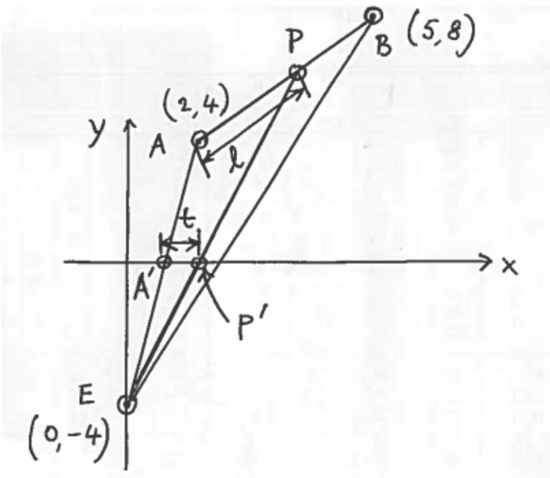
**Graphics (INFOGR)** **Practice Midterm**

**Duration: 2h; Total points: 50**

*No documents allowed. Use of electronic devices, such as calculators, smartphones, smartwatches is forbidden*

*Multiple choice questions: half of the total points will be deducted for each correct answer not selected, or for each wrong answer selected*

**Question 1.** [8 points] As shown in the picture below, a bar AB is placed on the two-dimensional plane, with the locations of A = $(2, 4)$ and B = $(5, 8)$. The bar is being viewed by the eye located at E = $(0, -4)$, and the view is being projected on the one-dimensional "screen", which is simply the $x$-axis. On the $x$-axis, A′ is the projection of A, P′ is the projection of P and so on. The distance AP is given by $l$ and the distance A′P′ is given by $t$.




The quantity $t$ relates to $l$ as ______________.

**Question 2.** [4 points] The unit vectors perpendicular to the triangular plane formed by A = $(4, -1, -3)$, B = $(4, 1, -2)$ and C = $(3, -3, -3)$ is given by

Your answer: ______________.

**Question 3.** [3+3=6 points] Consider the surface of the sphere given by the equation $(x-3)^2 + (y-4)^2 + z^2 = 25$. You shoot a ray from the point $(8, 4, 0)$ along the vector

$$\vec{v} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

The *outward unit* vectors normal to the surface of the sphere at the intersection points of the ray and the sphere are

Your answer: ________________________________.

**Question 4.** **[(1+1)+(5+5)+4=16 points]** Consider Fig. 1 in two dimensions. The equation of circle C is $x^2 + (y-8)^2 = 25$. The eye is located at E $= (0, -5)$. The **maximal** circular arc visible to the eye is AB, which is then being projected on to the one-dimensional "screen" as A$'$B$'$.

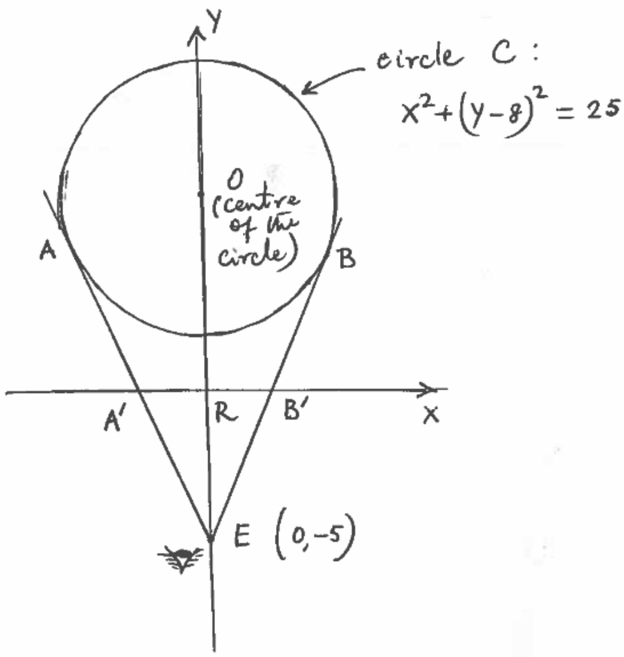



Figure 1: Figure for question 5.

(a) What are the co-ordinates of a point P on circle C in parametric form?

Your answer: ________________________________.

(b) The co-ordinates of the points A and B are: ________________________________.

(c) The length of the segment A$'$B$'$ is given by ______________.

**Question 5.** **[3+2+4=9 points]** Consider the plane $2x + y + 2z - 3 = 0$ in three dimensions, and the point P $= (0, 0, 3)$. Project the point on the plane at Q (meaning that the line PQ is perpendicular to the plane).

(a) The length of the line segment PQ is ________.

(b) The co-ordinates of the point Q is ________________.

(c) The unit vector $\hat{u} = \frac{1}{3}\begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix}$ is parallel to the plane. Obtain the second unit vector $\hat{v}$ that is parallel to the plane and perpendicular to $\hat{u}$.

Your answer: ________________.

**Question 6.** **[2 points]** There are two vectors

$$\vec{v} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \quad \text{an d} \quad \vec{w} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}.$$

The quantity $u = \vec{v} \cdot (\vec{v} \times \vec{w})$ equals ________.

**Question 7.** **[3 points]** As shown in Fig. 2 below, the angle between the two vectors $\vec{u}$ and $\vec{v}$ is $\phi$.

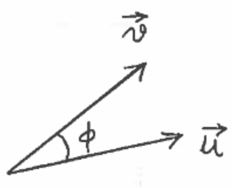


Figure 2: Figure for question 7.

The angle between the vector $\vec{w} = (\vec{u} \times \vec{v}) \times \vec{u}$ and $\vec{v}$ is ______________.