

Duration: 1h30m; Total points: 43

No documents allowed. Use of electronic devices, such as calculators, smartphones, smartwatches is forbidden

Question 1. [5 points] Consider three points $P = (2, 2, 2)$, $Q = (5, 4, 5)$ and $R = (3, 5, 4)$ in $\mathbb{R}^3$. Find the implicit equation of a plane passing through these points.

Your answer:

Question 2. [(2+1+2)+5+2+2+2=16 points] Consider the point $P = (6, 4, 6)$ and the plane $L: 6x + 3y + 2z - 11 = 0$ in $\mathbb{R}^3$, measured in co-ordinate system 1.

- (a) We translate the origin of the co-ordinate system to point $(1, 1, 1)$ of co-ordinate system 1 to define a (new) co-ordinate system 2. Write down the transformation matrix M_t that transforms the co-ordinates of the point X , which is (x, y, z) in co-ordinate system 1 to (x', y', z') in co-ordinate system 2. Apply this transformation to obtain the location of point P as well as the equation of the plane L in the co-ordinate system 2.

Your answer:

- (b) Write down the transformation matrix M_p that projects the point P on to the plane L in co-ordinate system 2. How would you characterise such a matrix?

Your answer:

- (c) The point P projected on the plane is denoted by P' . Calculate P' in co-ordinate system 2.

Your answer:

- (d) Now translate the origin of co-ordinate system 2 back to the location of the origin in co-ordinate system 1. Write down the transformation matrix M'_t that transforms the co-ordinates of the point X , which is (x', y', z') in co-ordinate system 2 to (x, y, z) in the co-ordinate system 1. Obtain the location of the projected point P' in co-ordinate system 1.

Your answer:

- (e) In co-ordinate system 1 if you were to obtain the location of P' directly from P using a transformation matrix M , then write down the expression of M in terms of M_t , M_p and M'_t .

Your answer:

Question 3. [3+3=6 points] Consider again the point $P = (6, 4, 6)$ and the plane $L: 6x + 3y + 2z - 11 = 0$ in $\mathbb{R}^3$.

- (a) Use the method of shooting a ray from point P to project it on plane L , and obtain the location of the projected point P' .

Your answer:

- (b) Reflect the point P on the plane L , and obtain the location of the reflected point Q .

Your answer:

Question 4. [3+3=6 points] Consider the line $2x - y + 3 = 0$ in $\mathbb{R}^2$.

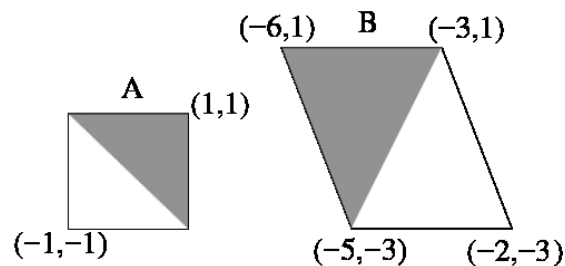
- (a) The 2×2 matrix that describes the reflection of a vector on the line is given by

Your answer: $\begin{bmatrix} -3/5 & 4/5 \\ 4/5 & 3/5 \end{bmatrix}$.

- (b) The unit vector that remains unchanged when reflected on this line is given by

Your answer:

Question 5. [4+6=10 points] You need to transform the square A to the rhombus B in the figure below (figure not to scale).



- (a) What are the elementary active transformations to you need to achieve this?

Your answer:

- (b) Write down the required transformation matrix.

Your answer: