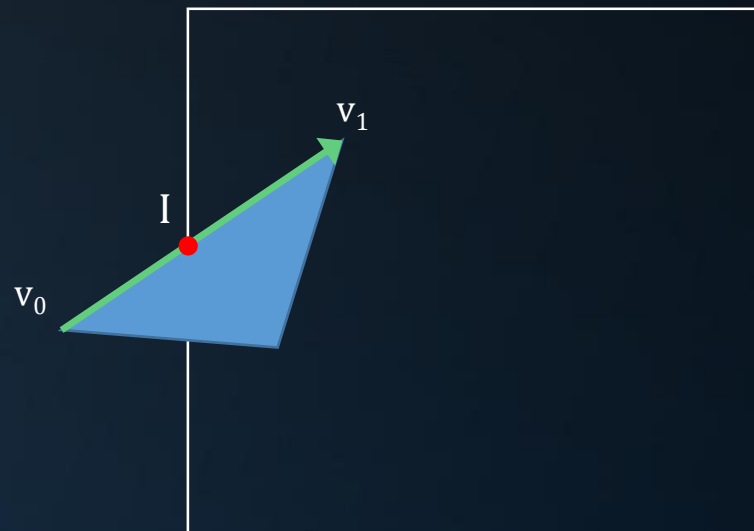
## Sutherland-Hodgeman

Calculating the intersections with plane  $ax + by + cz + d = 0$ :

$$dist_v = v \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} + d$$

$$f = \frac{|dist_{v_0}|}{|dist_{v_0}| + |dist_{v_1}|}$$

$$I = v_0 + f(v_1 - v_0)$$



After clipping, the input n-gon may have at most 1 extra vertex. We may have to triangulate it:

0,1,2,3,4  $\rightarrow$  0, 1, 2 + 0, 2, 3 + 0, 3, 4.



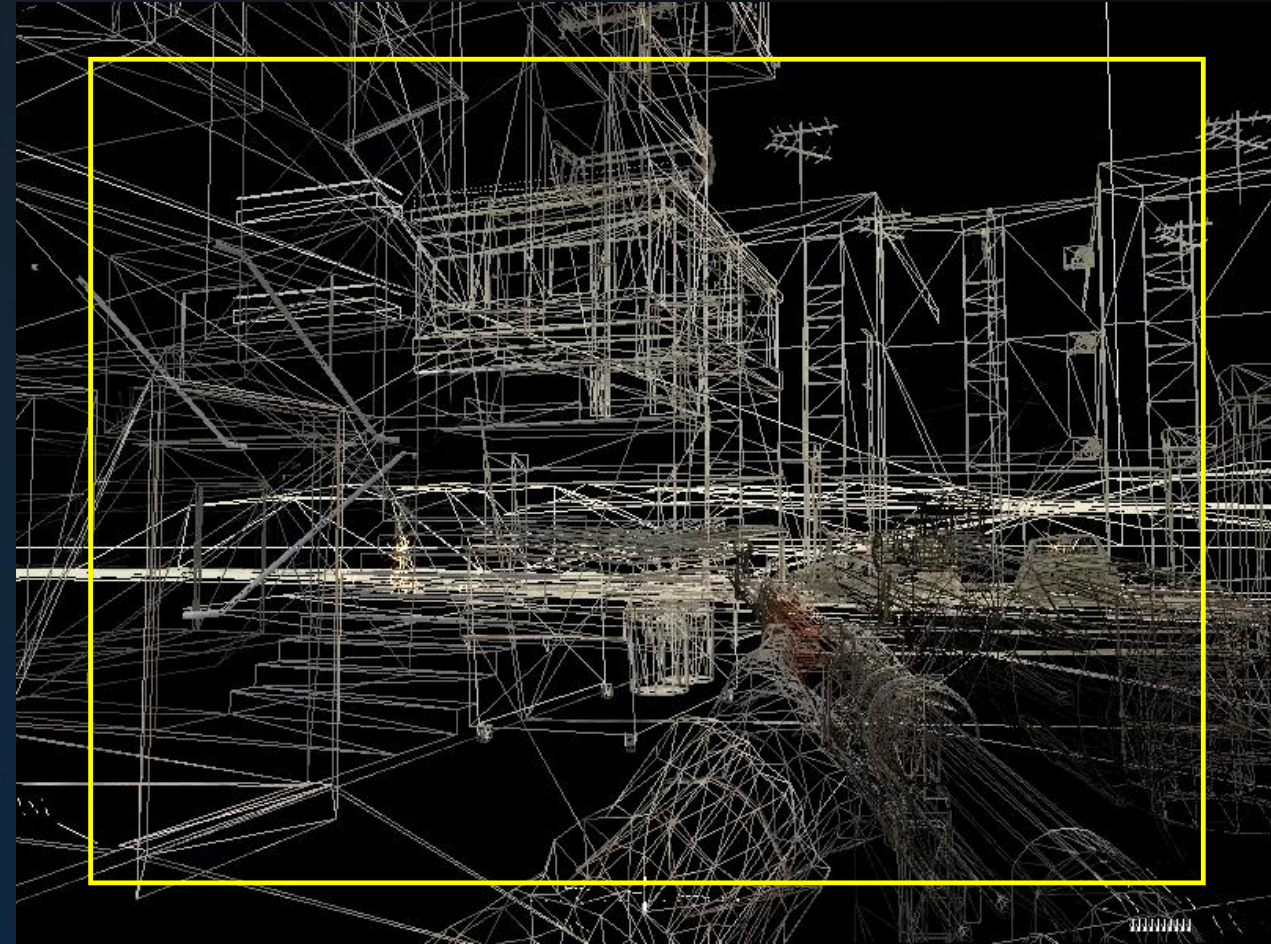


# Clipping

## Guard bands

To reduce the number of polygons that need clipping, some hardware uses *guard bands*: an invisible band of pixels outside the screen.

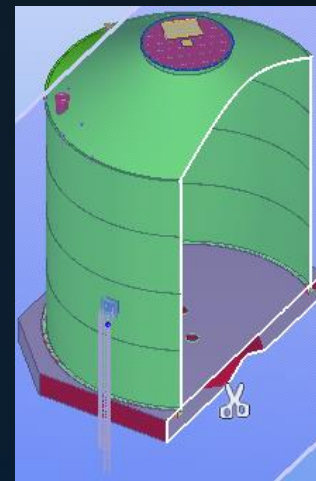
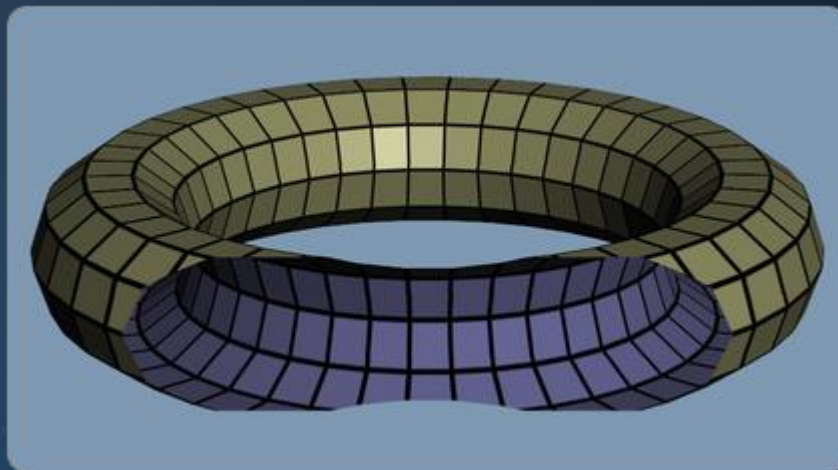
- Polygons outside the screen are discarded, even if they touch the guard band;
- Polygons partially inside, partially in the guard band are drawn without clipping;
- Polygons partially inside the screen, partially outside the guard band are clipped.



# Clipping

Sutherland-Hodgeman

Clipping can be done against arbitrary planes.





# Today's Agenda:

- Depth Sorting
- Clipping
- Visibility





Part of the tree is off-screen

Stuff that is too far to draw

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# Visibility

Only rendering what’s visible:

“Performance should be determined by visible geometry, not overall world size.”

- Do not render geometry outside the view frustum
- Better: do not *process* geometry outside frustum
- Do not render occluded geometry
- Do not render anything more detailed than strictly necessary



# Visibility

## Culling

### Observation:

50% of the faces of a cube are not visible.

On average, this is true for all meshes.

### Culling ‘backfaces’:

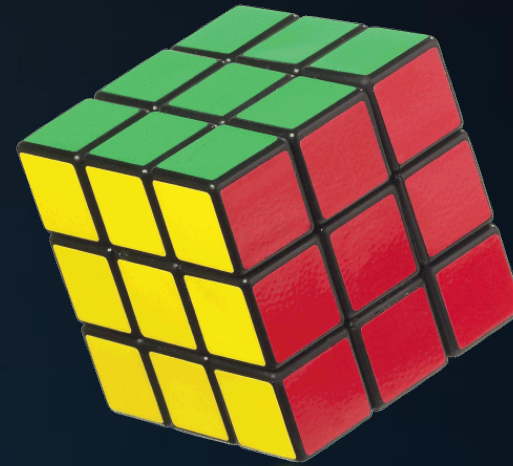
Triangle:  $ax + by + cz + d = 0$

Camera:  $(x, y, z)$

Visible: fill in camera position in plane equation.

$ax + by + cz + d > 0$ : *visible*.

**Cost: 1 dot product per triangle.**





# Visibility

## Culling

### Observation:

If the *bounding sphere* of a mesh is outside the view frustum, the mesh is not visible.

### But also:

If the *bounding sphere* of a mesh intersects the view frustum, the mesh may be not visible.

View frustum culling is typically a *conservative test*: we sacrifice accuracy for efficiency.

**Cost: 1 dot product per mesh.**





# Visibility

## Culling

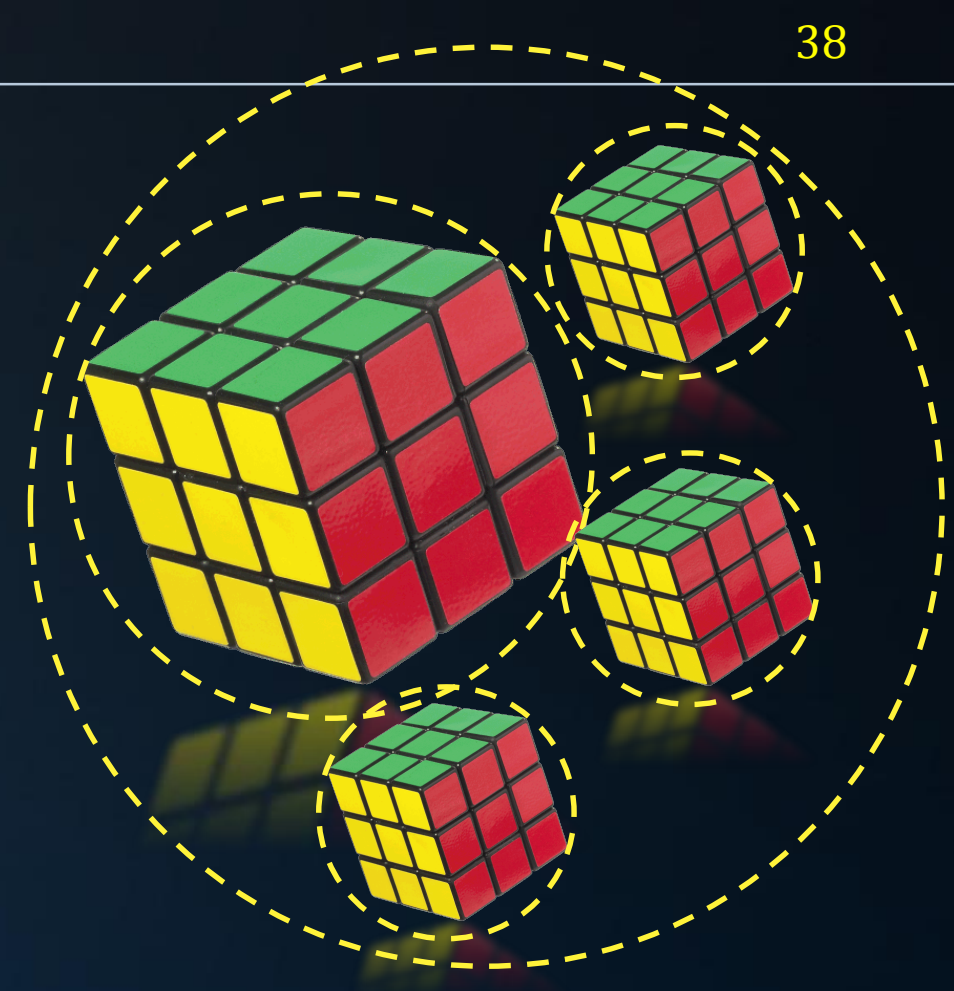
### Observation:

If the *bounding sphere* over a group of bounding spheres is outside the view frustum, a group of meshes is invisible.

We can store a bounding volume hierarchy in the scene graph:

- Leaf nodes store the bounds of the meshes they represent;
- Interior nodes store the bounds over their child nodes.

**Cost: 1 dot product per scene graph subtree.**



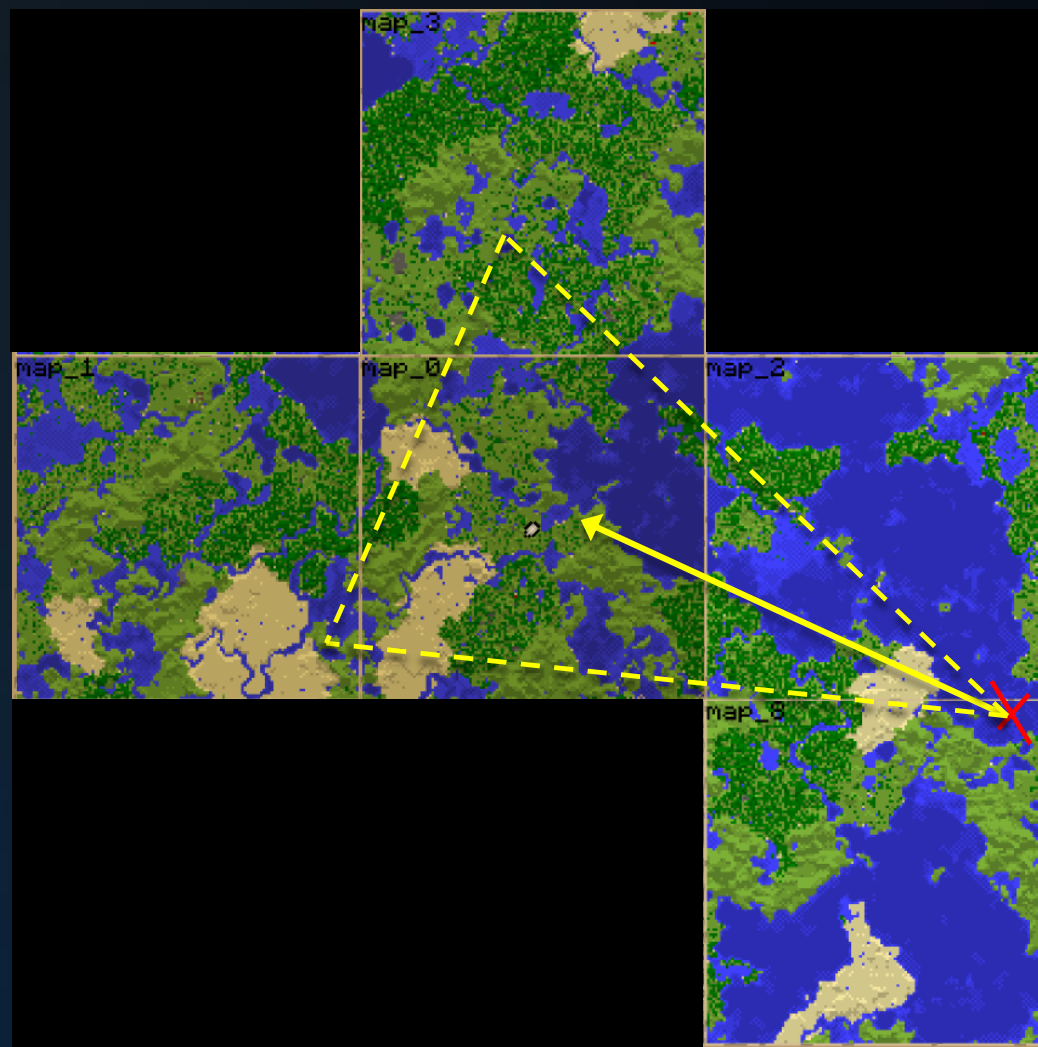
# Visibility

## Culling

### Observation:

If a grid cell is outside the view frustum, the contents of that grid cell are not visible.

**Cost: 0 for out-of-range grid cells.**





# Visibility

## Indoor visibility: Portals

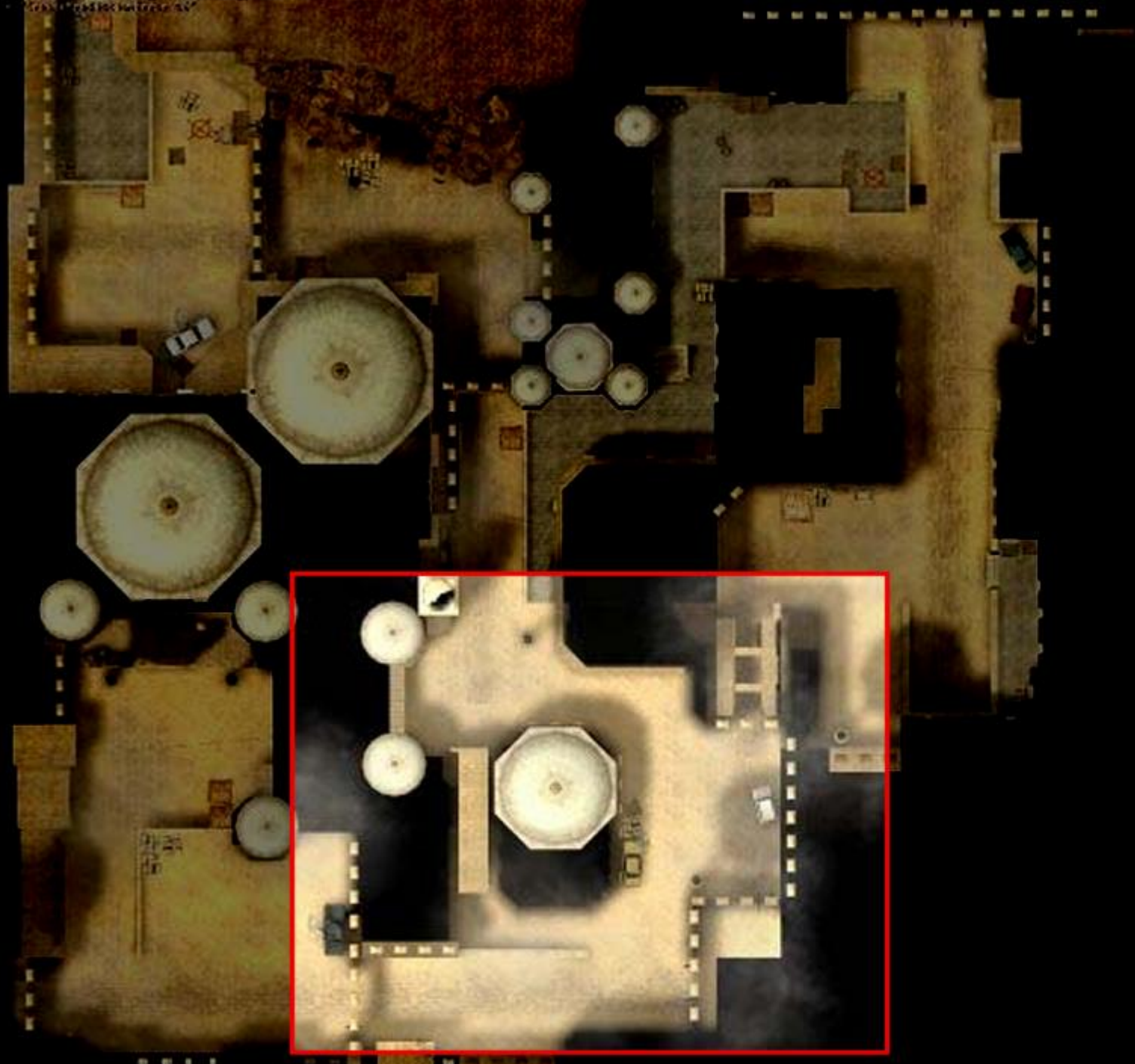
Observation: if a window is invisible, the room it links to is invisible.

```
...
    & (depth < MAXDEPTH)
...
    t = inside / (inside + outside);
    nt = nt / nc;
    cos2t = 1.0f - nnt;
    D, N );
    )
...
    at a = nt - nc, b = nt;
    at Tr = 1 - (R0 + (1 - R0) * t);
    Tr) R = (D * nnt - N * (a * t
...
    E * diffuse;
    = true;
...
    refl + refr)) && (depth < MAXDEPTH)
...
    D, N );
    -refl * E * diffuse;
    = true;
...
MAXDEPTH)
...
survive = SurvivalProbability( diffuse
estimation - doing it properly, closely
if;
radiance = SampleLight( @rand, I, M, Alignment
e.x + radiance.y + radiance.z) > 0) && (max
...
v = true;
at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
at3 factor = diffuse * INVPI;
at weight = Mis2( directPdf, brdfPdf );
at cosThetaOut = dot( N, L );
E * ((weight * cosThetaOut) / directPdf) * (radiance
...
random walk - done properly, closely following death
ive)
...
at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf
survive;
pdf;
n = E * brdf * (dot( N, R ) / pdf);
sion = true;
```























1

2

3

4











1

2

3

4



1

2

3

4





1

2

3

4

# Visibility

## Visibility determination

### Coarse:

- Grid-based (typically outdoor)
- Portals (typically indoor)

### Finer:

- Frustum culling
- Occlusion culling

### Finest:

- Backface culling
- Clipping
- Z-buffer





# Today's Agenda:

- Depth Sorting
- Clipping
- Visibility



# INFOGR – Computer Graphics

J. Bikker - April-July 2016 - Lecture 11: “Visibility”

## END of “Visibility”

next lecture: “Advanced Shading”

