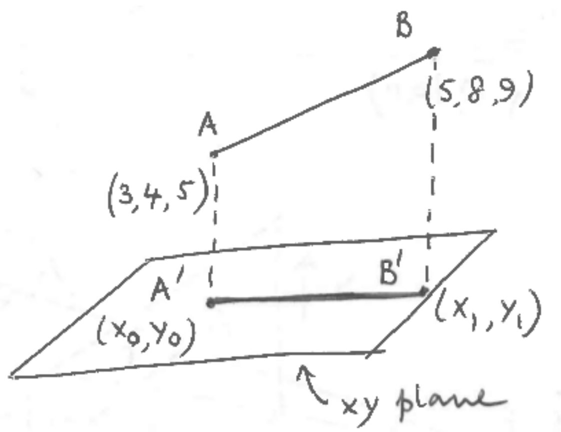
Tutorial 3 – vectors, coordinate systems, primitives and projections in 3D

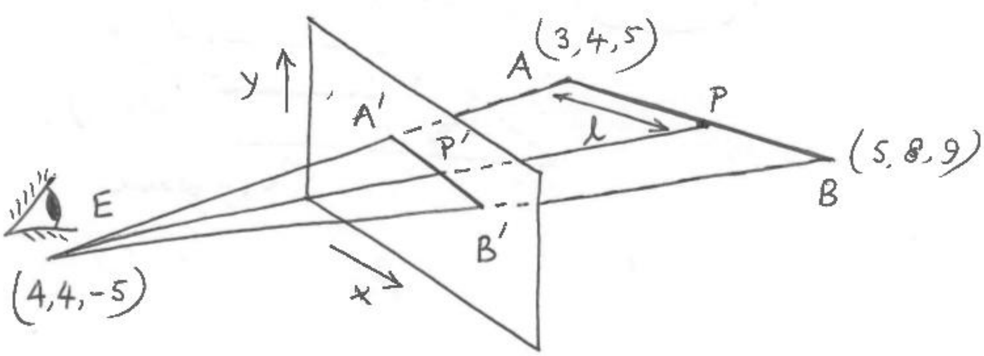
Exercise 1.

Express the equation of a line on the xy-plane (screen) $z = 0$ of a bar connecting (3,4,5) and (5,8,9) *in parametric form*. See figure. What are the values of (x_0, y_0) and (x_1, y_1)?



Exercise 2.

Consider the previous problem, but this time the bar is being looked at by the eye located at (4,4,-5). See figure. What are the co-ordinates of A' and B' on the xy-plane (screen) $z = 0$? Obtain the co-ordinates of the point P', projection of the point P on the bar, where AP = l.



Exercise 3.

Given: a sphere in $\mathbb{R}^3$ with radius $r = 3.14$ and center $c = (3,5,1)$.

a) What is the parametric representation of the sphere?
b) Using the parametric representation, calculate two opposing points on the sphere.

c) What is the distance between the points calculated in b)?

Exercise 4.

Given: a sphere in $\mathbb{R}^3$, with center $c = (3,3,3)$ and a point on the surface of the sphere, $p = (2,5,1)$.

a) Determine the implicit representation of the sphere.
b) Calculate the distance of the sphere to point $q = (4,9,-1)$ (note: this is not the distance to the centre of the sphere).
c) Define a plane that has p as the only intersection point with the sphere (i.e., the tangent plane of the sphere at position p).
d) Define two perpendicular unit vectors in this plane, which are both also perpendicular to the normal of the sphere in p.

The vectors found in exercise 2, together with the normal of the sphere in p span up an orthonormal basis, which we call *tangent space*. We operate in this space e.g. for doing normal mapping.

Exercise 5.

Given: a sphere in $\mathbb{R}^3$, with center $c = (3,3,3)$ and radius 3.

a) Find the range of parametric angles (θ, φ) that represents the part of the sphere viewed from $\left(3,3,3 - 3\sqrt{2}\right)$.
a) For the same viewing position as in part (a) find the location of the point on the xy-plane (screen) $z = 0$ corresponding to the projection of a point on the sphere given by the parametric angles (θ, φ). Follow the derivation provided in the class.