Graphics (INFOGR) **Midterm Exam**

Duration: 2 hrs; Total points: 100

No documents allowed. Use of electronic devices, such as calculators, smartphones, smartwatches is forbidden.

Question 1 **[5 points]** Equation of the plane given that the vector

$$\vec{v} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

is a normal to it at point $(1, 1, 1)$ is

(A) $x + 2y + z - 4 = 0$
(B) $2x + y + z - 4 = 0$
(C) $x + y + 2z - 4 = 0$
(D) $x + y + z - 3 = 0$
(●) $x + 2y + 2z - 5 = 0$
(F) $2x + y + 2z - 5 = 0$

Question 4 **[12 points]** Consider the surface of the sphere given by the equation $(x-3)^2 + (y-4)^2 + z^2 = 25$. You shoot a ray from the point $(8, 4, 0)$ along the vector

$$\vec{v} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$

The *outward* unit vectors normal to the surface of the sphere at the intersection points of the ray and the sphere are

(A) $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$

(B) $\begin{pmatrix} 2/3 \\ 2/3 \\ 1/3 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

(C) $\begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$

(D) $\begin{pmatrix} 1/3 \\ 2/3 \\ 2/3 \end{pmatrix}$ and $\begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$

(E) $\begin{pmatrix} 2/3 \\ 2/3 \\ 1/3 \end{pmatrix}$ and $\begin{pmatrix} 2/3 \\ 1/3 \\ 2/3 \end{pmatrix}$

[redacted] $\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

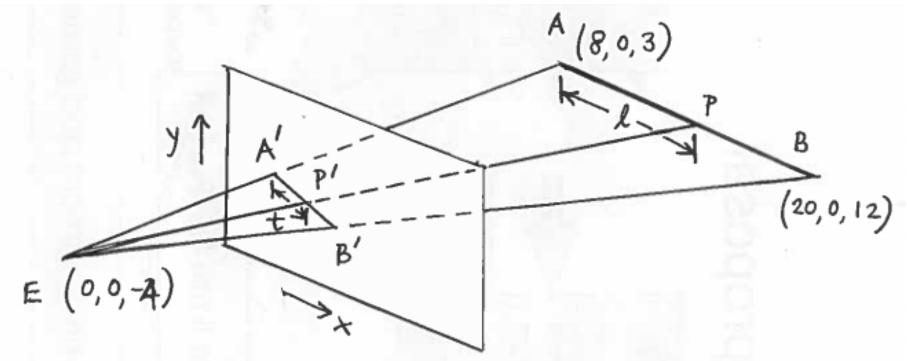
Question 6 **[3 points]** There are two vectors

$$\vec{v} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \vec{w} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}.$$

The quantity $u = \vec{v} \cdot (\vec{v} \times \vec{w})$ equals

(A) -1 [redacted] 0 (C) -2 (D) 3 (E) 2 (F) 1

Question 10 **[19 points]** As shown in the picture below, a bar AB is placed in three dimensions, with the locations of A = $(8, 0, 3)$ and B = $(20, 0, 12)$. The bar is being viewed by the eye located at E = $(0, 0, -4)$, and the view is being projected on the two-dimensional screen, which is simply the xy-plane. On the screen, A′ is the projection of A, P′ is the projection of P and so on. The distance AP is given by l and the distance A′P′ is given by t. The quantity t relates to l as



⬤ $t = \frac{16l}{7(3l+35)}$ (C) $t = \frac{2l}{2l+15}$ (E) $t = \frac{8l}{l+3}$ (G) $t = \frac{5l}{3l+4}$

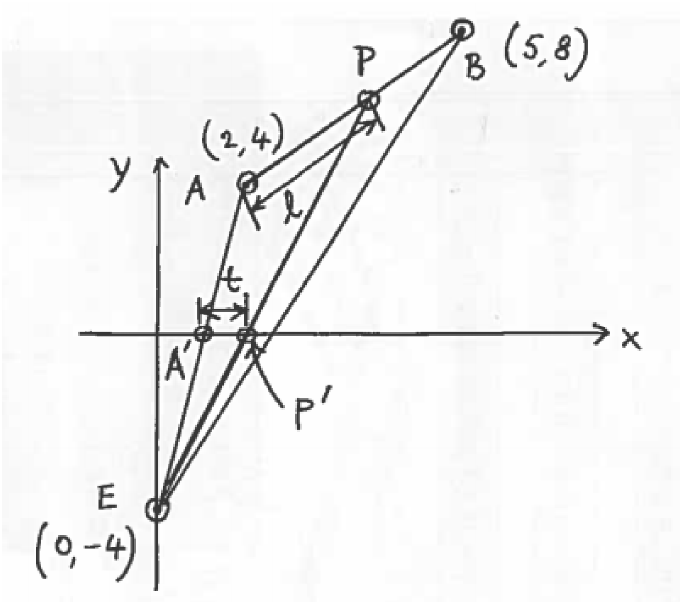
(B) $t = \frac{9l}{l+14}$ (D) $t = \frac{15l}{3l+21}$ (F) $t = \frac{15l}{2l+11}$ (H) $t = \frac{8l}{l+17}$

Question 11 **[8 points]** The unit vectors perpendicular to the triangular plane formed by A = $(4, -1, -3)$, B = $(5, -5, -2)$ and C = $(3, -3, -3)$ is

(A) $\pm \begin{pmatrix} -3/13 \\ 4/13 \\ 12/13 \end{pmatrix}$ (C) $\pm \begin{pmatrix} -2/3 \\ 1/3 \\ 2/3 \end{pmatrix}$ (E) $\pm \begin{pmatrix} 3/13 \\ 4/13 \\ 12/13 \end{pmatrix}$

⬤ $\pm \frac{1}{\sqrt{41}} \begin{pmatrix} 2 \\ -1 \\ -6 \end{pmatrix}$ (D) $\pm \begin{pmatrix} 3/13 \\ -4/13 \\ 12/13 \end{pmatrix}$ (F) $\pm \begin{pmatrix} 2/3 \\ -1/3 \\ 2/3 \end{pmatrix}$

Question 12 **[15 points]** As shown in the picture below, a bar AB is placed on the two-dimensional plane, with the locations of A = $(2, 4)$ and B = $(5, 8)$. The bar is being viewed by the eye located at E = $(0, -4)$, and the view is being projected on the one-dimensional “screen”, which is simply the x-axis. On the x-axis, A′ is the projection of A, P′ is the projection of P and so on. The distance AP is given by l and the distance A′P′ is given by t. The quantity t relates to l as



(A) $t = 2l/(l + 5)$ (C) $t = l/(l + 7)$ (E) $t = l/(l + 2)$ (G) $t = 2l/(l + 5)$

(B) $t = l/(l + 4)$ (D) $t = 3l/(l + 8)$ (F) $t = 3l/(l + 9)$ [redacted] $t = 2l/(l + 10)$

Question 14 **[6 points]** Take two points on the two-dimensional (x, y) plane: A = $(1, 2)$ and B = $(2, 3)$. Also consider a third point P = $(1, 5)$, from which you drop a perpendicular on to the line AB, intersecting it at point S. The co-ordinates of S and the *implicit form* equation for the line AB are respectively given by

(A) $(3, 7/2)$ and $3x - y - 1 = 0$

(B) $(5/2, 4)$ and $x - y + 1 = 0$

(C) $(7/2, 5/2)$ and $x + y - 1 = 0$

[redacted] $(5/2, 7/2)$ and $x - y + 1 = 0$

(E) $(3, 4)$ and $2x - y + 1 = 0$

(F) $(3, 7/2)$ and $x - y - 1 = 0$