TUTORIAL 5 - EXERCISES

MATRICES AND INTRODUCTIONTO TRANSFORMATIONS

PART 1: THEORY

EXERCISE 1

We start with some theoretical questions.

(a) What is a square matrix?
(b) What is the difference between a diagonal and a identity matrix?
(c) When can you sum two matrices? And when can you multiply them?
(d) Which conditions does a matrix need to satisfy in order to be able to be inverted?
(e) In class you have been shown that $A^{-1}A = AA^{-1} = I$, the identity matrix. Prove it for a generic 2×2 matrix A.
(f) Does any matrix have a determinant?
(g) What is a singular matrix? Which operation is not possible on singular matrices?
(h) What is the cofactor of an element of a matrix?

PART 2: OPERATIONS WITH MATRICES

EXERCISE 2

Given the following matrices:

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 2 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 5 & 2 \\ 3 & 2 & 1 \\ 5 & 3 & 0 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 2 \end{bmatrix} \quad F = \begin{bmatrix} 1 & 4 & 4 & 4 \\ 3 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

(a) Which matrices can be summed together?
(b) Compute those sums.
(c) Which matrices can be multiplied together? Does the order matter?
(d) Compute those multiplications.
(e) Compute the determinant for all matrices.
(f) Is it possible to compute the inverse of every matrix? If not, why?
(g) Compute the inverse matrices A^{-1} and B^{-1}.

EXERCISE 3

Given the following matrices:

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 3 \\ 4 & 1 \end{bmatrix} \quad J = \begin{bmatrix} 5 & 3 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 3 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad L = \begin{bmatrix} 3 & 0 & 0 & 1 \\ 0 & 2 & 0 & 2 \\ 1 & 5 & 0 & 3 \\ 1 & 8 & 2 & 3 \end{bmatrix}$$

For each of them compute:

(a) The determinant
(b) The transpose matrix.
(c) The cofactor matrix.
(d) The adjoint matrix.
(e) The inverse matrix.
(f) For matrix G, you could have answered all the previous question without doing any calculations. Why?

PART 3: GEOMETRICAL INTERPRETATION

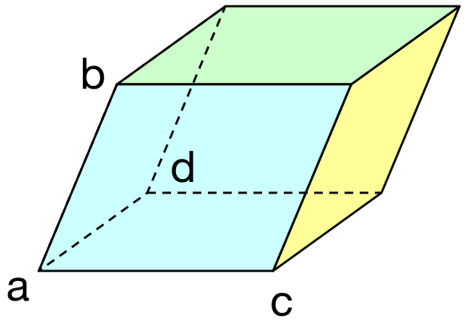
EXERCISE 4

Given the lists of vertices, compute the area of the following parallelograms:

$$A_1 = \{(3,1),(2,-3),(5,-2),(0,0)\} \qquad A_2 = \{(1,3),(3,1),(5,-1),(7,-3)\}$$

$$A_3 = \{(-1,-3),(4,-2),(0,-6),(5,-5)\} \qquad A_4 = \{(1,5),(-1,9),(2,9),(4,5)\}$$

EXERCISE 5



Given the list of vertices, compute the volume of the following parallelepipeds.

$$\pi_1 = \{(3,1,0),(2,-3,0),(5,-2,0),(0,0,-4)\} \qquad \pi_2 = \{(1,7,2),(3,7,1),(5,7,0),(-1,-1,-1)\}$$
$$\pi_3 = \{(2,1,5),(2,-1,9),(2,2,9),(-3,4,5)\} \qquad \pi_4 = \{(-1,2,-3),(4,2,-2),(0,2,-6),(5,-1,-5)\}$$

The points are ordered $\pi = \{a,b,c,d\}$ *and connected as in figure.*

EXERCISE 6

Two of the figures in the previous exercises have a zero oriented area or volume.

(a) What is the geometrical meaning of having a zero oriented area or volume?
(b) What restriction can you put on the vertices in order to avoid this case?