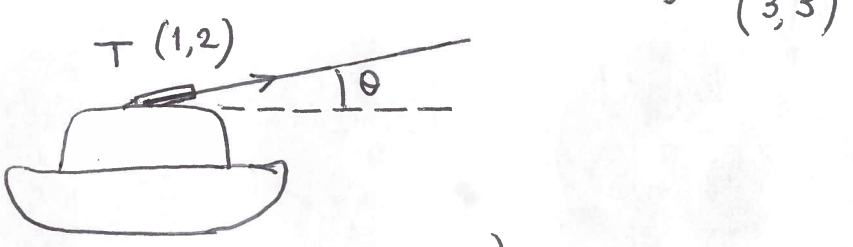
Slide 8 → Tank, bullet and target example in 2D




direction (unit vector) of the bullet?

Vector $\vec{TP} = \begin{bmatrix} 3-1 \\ 3-2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

unit vector $= \frac{1}{\sqrt{2^2+1^2}} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Angle wrt the ground?

$\tan\theta = \frac{1}{2}$ ; $\theta \simeq 26^\circ$; $\cos\theta = \frac{2}{\sqrt{5}}$, $\sin\theta = \frac{1}{\sqrt{5}}$

$\| \vec{TP}_x \| = \| TP \| \cos\theta = \sqrt{5} \times \frac{2}{\sqrt{5}} = 2$

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→ (a) $(x_0, y_0) = (1, 2)$, $\hat{v} = \begin{bmatrix} 3/5 \\ 4/5 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + t \begin{bmatrix} 3/5 \\ 4/5 \end{bmatrix} \Rightarrow \begin{aligned} x &= 1 + \frac{3t}{5} \\ y &= 2 + \frac{4t}{5} \end{aligned}$$

parametric form: $x - 1 = \frac{3t}{5}$, $y - 2 = \frac{4t}{5}$

$$t = \frac{5(x-1)}{3} = \frac{5(y-2)}{4}$$

$\Rightarrow 4(x-1) = 3(y-2) \Rightarrow y = \frac{4x}{3} + \frac{2}{3}$ : slope-intercept form

$4x - 3y + 2 = 0$ : implicit form

(b) $\hat{v} = \begin{bmatrix} 3/5 \\ 4/5 \end{bmatrix}$

Want to use parametric form again,
i.e., need to find $\hat{u} \perp \hat{v} \Rightarrow \hat{v} \cdot \hat{u} = 0$

$$\Rightarrow \hat{u} = \pm \begin{bmatrix} 4/5 \\ -3/5 \end{bmatrix}$$

$x - 4 = 4t/5$, $y - 6 = -\frac{3t}{5}$ : parametric form of the line $\perp$ to $L$ at $(4, 6)$

$$\Rightarrow t = \frac{5(x-4)}{4} = -\frac{5(y-6)}{3}$$

$\Rightarrow y = -\frac{3x}{4} + 9$ : slope-intercept form

$\Rightarrow 3x + 4y - 36 = 0$

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→ $(x_0, y_0) = (1, 2)$

$$\hat{v} = \begin{bmatrix} 3/5 \\ 4/5 \end{bmatrix}$$

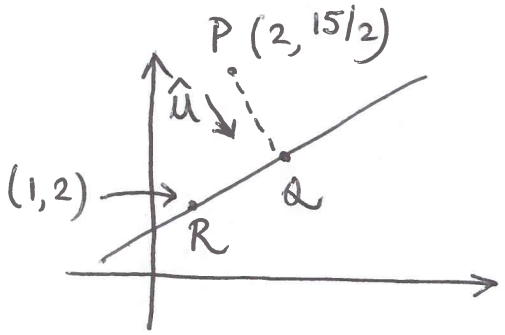
$$(x_1, y_1) = (2, 15/2)$$

Equation for RS in implicit form:

$$\underbrace{4}_{A}x - \underbrace{3}_{B}y + 2 = 0$$

Need to find $\hat{u} \perp \hat{v} \Rightarrow \hat{v} \cdot \hat{u} = 0$

$$\hat{u} = \pm \begin{bmatrix} 4/5 \\ -3/5 \end{bmatrix} \longrightarrow \frac{u_y}{u_x} = -\frac{3}{4} = \frac{B}{A}$$




$$\hat{u} = \begin{bmatrix} 4/5 \\ -3/5 \end{bmatrix}$$

choose this since $\hat{u}$ points in the $-y$-direction

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 15/2 \end{bmatrix} + t \begin{bmatrix} 4/5 \\ -3/5 \end{bmatrix} : \text{line PQ}$$

Also, $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + l \begin{bmatrix} 3/5 \\ 4/5 \end{bmatrix} : \text{line RS}$

They intersect at Q.

$$x_Q = 2 + 4t/5 = 1 + 3l/5$$

$$y_Q = \frac{15}{2} - 3t/5 = 2 + 4l/5$$

Solve $l$ and $t$: $l = 5, \quad t = \frac{5}{2}$

$$(x_Q, y_Q) = (4, 6)$$

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→ $(x_0, y_0) = (1, 2)$, $\hat{v} = \begin{bmatrix} 3/5 \\ 4/5 \end{bmatrix}$

RS: $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 3/5 \\ 4/5 \end{bmatrix}$ (parametric form)

or $y = \frac{4x}{3} + \frac{2}{3}$ (slope-intercept form)

$\Rightarrow y_Q = \frac{4}{3} x_Q + \frac{2}{3}$ or $f(x_Q, y_Q) = 0$

Now $f(x_1, y_1) = f(x_1, y_1) - f(x_Q, y_Q)$

$= (y_1 - y_Q) - \frac{4}{3}(x_1 - x_Q)$

PQ: $\begin{bmatrix} x_Q \\ y_Q \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + t \underbrace{\begin{bmatrix} 4/5 \\ -3/5 \end{bmatrix}}_{\hat{u}}$

$y_Q - y_1 = -\frac{3t}{5}$, $x_Q - x_1 = 4t/5$; $t > 0$

$\Rightarrow f(x_1, y_1) = \frac{3t}{5} + \frac{16t}{15} > 0$