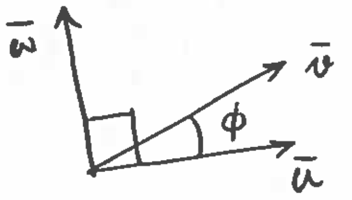
1. $\bar{t} = \bar{u} \times \bar{v}$ is a vector perpendicular to both $\bar{u}$ and $\bar{v}$, i.e., $\bar{t}$ is perpendicular to the plane formed by $\bar{u}$ and $\bar{v}$.

$\bar{w} = \bar{t} \times \bar{u}$ is similarly perpendicular to both $\bar{t}$ and $\bar{u}$ and is parallel to the plane formed by $\bar{u}$ and $\bar{v}$.
So we have the following scenario:



where $\bar{w} \perp \bar{u}$, i.e., the angle between $\bar{w}$ and $\bar{v}$ is $\boxed{(90^\circ - \phi)}$

2. Eqn. of the line $A'B'$ is $x - 2y + 1 = 0$
So the vector AA' is (anti) parallel to $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

the line AA' passes through $(0,3)$;
so we use the parametric form of the line AA':

$$AA' = \begin{pmatrix} 0 \\ 3 \end{pmatrix} + \frac{t}{\sqrt{5}} \begin{pmatrix} 1 \\ -2 \end{pmatrix};$$ using the unit vector $\frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\|\overline{AA'}\| = t$

At A': $$\left(0 + \frac{t}{\sqrt{5}}\right) - 2\left(3 - \frac{2t}{\sqrt{5}}\right) + 1 = 0$$

$$\Rightarrow \sqrt{5}\,t - 5 = 0 \quad \Rightarrow \boxed{t = \sqrt{5}}$$

The location of A' is obtained by using this value of t in the equation for AA'; it is $\boxed{(1,1)}$

The unit vector $\hat{u}$ along $\overline{AB}$ is given by $\frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$
(Eq. of the line AB is $y - 2x + 3 = 0$)
The unit vector $\hat{v}$ along $\overline{A'B'}$ is given by $\frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$
(Eq. of the line $A'B'$ is $x - 2y + 1 = 0$)
cosine of the angle ϕ between the two is

$$\cos\phi = \frac{4}{5} = (\hat{u} \cdot \hat{v})$$

$$\Rightarrow \|\overline{A'B'}\| = \|\overline{AB}\| \cos\phi \boxed{= \frac{4l}{5}}$$

3. Centre of the sphere is located at $(1, 3, 1)$ and its radius is 6.

Let us denote the centre of the sphere by O.

The unit vector $\hat{OP}$ is given by

$$\hat{u} = \frac{1}{3}\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

The plane perpendicular to this unit vector passing through P is the tangent plane.

The eqn. of the plane is

$$2x + y + 2z + a = 0 \quad \text{for some } a.$$

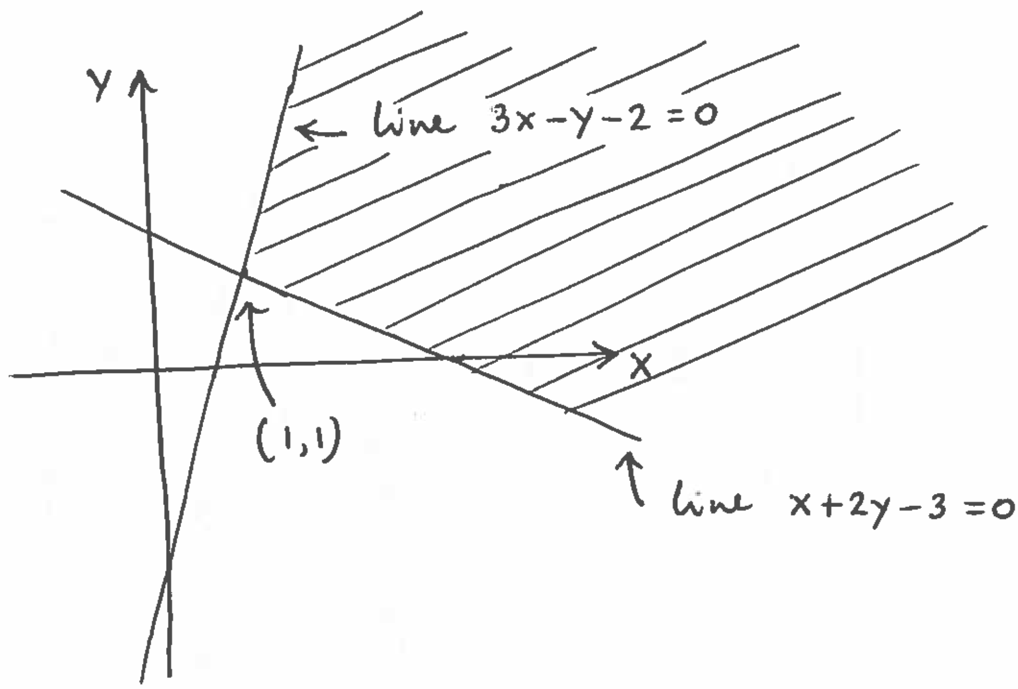
Since the plane passes through $(5, 5, 5)$

$$a = -2\times 5 - 5 - 2\times 5 = 25.$$

So the eqn. of the plane is

$$\boxed{2x + y + 2z - 25 = 0}$$

4.



5. The centre of the circle is located at $O = (0, 8)$

Let us take the triangle OEB. Since AB is the maximal arc of the circle visible to the eye, $\angle OBE = 90°$, i.e., EB is a tangent to the circle.

We work in terms of the parametric form of the circle, i.e., the point $P=(x,y)$ on the circle is given by

$$\boxed{\begin{cases} x = 5\cos\theta \\ y = 8+5\sin\theta \end{cases}}$$

where θ is the angle subtended by the line OP w.r.t. to x-axis. We use parametric representation of point B. The vector $\overline{OB}$ is perpendicular the vector $\overline{EB}$.

Location of $B = (x_B, y_B) = (5\cos\theta_B, 8+5\sin\theta_B)$

$$\overline{OB} = 5\begin{pmatrix}\cos\theta_B \\ \sin\theta_B\end{pmatrix} \qquad \overline{EB} = \begin{pmatrix} 5\cos\theta_B \\ 13+5\sin\theta_B\end{pmatrix}$$

$\overline{OB}\cdot\overline{EB} = 0$

$$\Rightarrow 25\cos^2\theta_B + 5\sin\theta_B\,(13+5\sin\theta_B) = 0$$

$$\Rightarrow \sin\theta_B = -\frac{5}{13}; \quad \cos\theta_B = \sqrt{1-\sin^2\theta_B} = \frac{12}{13}.$$

Location of $B = \boxed{\left(\frac{60}{13}, \frac{79}{13}\right)}$.

Location of A (by symmetry) $= \boxed{\left(-\frac{60}{13}, \frac{79}{13}\right)}$

Note that the two triangles EAB and EA′B′ are similar.

$$\Rightarrow \frac{|A'B'|}{|AB|} = \frac{|ER|}{|EQ|},$$ where Q is the intersection point of AB and the y-axis

$$|EQ| = 5 + \frac{79}{13} = \frac{144}{13} \qquad |AB| = \frac{120}{13}$$

$$|ER| = 5$$

$$\boxed{|A'B'| = |AB|\,\frac{65}{144} = \frac{25}{6}}$$

6. The unit vector $\hat{\omega}$ (anti) parallel to $\overline{QP}$ is given by $\frac{1}{3}\begin{pmatrix}2\\1\\2\end{pmatrix}$. The line QP has the eqn.

$$\begin{pmatrix}0\\0\\3\end{pmatrix} + \frac{t}{3}\begin{pmatrix}2\\1\\2\end{pmatrix}$$

It intersects the plane at Q;

$$2\times\frac{2t}{3} + \frac{t}{3} + 2\times\left(3+\frac{2t}{3}\right) \quad -3=0$$

$$\Rightarrow 3t+3 = 0 \Rightarrow t=-1$$

length of the line segment QP = $\boxed{|t|=1}$.

location of $\boxed{Q = \left(-\frac{2}{3}, -\frac{1}{3}, \frac{7}{3}\right)}$

$\hat{u}\cdot\hat{\omega}=0 \Rightarrow \hat{u}$ is parallel to the plane.

The unit vector parallel to the plane and perpendicular to $\hat{u}$ is $\hat{\omega}\times\hat{u} = \frac{1}{3}\begin{pmatrix}2\\1\\2\end{pmatrix}\times\frac{1}{3}\begin{pmatrix}-1\\-2\\2\end{pmatrix}$

$$=\boxed{\frac{1}{3}\begin{pmatrix}2\\-2\\-1\end{pmatrix}}$$

7. The eqn. for the surface of the cylinder is $\boxed{(x-3)^2+(y-5)^2=25}$

Eqn. of the ~~line~~ ray is

$$\begin{pmatrix}0\\-3\\-4\end{pmatrix} + t\begin{pmatrix}2\\2\\1\end{pmatrix}$$

Note: no unit vector to keep the calculations simple

The intersection points satisfy

$$(2t-3)^2 + \left(\frac{2t}{} - 8\right)^2 = 25$$

$$\Rightarrow 8t^2 - 44t + 48 = 0$$

$$\Rightarrow 2t^2 - 11t + 12 = 0$$

$$\Rightarrow (2t-3)(t-4) = 0$$

$$\Rightarrow t = 3/2 \text{ or } t = 4.$$

Intersection points

$\left(3, 0, -\frac{5}{2}\right)$, $(8, 5, 0)$

$t=\frac{3}{2}$ $\quad$ $t=4$

tangent planes are

zx - plane

$t = \frac{3}{2}$

surface normal $-\hat{y}$

and

a plane parallel to yz - plane passing through $x = 8$

$t = 4$

surface normal $\hat{x}$

They intersect on the line $x = 5$, $y = 0$.