Slides 5–11

$\rightarrow$ Plane $3x + 12y + 4z = 39$

Point P: $(1, 5, 4)$

$$\hat{n} = \pm \frac{1}{13}\begin{bmatrix} 3 \\ 12 \\ 4 \end{bmatrix}$$

Line spanning P to P_1: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix} + \frac{a}{13}\begin{bmatrix} 3 \\ 12 \\ 4 \end{bmatrix}$

P_1 on plane $3x+12y+4z=39$

$$\Rightarrow 3\left(1+\frac{3a}{13}\right) + 12\left(5+\frac{12a}{13}\right) + 4\left(4+\frac{4a}{13}\right) = 39$$

$$\Rightarrow a = -\frac{40}{13}$$

$$P_1 : \left(\frac{49}{169}, \frac{365}{169}, \frac{516}{169}\right)$$

Continue to slides 12–17

$\rightarrow$ Point Q: $(x_Q, y_Q, z_Q) = (1+lu_x, 5+lu_y, 4+lu_z)$

Line spanning Q to Q_1: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_Q \\ y_Q \\ z_Q \end{bmatrix} + \frac{b}{13}\begin{bmatrix} 3 \\ 12 \\ 4 \end{bmatrix}$

take $\hat{u} = \frac{1}{3}\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

$$= \begin{bmatrix} 1+\frac{l}{3}+\frac{3b}{13} \\ 5+\frac{2l}{3}+\frac{12b}{13} \\ 4+\frac{2l}{3}+\frac{4b}{13} \end{bmatrix}$$

Q_1 on plane $3x+12y+4z=39$

$$\Rightarrow 3\left(1+\frac{l}{3}+\frac{3b}{13}\right) + 12\left(5+\frac{2l}{3}+\frac{12b}{13}\right) + 4\left(4+\frac{2l}{3}+\frac{4b}{13}\right) = 39$$

$$\Rightarrow b = -\frac{40}{13} - \frac{35l}{39}$$

$$Q_1 : \left(\frac{49}{169}+\frac{64l}{507}, \frac{365}{169} - \frac{82l}{507}, \frac{516}{169} + \frac{66l}{169}\right)$$

Vector spanning P_1 to Q_1 : $\frac{l}{507}\begin{bmatrix} 64 \\ -82 \\ 198 \end{bmatrix}$

$$t = \|\overline{P_1 Q_1}\| = \frac{2\sqrt{74}}{39} l$$

Slides 18–26

→ Camera location c : $(0, 0, -4) = (x_c, y_c, z_c)$

$$P : (8, 0, 3) = (x_0, y_0, z_0)$$

$$\hat{u} = \frac{1}{5}\begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix}$$

$$\hat{v} = \frac{1}{\sqrt{8^2+7^2}}\begin{bmatrix} 8 \\ 0 \\ 7 \end{bmatrix} = \frac{1}{\sqrt{113}}\begin{bmatrix} 8 \\ 0 \\ 7 \end{bmatrix}$$

Screen is the (x,y)-plane : $z = 0$

or $0 \times x + 0 \times y + 1 \times z = 0$

Vector spanning camera to P_1 $= \begin{bmatrix} 0 \\ 0 \\ -4 \end{bmatrix} + \frac{a}{\sqrt{113}}\begin{bmatrix} 8 \\ 0 \\ 7 \end{bmatrix}$

for P_1, $-4 + \frac{7a}{\sqrt{113}} = 0 \Rightarrow a = \frac{4\sqrt{113}}{7}$

$$P_1 : \left(\frac{8a}{\sqrt{113}}, 0, -4 + \frac{7a}{\sqrt{113}}\right)$$

$$= \left(\frac{32}{7}, 0, 0\right)$$

$$Q : \left(8 + \frac{4l}{5}, 0, 3 + \frac{3l}{5}\right)$$

$$\hat{w} = \frac{1}{\sqrt{\left(8+\frac{4l}{5}\right)^2 + \left(7+\frac{3l}{5}\right)^2}}\begin{bmatrix} 8 + 4l/5 \\ 0 \\ 7 + \frac{3l}{5} \end{bmatrix}$$

Vector spanning camera to $Q_1 = \begin{bmatrix} 0 \\ 0 \\ -4 \end{bmatrix} + l\,\hat{\omega}$

$$\text{for } Q_1, \quad -4 + \frac{\left(7+\frac{3l}{5}\right) l}{\sqrt{\left(8+\frac{4l}{5}\right)^2 + \left(7+\frac{3l}{5}\right)^2}} = 0$$

$$\Rightarrow l = \frac{4\sqrt{\left(8+\frac{4l}{5}\right)^2 + \left(7+\frac{3l}{5}\right)^2}}{7+\frac{3l}{5}}$$

$$Q_1 : \left(\frac{(8+4l/5)\, l}{\sqrt{\left(8+\frac{4l}{5}\right)^2 + \left(7+\frac{3l}{5}\right)^2}},\ 0,\ -4 + \frac{\left(7+\frac{3l}{5}\right) l}{\sqrt{\left(8+\frac{4l}{5}\right)^2 + \left(7+\frac{3l}{5}\right)^2}} \right)$$

$$= \left(\frac{4\,(8+4l/5)}{7+\frac{3l}{5}},\ 0,\ 0 \right)$$

$$t = \|\overline{P_1 Q_1}\| = \frac{4\,(8+4l/5)}{7+3l/5} - \frac{32}{7} = \frac{16\,l}{7\,(3l+35)}$$