Tutorial 2 – vectors, coordinate systems, primitives and projections in 2D continued, and vectors, coordinate systems, primitives and projections in 3D

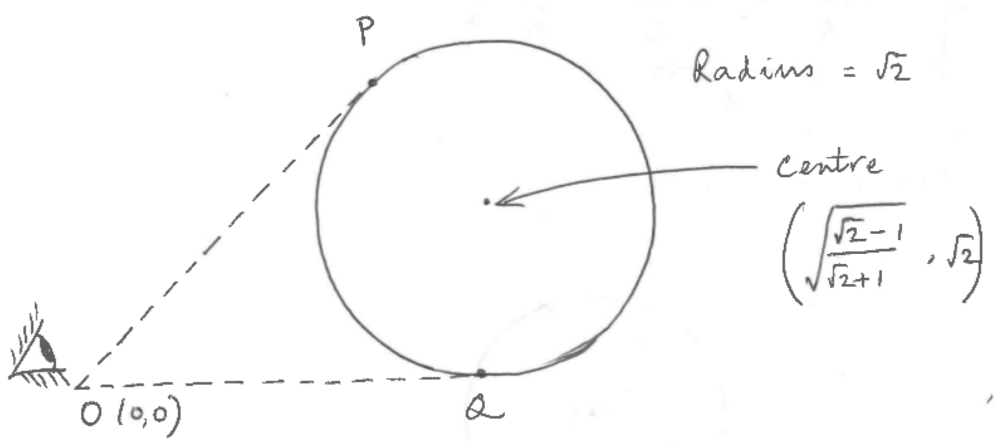
Exercise 1.

Given: a circle in $\mathbb{R}^2$ with radius $r = 4$, and center $c = (5,1)$.

a) What is the implicit representation of the circle?
b) What is the parametric representation of the circle?
c) What is the location of the point on the circle at angle $\pi/4$? What is the unit radial vector at angle $\pi/4$?
d) What is the location of the point on the circle at angle $3\pi/4$? What is the unit tangent vector at angle $3\pi/4$?

Exercise 2.

You are viewing a circle, with your eye at the origin (0,0) in $\mathbb{R}^2$. See figure below.



a) Obtain the parametric representation of the points P and Q.
b) Calculate the tangent vectors at the points P and Q; what are their relations to the lines OP and OQ respectively?

Exercise 3.

Given: a line in $\mathbb{R}^3$ through two points, (-4,1,1) and (2, 2, 5).

a) What is the length of this line segment?
b) What is the *parametric* representation of this line?
c) Determine two normals for the line, with different directions.
d) How many normals of unit length exist for this line, and how are they organized?
e) What is the equation of the plane perpendicular to this line?

Exercise 4.

Given: a line in $\mathbb{R}^4$ through two points, (-2,0,1,1) and (4, 1, 5, -1).

a) Determine two normals for the line, with different direction.
b) How many normals of unit length exist for this line, and how are they organized?

Exercise 5.

Given: a plane in $\mathbb{R}^3$, for which the following is true: the x-coordinate of all points on the surface is 1.

a) Determine two linearly independent vectors in this plane.
b) Determine the normal of the plane.
c) Calculate the distance of point $p = (5,1,1)$ to the plane.
d) How many planes exist that are perpendicular to the plane, and have a distance $d = 0$ to the origin?

Exercise 6.

a) Give a parametric equation for the plane V in $\mathbb{R}^3$ through the points $p_1 = (0,7,6)$, $p_2 = (8,0,8)$, and $p_3 = (12,10,0)$.
b) Give an implicit equation for the plane from the previous sub-problem.
c) Verify that the tree points are indeed part of the plane.
d) Now give a normal vector to the plane.

Exercise 7.

It's generally not possible to create an implicit representation of "1-dimensionalish" curves (e.g. lines, circles, spirals) in 3D. But with the parametric representation we can. For example, we can create a spiral using the sin and cos functions. Another (rather simple, but very useful) example is a line in $\mathbb{R}^3$.

a) Construct the parametric equation of a line l_1 in $\mathbb{R}^3$ that goes through the points $p_1 = (0,7,6)$ and $p_2 = (8,0,8)$.
b) Now create the parametric equation of a line l_2 in $\mathbb{R}^3$ that goes through $p_1 = (0,7,6)$ and $p_3 = (12,10,0)$.
c) What is the intersection point of both lines? You can either calculate it, or write it down directly. In the latter case, you have to argue why this is an intersection point, and also why it is the only one.

Exercise 8.

Given:

- a screen with a resolution of 1024×768 pixels and pixel x-coordinates $0..1023$ (left to right) and y-coordinates $0..767$ (top to bottom);
- a rectangle in $\mathbb{R}^2$, with corners $(-2,1.5)$ (top left corner), $(2, 1.5)$ (top right corner), $(2, -1.5)$ (bottom right corner) and $(-2, -1.5)$ (bottom left corner).

a) Calculate x_{screen}, y_{screen} based on parameters x_{world} and y_{world}.
b) Verify your solution for the four corners, e.g. $(2, -1.5)$ should yield $(1023{,}767)$.
c) Verify your solution for the world origin $(0{,}0)$.
d) Generalize your solution for a screen of $W \times H$ pixels, and a world rectangle of $x_1..x_2, y_1..y_2$.
e) For a screen of $W \times H$ pixels and a world rectangle of $x_1..x_2, y_1..y_2$, what is the size of a pixel in world coordinates?
f) Modify your solution for d) so that it only requires $x_1..x_2$; use the screen aspect ratio to determine $y_1..y_2$.
g) For the general solution of f), calculate the inverse operation, i.e. determine x_{world} and y_{world} based on parameters x_{screen} and y_{screen}.