TUTORIAL 1 - EXERCISES

VECTORS AND VECTOR ALGEBRA

PART 1: THEORY

EXERCISE 1

We start with some theoretical questions.

(a) Explain in your own words the difference between a scalar, a point and a vector, using an example in daily life.
(b) What is a unit vector?
(c) What is a 'basis' for a coordinate system?
(d) What is the 'norm' of a vector?
(e) Write down Pythagoras' theorem and explain it in your own words.

PART 2: TWO-DIMENSIONAL VECTORS, INNER PRODUCTS AND NORMS

EXERCISE 2

Given two vectors $\vec{a} = \begin{bmatrix}1\\2\end{bmatrix}$ and $\vec{b} = \begin{bmatrix}3\\2\end{bmatrix}$. Use the regular 2D Cartesian coordinate system (the x, y-system you know from highschool).

(a) Draw $\vec{a}$ and $\vec{b}$.
(b) Draw $\vec{a} + \vec{b}$.
(c) Draw $\vec{a} - \vec{b}$.
(d) Draw $2(\vec{a} - \vec{b})$.

EXERCISE 3

Given the following vectors: $\vec{a} = \begin{bmatrix}1\\2\end{bmatrix}, \vec{b} = \begin{bmatrix}2\\3\end{bmatrix}, \vec{c} = \begin{bmatrix}6\\8\end{bmatrix}, \vec{d} = \begin{bmatrix}4\\1\end{bmatrix}, \vec{e} = \begin{bmatrix}1\\6\\2\end{bmatrix}, \vec{f} = \begin{bmatrix}0\\2\\4\end{bmatrix}, \vec{g} = \begin{bmatrix}5\\4\\3\\2\\1\end{bmatrix}, \vec{h} = \begin{bmatrix}2\\3\\8\\1\\2\end{bmatrix}, \vec{i} = \begin{bmatrix}1\\5\\2\\1\\6\end{bmatrix}$.

Calculate the following vector problems:

(a) $\vec{a} + \vec{b}$
(b) $\vec{g} - \vec{h}$
(c) $\vec{a} + \vec{b} - \vec{c}$
(d) $||\vec{a} - \vec{b}||^2$
(e) $(||\vec{h}||^5 - ||\vec{i}||^3)\,(\vec{e} - \vec{f})$

EXERCISE 4

Given: two vectors in $\mathbb{R}^2$: $\vec{a} = \begin{bmatrix}1\\2\end{bmatrix}, \vec{b} = \begin{bmatrix}3\\4\end{bmatrix}$ and $\vec{c} = \begin{bmatrix}5\\6\end{bmatrix}$. Calculate the following:

(a) $\vec{a} + \vec{b}$
(b) $\vec{b} - \vec{c}$
(c) $5\vec{b}$
(d) $||\vec{c}||^2$
(e) $||\vec{a}||$

Now in general form, set $\vec{a} = \begin{bmatrix}a_1\\a_2\end{bmatrix}, \vec{b} = \begin{bmatrix}b_1\\b_2\end{bmatrix}$ and $\vec{c} = \begin{bmatrix}c_1\\c_2\end{bmatrix}$. Redo exercises (a)-(e).

EXERCISE 5

Given: two vectors in $\mathbb{R}^2$: $\vec{a} = \begin{bmatrix}3\\1\end{bmatrix}, \vec{b} = \begin{bmatrix}-1\\3\end{bmatrix}$ and $\vec{c} = \begin{bmatrix}-3\\-1\end{bmatrix}$. Calculate the following, and explain your answer geometrically.

(a) The length of $\vec{a}, \vec{b}$ and $\vec{c}$. For each of these calculations, draw an xy-plane and draw the triangles you implicitly use with Pythagoras' theorem.
(b) $\vec{a} + \vec{c}$
(c) Normalize $\vec{a}, \vec{b}$ and $\vec{c}$.

PART 3: NORMS, UNIT VECTORS AND ANGLES

EXERCISE 6

Draw the following vectors, then calculate the norm of each of them and write down the corresponding unit vectors:

(a) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} 7 \\ 9 \end{bmatrix}$

(c) $\begin{bmatrix} \frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(e) $\begin{bmatrix} 3 \\ 1 \\ 7 \end{bmatrix}$

(f) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

EXERCISE 7

Given: three vectors in $\mathbb{R}^2$: $\vec{a} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\vec{c} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

(a) Draw the vectors on a sheet of paper (using a regular 2D-Cartesian x,y-grid)
(b) Compute the sum of $\vec{a}$ and $\vec{b}$. Draw the result.
(c) Scale vector $\vec{a}$ by the factors 1, 2, -1 and $\sqrt{2}$. Draw the results.
(d) Determine $\vec{b} - \vec{c}$ graphically, using the rule from the lecture: first, join the starting points of b and c, then draw an arrow from the tip $\vec{c}$ to the tip of $\vec{b}$, which gives you the result.
(e) Compute and draw $2(\vec{a} + \vec{c})$ and $2\vec{a} + 2\vec{c}$. Visualize how this gives the same result (this is called the distributive rule).
(f) Build a linear combination $\vec{v}$ of the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$. That is, $\vec{v} = \lambda_1 \vec{a} + \lambda_2 \vec{b} + \lambda_3 \vec{c}$.