

Tutorial 1 – vectors, coordinate systems, primitives and projections in 2D

Introduction

The tutorials are designed to give you a way to practice the material discussed in the lectures. Assistance on these tutorials is provided by the student assistants during the working colleges.

The exercises in these tutorials are representative for the exams (one halfway the block, one at the end).

Note that the answers to the exercises are not always directly available from the slides: it may (and will) require some tinkering to apply concepts. Feel free to ask for help doing this during the tutorial sessions, or on the forum:

infogr2017.slack.com

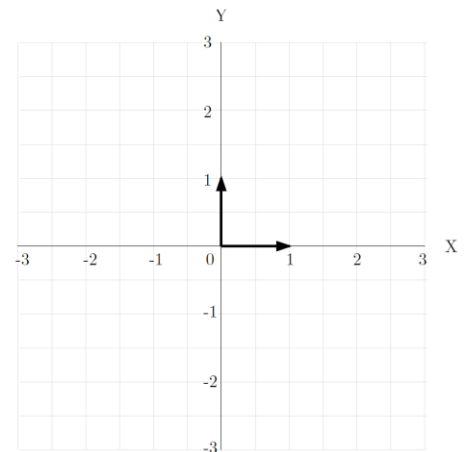
These tutorials are partially derived from materials by Michael Wand and Wolfgang Hürst.

Basic vectors

Exercise 1.

Given: three vectors in $\mathbb{R}^2$: $a = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $c = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

- Draw the vectors on a sheet of paper (using a grid such as the one on the right).
- Compute the sum of a and b . Draw the result.
- Scale vector a by the factors $1, 2, -1$ and $\sqrt{2}$. Draw the results.
- Determine $b - c$ graphically, using the rule from the lecture: first, join the starting points of b and c , then draw an arrow from the tip c of to the tip of b , which gives you the result.
- Compute and draw $2(a + c)$ and $2a + 2c$. Visualize how this gives the same result (distributive rule).
- Build a linear combination v of the vectors a, b, c , i.e.
$$v = \lambda_1 a + \lambda_2 b + \lambda_3 c.$$



Remark: This first assignment is only meant to familiarize yourself with geometric vectors. There is no big insight here. If you already took vector algebra in high-school, this should be very easy.

Exercise 2.

Given: three vectors $\vec{a} = (1,5,4)$, $\vec{b} = (2, -5,1)$ and $\vec{c} = (1,2,1)$ in $\mathbb{R}^3$.

- a) What is the sum of vectors $\vec{a}$ and $\vec{b}$?
- b) What is the length of vector $\vec{a}$?
- c) Calculate unit vector $\vec{u}$, which has the same direction as vector $\vec{a}$.
- d) Calculate the dot product of $\vec{a}$ and $\vec{b}$.
- e) Calculate the cross product of $\vec{a}$ and $\vec{c}$.
- f) Create a vector $\vec{p}$ with magnitude 2, parallel to $\vec{b}$.
- g) Create a vector perpendicular to $\vec{b}$.
- h) Create a normalized vector that is perpendicular to $\vec{b}$ and $\vec{c}$.

Exercise 3.

What is the relation between the magnitude of a vector and the dot product of the vector with itself?

Exercise 4.

Given: two unit vectors $\vec{a}$ and $\vec{b}$.

- a) If $\vec{a} \cdot \vec{b} = 0$, what do we know about the angle between $\vec{a}$ and $\vec{b}$?
- b) If $\vec{a} \cdot \vec{b} < 0$, what do we know about the angle between $\vec{a}$ and $\vec{b}$?
- c) If $\vec{a} \cdot \vec{b} = 1$, what do we know about the angle between $\vec{a}$ and $\vec{b}$?

Coordinate systems

Exercise 5.

Given: two vectors $\vec{a}$ and $\vec{b}$.

- a) The vectors form a 2D basis. What can you say about the dot product $\vec{a} \cdot \vec{b}$?
- b) The vectors form an orthonormal basis. What can you say about the magnitude of $\vec{b}$?
- c) Vector $\vec{a} = \begin{pmatrix} \frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} \end{pmatrix}$. Assuming orthonormality, write down all possibilities for vector $\vec{b}$.
- d) Point $C = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. What are the coordinates of point C in the 2D coordinate system defined by $\vec{a}$ and $\vec{b}$? Specify which $\vec{b}$ you used.

Basic geometric entities

Exercise 6.

Given: a line in $\mathbb{R}^2$ through two points $(-3,0)$ and $(2,2)$.

- a) What is the slope-intersect form of this line?
- b) Verify your answer for a) for the two given points.
- c) What is the implicit form of the line?
- d) Determine two normals for the line, with different directions.
- e) What is the relation between the distance of the line to the origin, 'C' in the general representation, and the magnitude of the normal?

Exercise 7.

Let l be a line in $\mathbb{R}^2$ through the origin, and let $\vec{n}$ be a normal vector for l .

- a) Show geometrically that all points p that lie on the line satisfy $\vec{n} \cdot \vec{p} = 0$.

Now, let l be a line in $\mathbb{R}^2$ that does not go through the origin, and let $\vec{n}$ be a normal vector for l .

- b) Show geometrically that all points p that lie on the line satisfy $\vec{n} \cdot \vec{p} - \vec{n} \cdot \vec{p}' = 0$, where p' is an arbitrary point on the line.

Note: "geometrically" here means that you can solve this question by drawing an image and using it to explain the solution.)

Exercise 8.

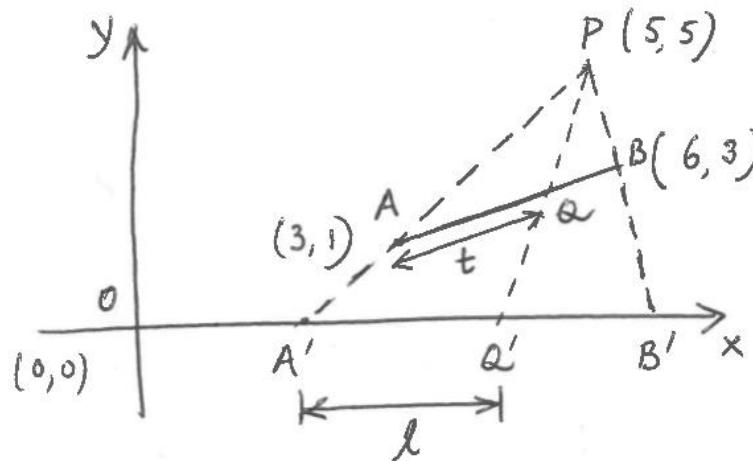
In the lecture, we discussed three different kinds of representations of a line: the parametric representation, the implicit representation, and the slope-intercept representation. Write down the general form of these equations and explain how we can do a conversion from one form to another. Take an example and transfer it into the other representations.

Exercise 9.

Let l be a line in $\mathbb{R}^2$ with implicit equation $f(x, y) = ax + by + c = 0$. For points that do not lie on the line, we have $f(x, y) \neq 0$. We say that the line l splits $\mathbb{R}^2$ in two half-spaces: the positive half-space l^+ , containing the points in $\mathbb{R}^2$ that lie on the side of l to which the normal vector (a, b) points, and the negative half-space l^- , containing the remaining points not on l . Show that the name *positive half-space* is appropriate by proving that $f(x_p, y_p) > 0$ for any point $p = (x_p, y_p)$ that lies in the halfspace to which the normal vector (a, b) points.

Exercise 10.

As shown in the picture, a light source P located in two-dimensions at $(5,5)$ casts a shadow of the bar AB on to the x -axis as $A'B'$. The locations of A and B respectively are $(3,1)$ and $(6,3)$.



- Find the locations of A' and B' .
- The shadow of the point Q is cast on the x -axis at point Q' . The distance between A and Q is t , and the distance between A' and Q' is l . Express l as a function of t .