Graphics (INFOGR), 2016-17, Block IV, Maths lecture 7

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Today: Grand Recap on maths

Welcome

Summary: maths lecture 1

- Vectors
  - coordinate systems
  - addition and scalar multiplication
  - vector length $||\vec{v}||$ and unit vector $\hat{v} = \vec{v}/||\vec{v}||$
  - vector length and unit vector
  - dot product: $\cos\alpha = \vec{v} \cdot \vec{w}/(||\vec{v}||||\vec{w}||) = \hat{v} \cdot \hat{w}$
  - basis and components
  - elementary trigonometry: $v_x = \vec{v} \cdot \hat{x} = ||\vec{v}|| \cos\alpha$ etc.
- Lines in 2D
  - equation: $\vec{X} = \vec{X}_0 + t\vec{u}$ (parametric),
    $y = mx + c$ (slope-intercept), $Ax + By + C = 0$ (implicit)
  - tangent $\vec{u}$ and normal vector (A, B)
  - projections of lines as applications

Summary: maths lecture 2

- Primitives and projections in 2D (contd.)
  - circle ($x^2 + y^2 = a^2$) and ellipse ($x^2/a^2 + y^2/b^2 = 1$); implicit forms
  - circular coordinates: $(x, y) = R(\cos\phi, \sin\phi)$; parametric form
  - normal ($\hat{n}$) and tangent ($\hat{t}$) for circles; $\hat{n} \cdot \hat{t} = 0$
  - projections: applications of vector algebra with primitives in 2D
- Vectors, primitives and projections in 3D
  - cross product: $\vec{v} \times \vec{w} = ||\vec{v}||||\vec{w}||\hat{t} \sin\alpha$; $\hat{t} \cdot \vec{v} = \hat{t} \cdot \vec{w} = 0$
  - straight line in 3D: $\vec{X} = \vec{X}_0 + t\vec{u}$
    or $(x - x_0)/u_x = (y - y_0)/u_y = (z - z_0)/u_z$
  - planes (and hyperplanes): $Ax + By + Cz + Dw + \ldots + p = 0$
    $(A, B, C, D, \ldots)$ is a normal to the (hyper)plane
  - circles, cylinders, their tangent planes and normal vectors
  - projections as applications

Summary: maths lecture 3

- Other coordinate systems in 3D
  - cylindrical: same as circular in xy-plane using ϕ and z
  - spherical: (using θ, ϕ)
  - corresponding unit vectors
  - projections as applications
- Properties of light: direction and intensity
  - (specular) reflection: $\vec{R} = \vec{D} - 2(\vec{D} \cdot \hat{n})\hat{n}$
  - refraction: $\eta_1 \sin\theta_1 = \eta_2 \sin\theta_2$ (η is the refractive index)

    $\rightarrow$ critical angle of TIR: $\theta_c = \sin^{-1}(\eta_2/\eta_1)$; $\eta_2 < \eta_1$

    $\rightarrow \hat{T} = \frac{\eta_1}{\eta_2}\hat{D} + \left(\frac{\eta_1}{\eta_2}\cos\theta_1 - \sqrt{k}\right)\hat{n}$; $k = 1 - \frac{\eta_1^2}{\eta_2^2}\sin^2\theta_1$ ($k < 0 \Rightarrow \text{TIR}$)

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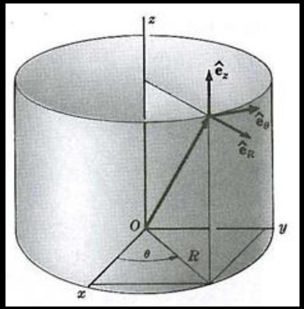

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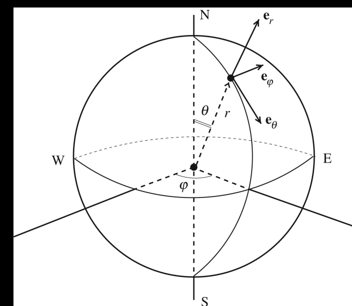

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Summary: maths lecture 4

- Matrices
  - definition, addition, multiplication particularly $AB \neq BA$
  - special matrices: diagonal, identity and null
  - transpose, cofactors and determinant (Laplace, Sarrus)
  - cofactor, adjoint, inverse and singular matrices
  - geometric interpretation of determinant: oriented area and volume
- Transformations: matrix operations on vectors
  - projection
  - (non-)uniform scaling
  - reflection
  - shearing
  - rotation

Summary: maths lecture 5

- Transformation: a tool to "talk across" different coordinate systems
  - rotation revisited
  - generally, switching of multiplication order, again $AB \neq BA$
  - translation
- We need transformation to get the world on to the screen

$$\begin{array}{ccccccc} & PM_c & & M_{\text{ortho}} & & M_{\text{vp}} & \\ \text{world} & \rightarrow & \text{camera} & \rightarrow & \text{canonical} & \rightarrow & \text{screen} \\ \{X_w\} & & \{X_m\} & & \{X_c\} & & \{X_s\} \end{array}$$

i.e., finally, $\{X_s\} = M_{\text{vp}} M_{\text{ortho}} P M_c \{X_w\}$

Summary: maths lecture 6

- Mathematical/statistical tricks for postprocesing
  - Averages and weighted averages
  - probability (density)
  - measure
  - expectation
  - law of large numbers and convergence of expectation values
  - calculating expectation using Monte Carlo
  - sampling from various probability distributions
  - importance sampling and stratified sampling