TUTORIAL 4 - EXERCISES

PRIMITIVES (CONTINUED) AND PROJECTIONS IN 3D

PART 1: THEORY

EXERCISE 1

(a) Give the general implicit formula of a sphere.
(b) Give the general parametric formula of a sphere.
(c) Given the implicit equation for a plane in 3D, give a general formulation of a normalized normal vector to this plane.

PART 2: PLANES AND PROJECTIONS IN 3D

EXERCISE 2

Given coordinates $A = (-1, 1, 0)$, $B = (1, -3, 1)$ and $C = (-2, -2, -2)$. (See exercise 10 in tutorial 3 as a reference.)

(a) Determine the equation of the plane through A, B and C (parametric form).
(b) Determine the equation of this plane in the implicit form.

EXERCISE 3

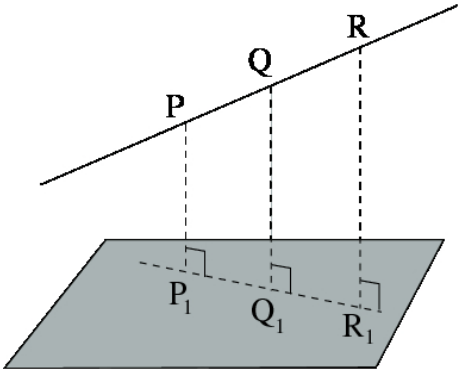
Given a plane that is given by $3x + 1y + 6z - 2 = 0$. Calculate the distance of the following points to this plane:

(a) $A = (0, 0, 0)$
(b) $B = (1, 1, 1)$
(c) $C = (2, -5, 3)$

EXERCISE 4

Given two points $P = (3, 4, 5)$ and $Q = (5, 8, 9)$.

(a) Calculate the line l going through P and Q in parametric form.
(b) Then project the piece of line between P and Q on the xy-plane (see figure; their projections are P_1 and Q_1). Calculate the length of this projected piece of line.
(c) Now consider another point R on line l. Say that $QR = t$, calculate the length of Q_1R_1 in terms of t.



EXERCISE 5

Given two points $P = (3, 4, 5)$ and $R = (5, 8, 9)$, and camera at point $E = (4, 4, -5)$. The xy-plane is the screen.

(a) Project PR on the screen as seen by the camera (see figure). Obtain the coordatines of P_1 and R_1.
(b) Given $PQ = l$, calculate the coordinates of point Q_1 in the xy-plane.

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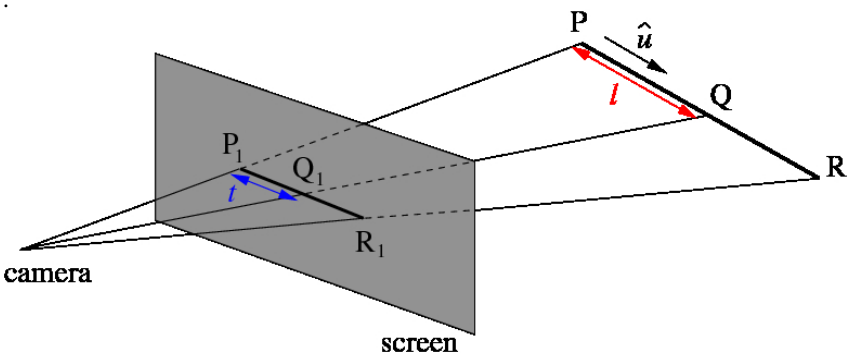

PART 3: SPHERES

EXERCISE 6

Given a sphere in $\mathbb{R}^3$ with radius $r = 3.14$ and center $C = (3,5,1)$

(a) What is the parametric representation of this sphere?
(b) Using the parametric representation, calculate the coordinates two opposing points on the sphere.
(c) Calculate the distance between these opposing points, and verify this using the diameter of the sphere.

EXERCISE 7

Given a sphere in $\mathbb{R}^3$ with center $C = (3,3,3)$ and radius 3.

(a) Find the range of parametric angles (θ, ϕ) that represents the part of the sphere viewed from $(3,3,3-3\sqrt{2})$
(b) Choose a point P on the sphere with $3 < z < 6$ and determine the the parametric form of the line that passes through this point and the origin of the sphere. Call this line l.
(c) Calculate the parametric angle (θ, ϕ) of the chosen point P with respect to the sphere.
(d) Calculate the intersection of l with the xy-plane.