Slides 12-18

$\rightarrow$ $(h, k) = (3, 3)$; $r = 3$

Equation of the circle $(x-3)^2 + (y-3)^2 = 9$

Eye at origin $(0, 0)$

unit vector $\hat{v}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + l \begin{bmatrix} v_x \\ v_y \end{bmatrix} \Rightarrow x = l v_x, \ y = l v_y$$

$$(l v_x - 3)^2 + (l v_y - 3)^2 = 9$$

$$\Rightarrow l^2 \underbrace{(v_x^2 + v_y^2)}_{=1} - 6l(v_x + v_y) + 9 = 0$$

$$l^2 - 6l(v_x + v_y) + 9 = 0$$

$$l = \frac{6(v_x + v_y) \pm \sqrt{36(v_x+v_y)^2 - 36}}{2}$$

$$= 3(v_x + v_y) \pm \sqrt{9(\underbrace{v_x^2 + v_y^2}_{=1} + 2v_x v_y) - 9}$$

$$= 3(v_x + v_y) \pm 3\sqrt{2 v_x v_y}$$

$v_x v_y > 0 \rightarrow$ two solutions P1, P2
(P2 not visible to the eye)

$v_x v_y = 0 \rightarrow v_x = 0$, or $v_y = 0$
ray is tangent to the circle

$v_x v_y < 0 \rightarrow$ no ray-circle intersection

Slide 21

$\rightarrow \quad \hat{v} = \frac{1}{13}\begin{bmatrix} 3 \\ 4 \\ 12 \end{bmatrix} \; ; \quad (x_0, y_0, z_0) = (6, 7, 8)$

Shoot a ray: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 8 \end{bmatrix} + \frac{l}{13}\begin{bmatrix} 3 \\ 4 \\ 12 \end{bmatrix}$

parametric form: $x = 6 + \frac{3l}{13}$

$y = 7 + \frac{4l}{13}$

$z = 8 + \frac{12l}{13}$

Slides 22-24

$\rightarrow \quad \hat{n} = \frac{1}{13}\begin{bmatrix} 3 \\ 12 \\ 4 \end{bmatrix} \qquad (x_0, y_0, z_0) = (1, 2, 3)$: Point P on the plane

Another point P_1 on the plane: (x, y, z)

Vector $\bar{V}$ spanning P to P_1 : $\begin{bmatrix} x-1 \\ y-2 \\ z-3 \end{bmatrix}$

$\bar{V} \cdot \hat{n} = 0 \Rightarrow 3(x-1) + 12(y-2) + 4(z-3) = 0$

or, $3x + 12y + 4z = 39$

↑ implicit form

Slides 30-34

$\rightarrow \quad \bar{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \bar{v} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

$\bar{u} \times \bar{v} = \begin{bmatrix} 2\times 6 - 5\times 3 \\ 3\times 4 - 1\times 6 \\ 1\times 5 - 2\times 4 \end{bmatrix} = \begin{bmatrix} -3 \\ 6 \\ -3 \end{bmatrix} = \bar{w}$

$\bar{u} \cdot \bar{w} = 1\times(-3) + 2\times 6 + 3\times(-3) = 0 \qquad \bar{u} \times \bar{v} = -(\bar{v} \times \bar{u})$

$\bar{v} \cdot \bar{w} = 4\times(-3) + 5\times 6 + 6\times(-3) = 0$