

INFOGR – Computer Graphics

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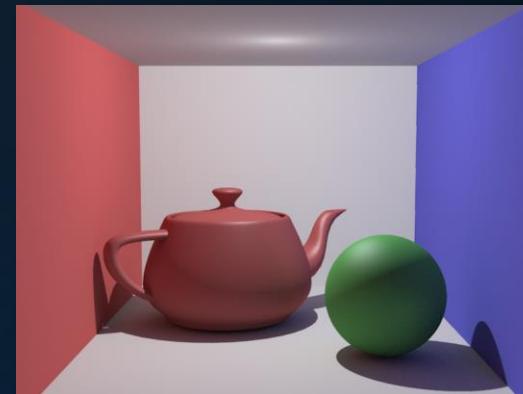
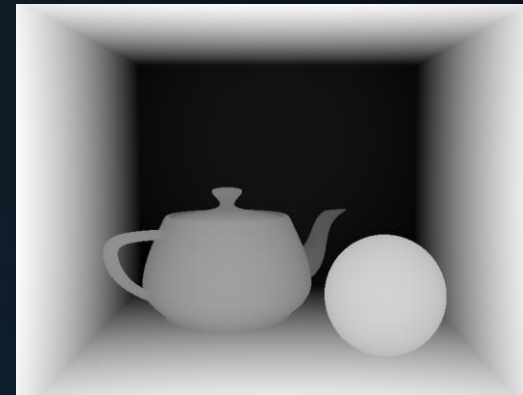
Lecture 6: “Ray Tracing”

Welcome!



Today's Agenda:

- Ray Tracing Recap
- Shading
- Textures



Ray Tracing

Ray Tracing:

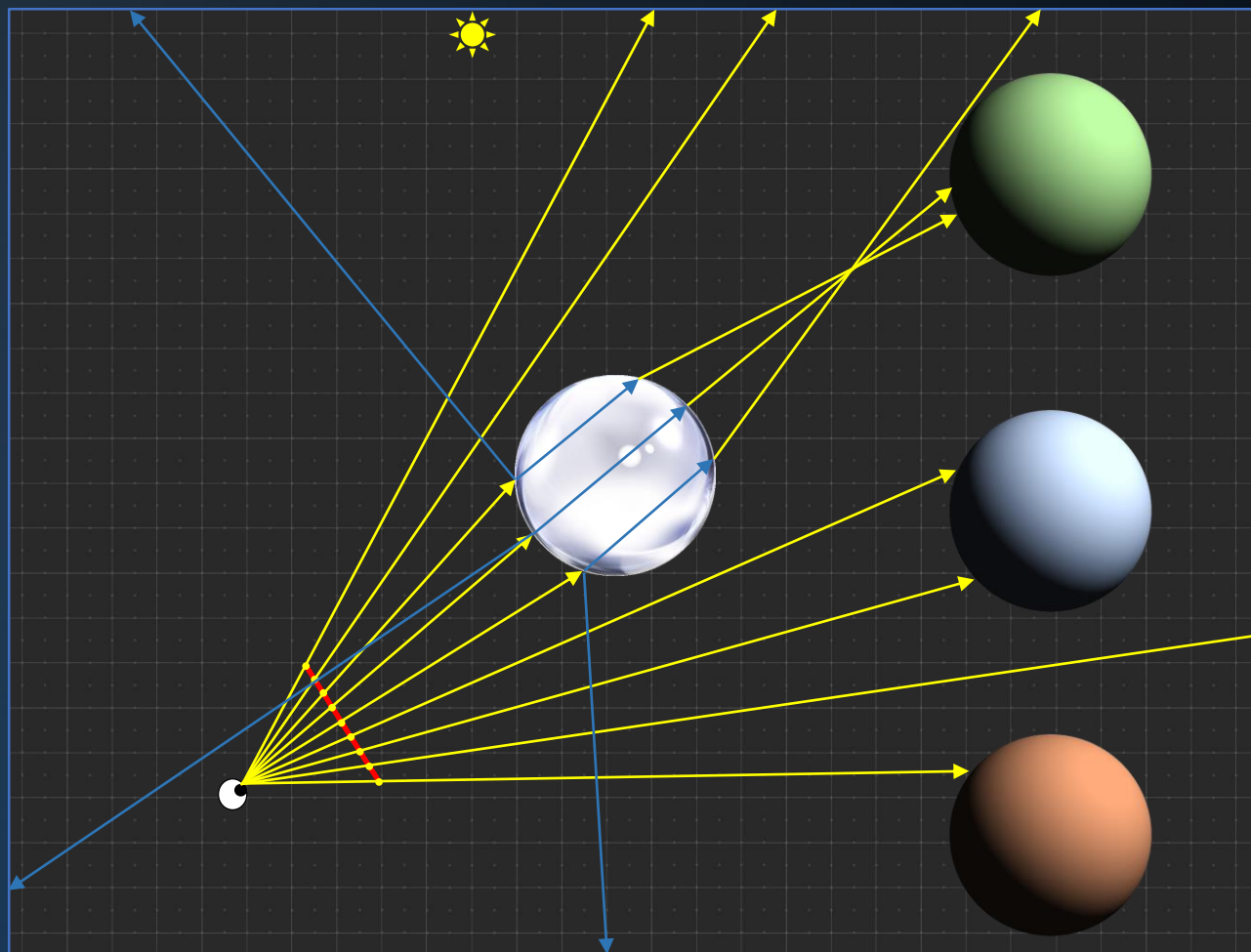
World space

- Geometry
- Eye
- Screen plane
- Screen pixels
- Primary rays
- Intersections
- Point light
- Shadow rays

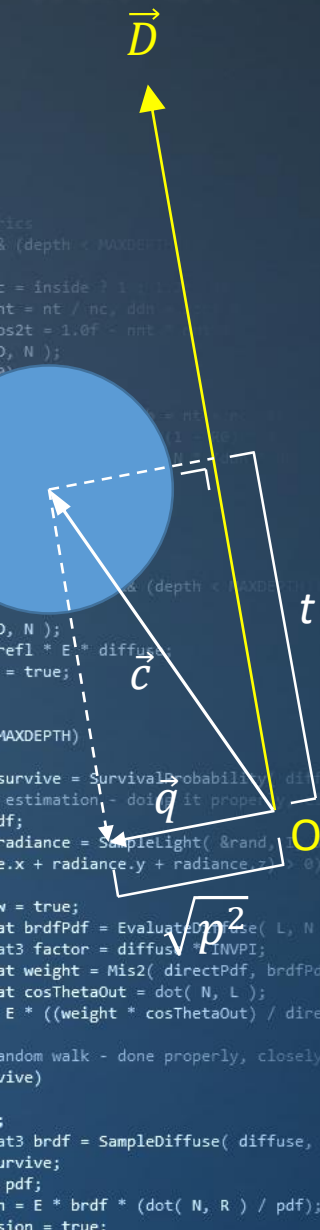
Light transport

- Extension rays

Light transport



Recap



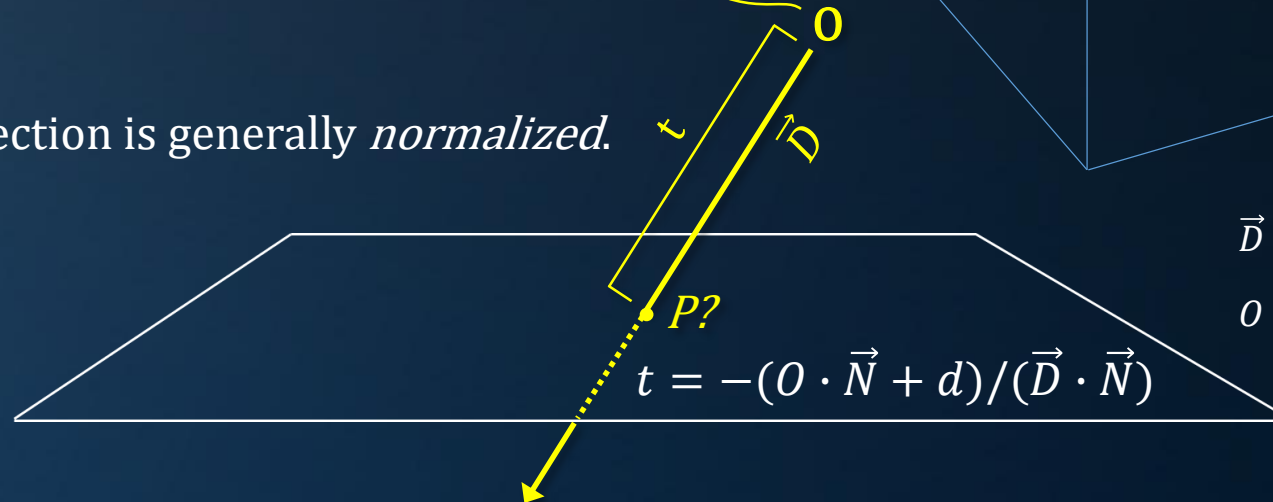
Ray definition

A ray is an infinite line with a start point:

$$p(t) = O + t\vec{D}, \text{ where } t > 0.$$

```
struct Ray
{
    float3 O;    // ray origin
    float3 D;    // ray direction
    float t;     // distance
};
```

The ray direction is generally *normalized*.

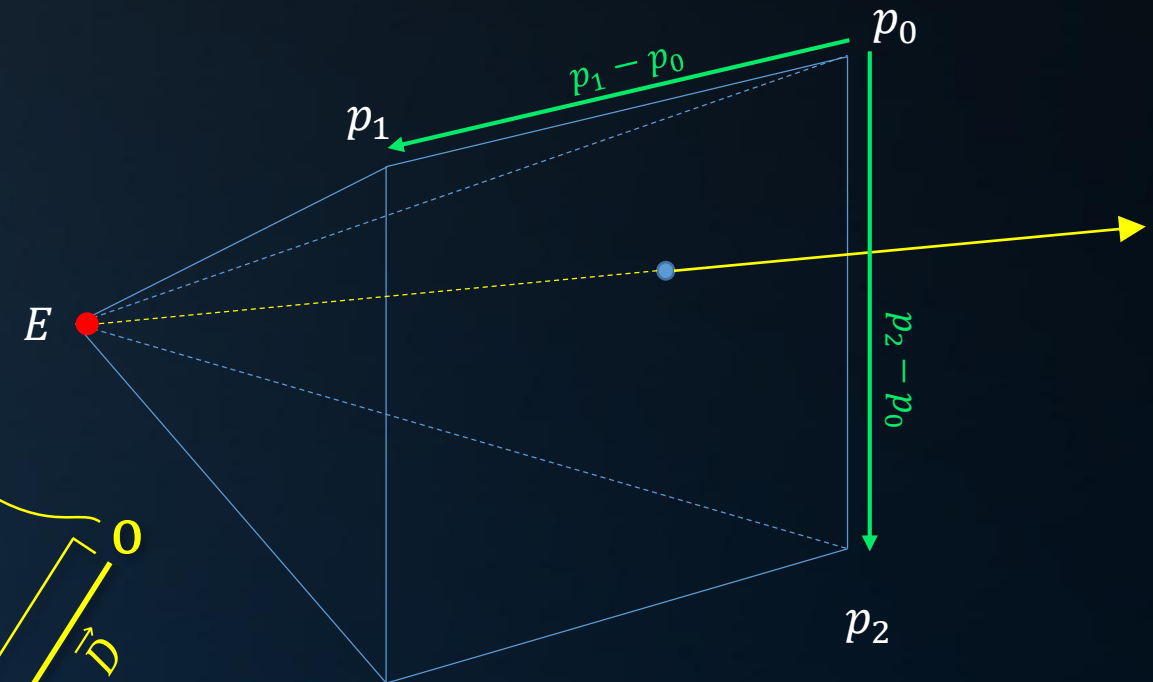


$$t = -(O \cdot \vec{N} + d) / (\vec{D} \cdot \vec{N})$$

Ray setup

Point on the screen: $p(u, v) = p_0 + u(p_1 - p_0) + v(p_2 - p_0)$, $u, v \in [0, 1]$

Ray direction (before normalization): $\vec{D} = p(u, v) - E$



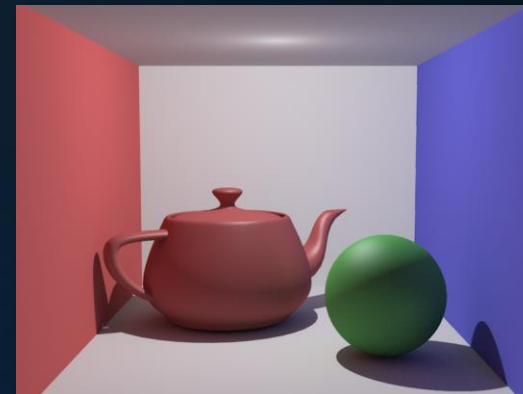
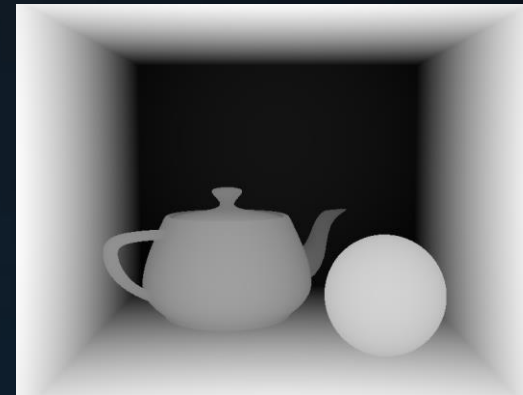
$$\vec{D} \cdot \vec{N} = \vec{D}_x \vec{N}_x + \vec{D}_y \vec{N}_y + \vec{D}_z \vec{N}_z$$

$$O \cdot \vec{N} = O_x \vec{N}_x + O_y \vec{N}_y + O_z \vec{N}_z$$



Today's Agenda:

- Ray Tracing Recap
- Shading
- Textures



Shading

The End

We used *primary rays* to find the *primary intersection* point.

Determining light transport:

- Sum illumination from all light sources
- ...If they are *visible*.

We used a primary ray to find the object visible through a pixel:

Now we will use a *shadow ray* to determine visibility of a light source.

```

ics
& (depth < MAXDEPTH) {
    // Inside?
    if (nt < 0) {
        // Inside?
        nt = nt / nc; ddn = ddn * nc;
        // Inside?
        cos2t = 1.0f - nnt * nnt;
        // Inside?
        D, N );
    }
    // Inside?
    at a = nt - nc, b = nt + nc;
    // Inside?
    at Tr = 1 - (R0 + (1 - R0) * a);
    // Inside?
    (Tr) R = (D * nnt - N * (ddn < 0) ? 1 : -1);
    // Inside?
    E * diffuse;
    // Inside?
    = true;
    // Inside?
    // Inside?
    refl + refr)) && (depth < MAXDEPTH) {
        // Inside?
        D, N );
        // Inside?
        refl * E * diffuse;
        // Inside?
        = true;
        // Inside?
        MAXDEPTH)
    }
    // Inside?
    survive = SurvivalProbability( diffuse );
    // Inside?
    estimation - doing it properly, closely following
    // Inside?
    if;
    // Inside?
    radiance = SampleLight( &rand, I, &L, &light,
    // Inside?
    e.x + radiance.y + radiance.z) > 0) && (depth <
    // Inside?
    w = true;
    // Inside?
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    // Inside?
    at3 factor = diffuse * INVPI;
    // Inside?
    at weight = Mis2( directPdf, brdfPdf );
    // Inside?
    at cosThetaOut = dot( N, L );
    // Inside?
    E * ((weight * cosThetaOut) / directPdf) * (radiance
    // Inside?
    random walk - done properly, closely following
    // Inside?
    survive)
    // Inside?
    ;
    // Inside?
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    // Inside?
    survive;
    // Inside?
    pdf;
    // Inside?
    n = E * brdf * (dot( N, R ) / pdf);
    // Inside?
    sion = true;

```



Shading

```
ics
& (depth < MAXDEPTH)
{
    t = inside ? 1.0f : 0.5f;
    nt = nt / nc, ddn = ddn * nt;
    float2t = 1.0f - nnt * ddn;
    D, N );
    0);
    at a = nt - nc, b = nt + nc;
    at Tr = 1 - (R0 + (1 - R0) * t);
    Tr) R = (D * nnt - N * (ddn > 0 ? 1 : -1));
    E * diffuse;
    = true;
    -
    efl + refr)) && (depth < MAXDEPTH)
    D, N );
    -refl * E * diffuse;
    = true;
    MAXDEPTH)
    survive = SurvivalProbability( diffuse );
    estimation - doing it properly, closely following
    if;
    radiance = SampleLight( &rand, I, &L, &lightPdf );
    e.x + radiance.y + radiance.z > 0) && (depth < MAXDEPTH)
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiance
    random walk - done properly, closely following Small's
    vive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
```

Shadow Ray

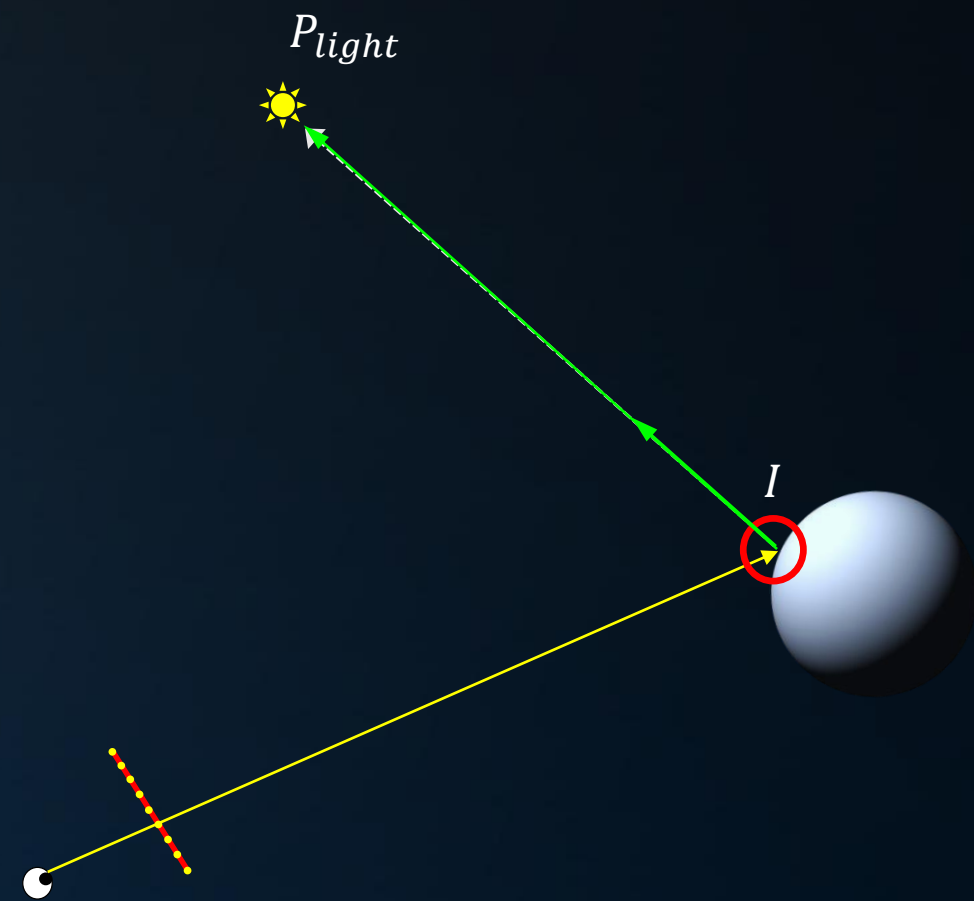
Constructing the shadow ray:

$$p(t) = O + t\vec{D}$$

Ray origin: the primary intersection point I .

Ray direction: $P_{light} - I$ (normalized)

Restrictions on t : $0 < t < ||P_{light} - I||$



Shading

Shadow Ray

Direction of the shadow ray: $\frac{P_{light} - I}{\|P_{light} - I\|}$

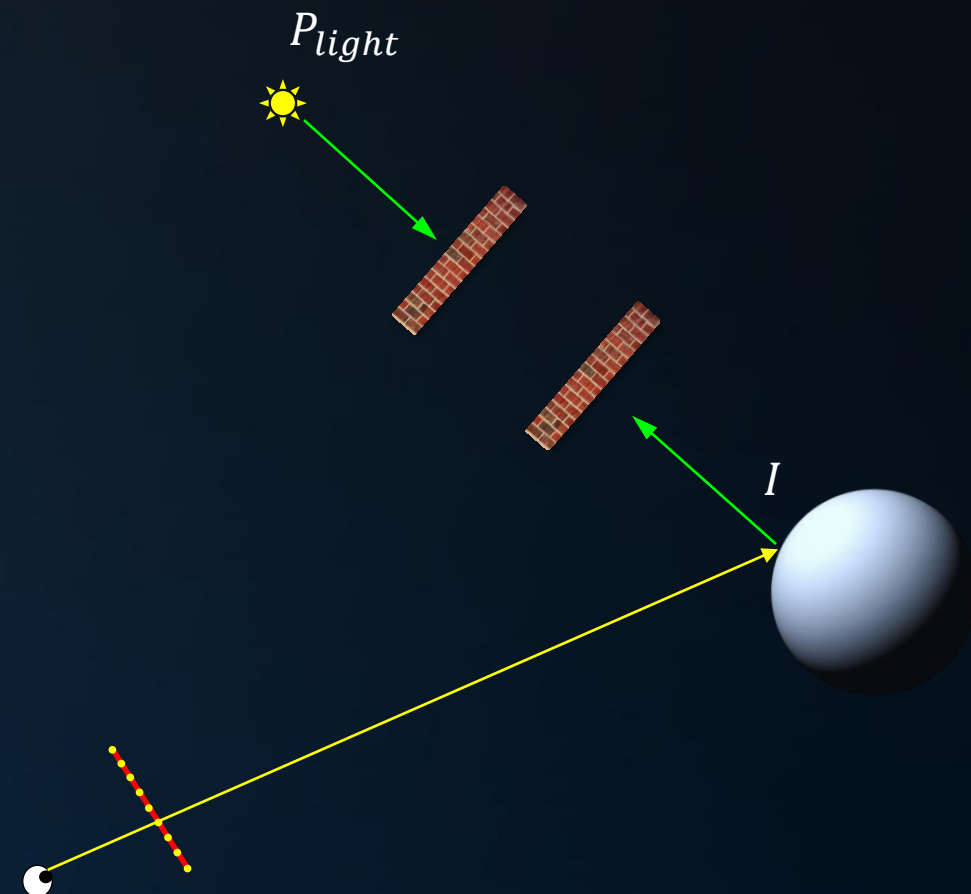
Equally valid: $\frac{I - P_{light}}{\|I - P_{light}\|}$ or $\frac{I - P_{light}}{\|I - P_{light}\|}$

Note that we get different intersection points depending on the direction of the shadow ray.

It doesn't matter: the shadow ray is used to determine *if* there is an occluder, not *where*.

This has two consequences:

1. We need a dedicated shadow ray query;
2. Shadow ray queries are (on average) twice as fast. (why?)



Shading

Shadow Ray

“In theory, theory and practice are the same. In practice, they are not.”

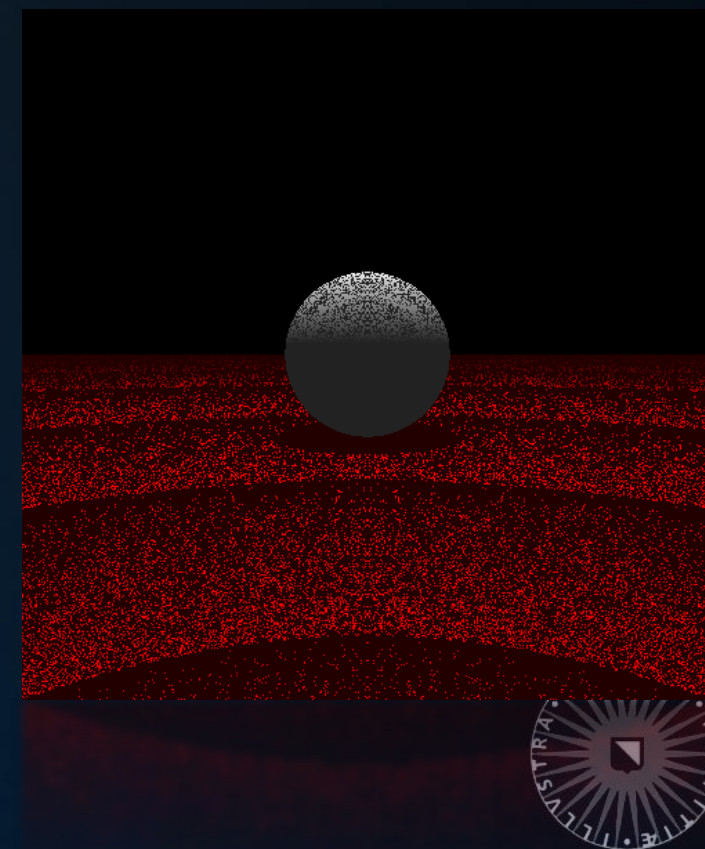
Problem 1:

Our shadow ray queries report intersections at $t = \sim 0$. Why?

Cause: the shadow ray sometimes finds the surface it originated from as an occluder, resulting in *shadow acne*.

Fix: offset the origin by ‘epsilon’ times the shadow ray direction.

Note: don’t forget to reduce t_{max} by epsilon.



Shading

Shadow Ray

“The most expensive shadow rays are those that do not find an intersection.”

Why?

(because those rays tested every primitive before concluding that there was no occlusion)

```
...ics
& (depth < MAXDEPTH) {
    // Inside?
    if (nt < 0) {
        nt = -nt;
        ddn = -ddn;
        cos2t = 1.0f - nnt * nnt;
        D, N );
    }
    // Refracted ray
    float a = nt - nc, b = nt + nc;
    float Tr = 1 - (R0 + (1 - R0) * cosTheta);
    float R = (D * nnt - N * (ddn < 0 ? 1 : -1));
    // Diffuse
    E * diffuse;
    = true;
    // Refracted ray
    D, N );
    refl * E * diffuse;
    = true;
    // Ray-traced ray
    MAXDEPTH)
    survive = SurvivalProbability( diffuse );
    // Estimation - doing it properly, closely following Small's
    if (
        radiance = SampleLight( &rand, I, &L, &light, &N, &N );
        e.x + radiance.y + radiance.z > 0) && (oct < 10) {
            w = true;
            // SampleDiffuse
            at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
            at3 factor = diffuse * INVPI;
            at weight = Mis2( directPdf, brdfPdf );
            at cosThetaOut = dot( N, L );
            E * ((weight * cosThetaOut) / directPdf) * (radiance
            // Random walk - done properly, closely following Small's
            vive)
            ;
            at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
            survive;
            pdf;
            n = E * brdf * (dot( N, R ) / pdf);
            ion = true;
```



Shading

Transport

Energy at distance r :

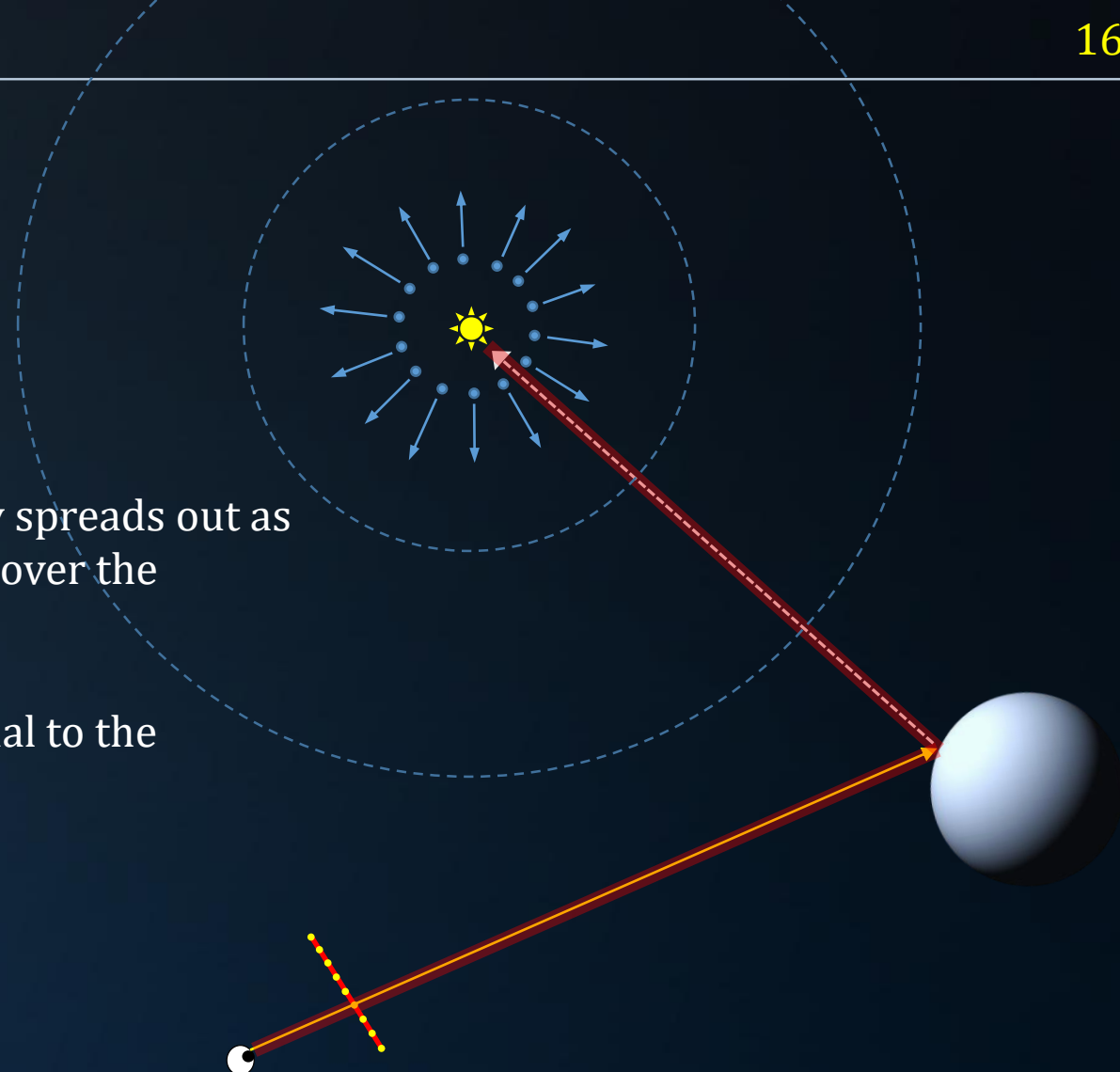
For a point light, a brief pulse of light energy spreads out as a growing sphere. The energy is distributed over the surface of this sphere.

Energy per unit area is therefore proportional to the inverse area of the sphere at distance r , i.e.:

$$E/m^2 = E_{light} \frac{1}{4\pi r^2}$$

Light energy thus dissipates at a rate of $\frac{1}{r^2}$.

This is referred to as *distance attenuation*.



Shading

Transport

Absorption:

Most materials absorb light energy. The wavelengths that are not fully absorbed define the ‘color’ of a material.

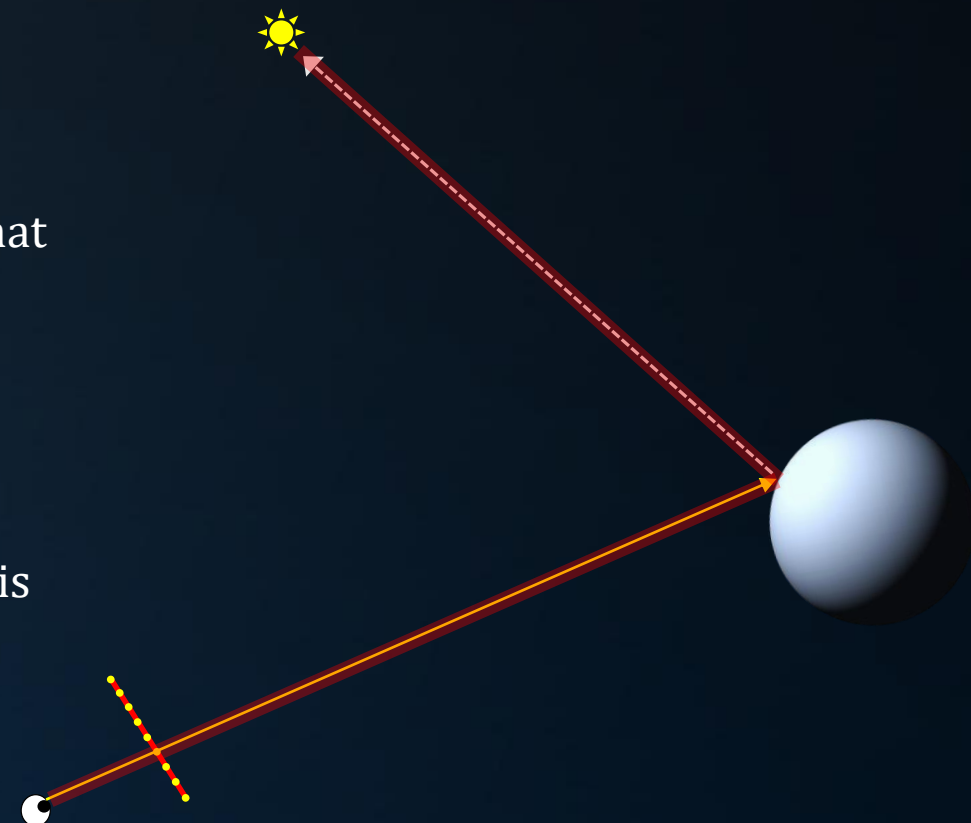
The reflected light is thus:

$$E_{reflected} = E_{incoming} \circ C_{material}$$

Note that $C_{material}$ cannot exceed 1; the reflected light is never *more* than the incoming light.

$C_{material}$ is typically a vector: we store r, g, b for all light transport.

$$A \circ B = \begin{bmatrix} A_x B_x \\ A_y B_y \\ A_z B_z \end{bmatrix} \quad (\text{'entrywise product'})$$



Shading

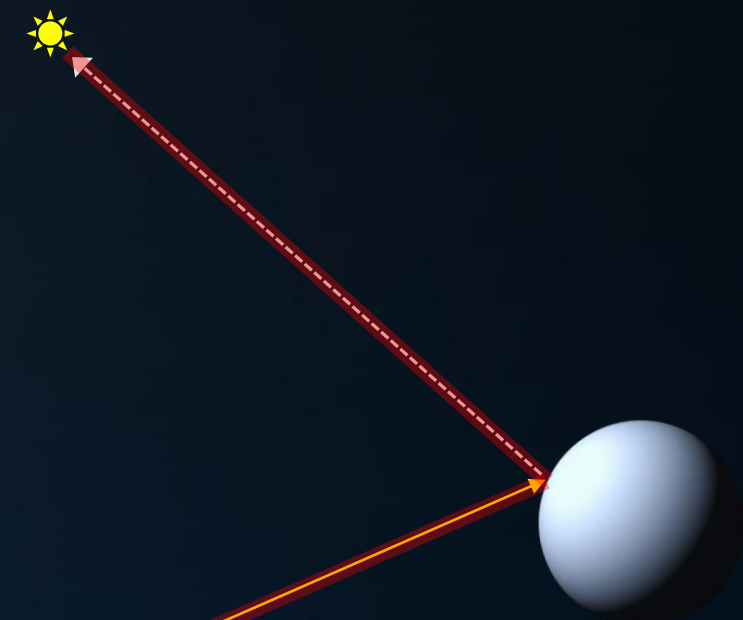
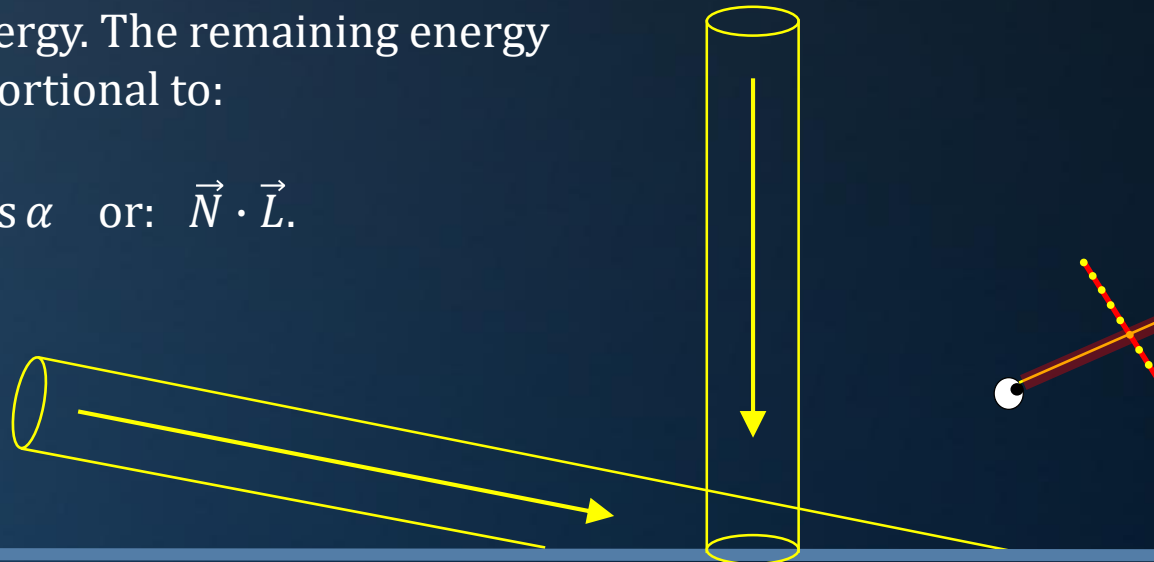
Transport

Energy arriving at an angle:

A small bundle of light arriving at a surface affects a larger area than the cross-sectional area of the bundle.

Per m^2 , the surface thus receives less energy. The remaining energy is proportional to:

$$\cos \alpha \quad \text{or: } \vec{N} \cdot \vec{L}.$$

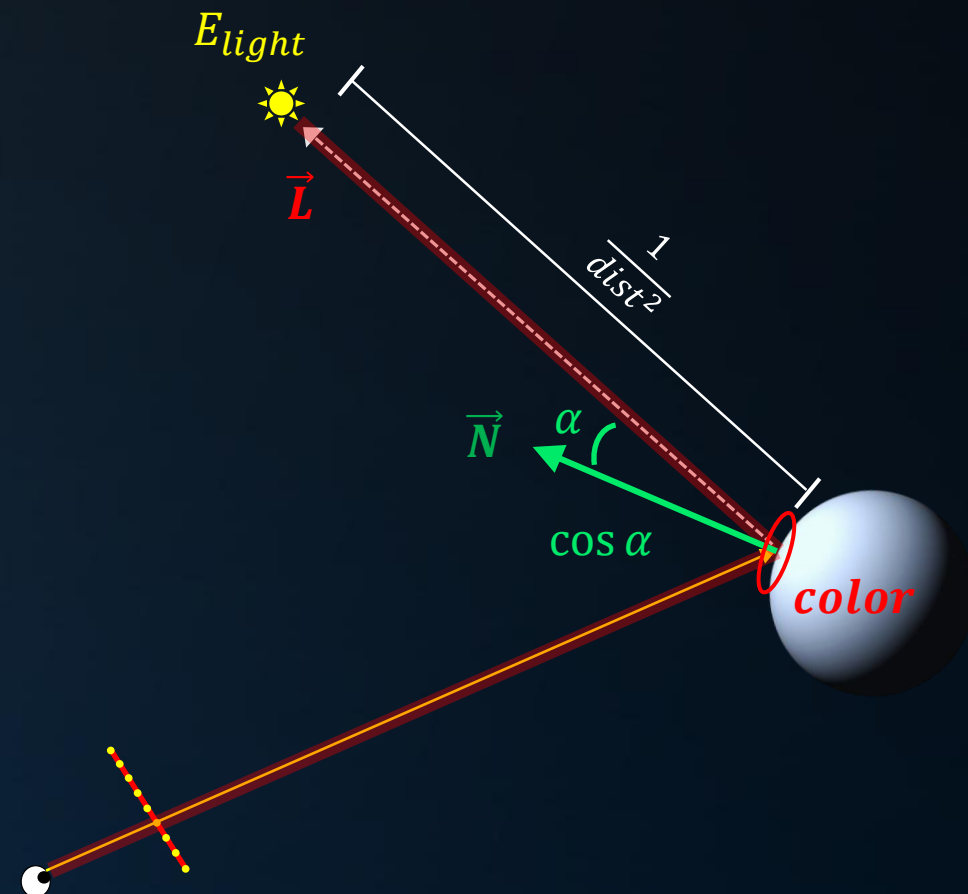


Shading

Transport

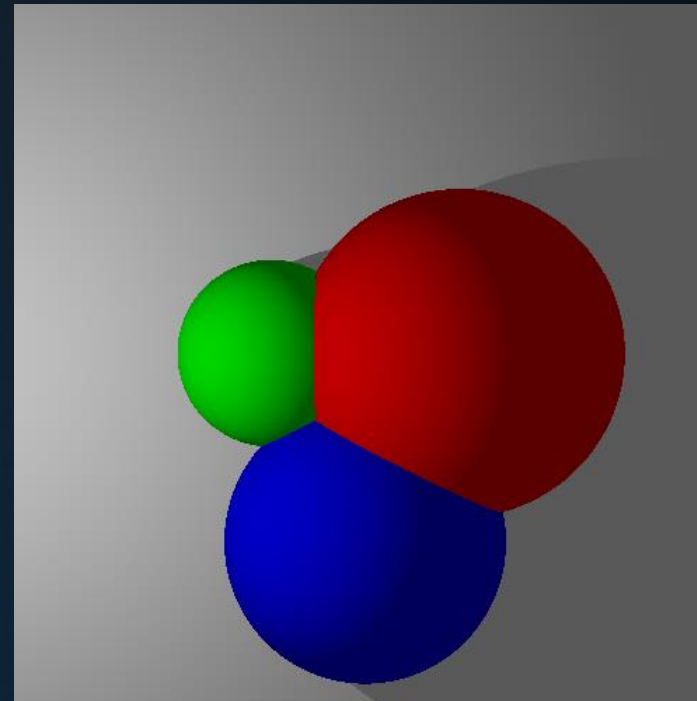
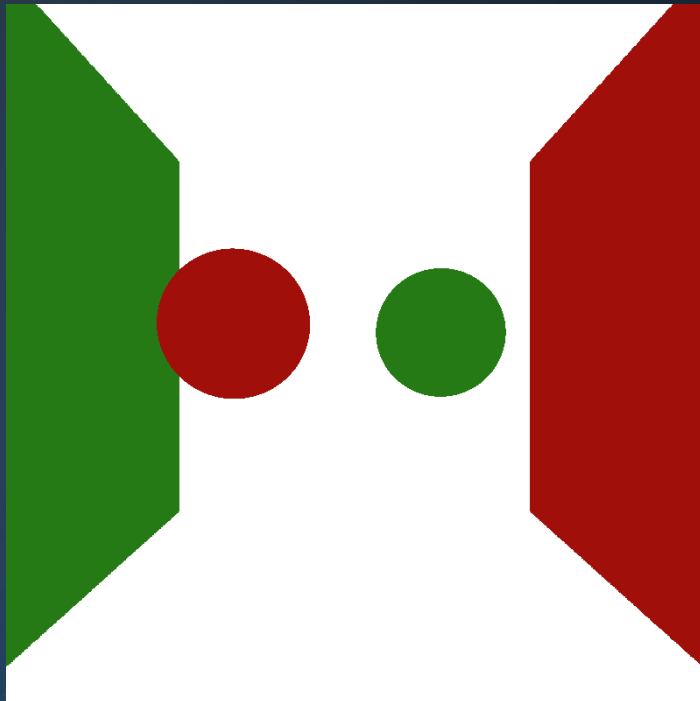
All factors:

- Emitted light : defined as RGB color, floating point
- Distance attenuation: $\frac{1}{r^2}$
- Absorption, modulate by material color
- $N \cdot L$



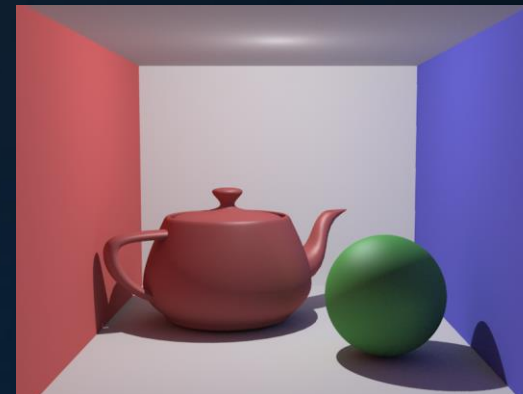
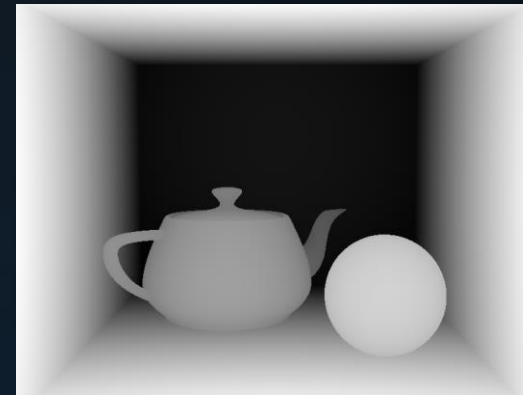
Shading

```
ics
& (depth < MAXDEPTH)
{
    if (inside) {
        nt = inside ? 1.0f : 0.0f;
        nt = nt / nc, ddn = dot(N, L);
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    at a = nt - nc, b = nt + nc;
    at Tr = 1 - (R0 + (1 - R0) * cos2t);
    Tr) R = (D * nnt - N * (ddn < 0 ? 1 : -1));
    E * diffuse;
    = true;
    refl + refr)) && (depth < MAXDEPTH)
    D, N );
    refl * E * diffuse;
    = true;
    MAXDEPTH)
    survive = SurvivalProbability( diffuse );
    estimation - doing it properly, closely following
    df;
    radiance = SampleLight( &rand, I, &L, &align );
    e.x + radiance.y + radiance.z > 0) && (depth <
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiance
    random walk - done properly, closely following
    survive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    ion = true;
```



Today's Agenda:

- Ray Tracing Recap
- Shading
- Textures



Textures

Texturing a Plane

Given a plane: $y = 0$ (i.e., with a normal vector $(0,1,0)$).

Two vectors on the plane define a basis:

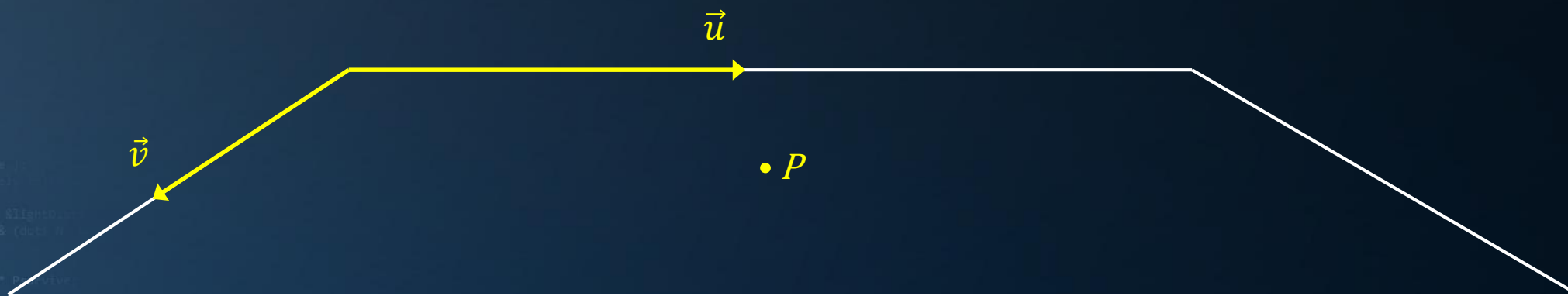
$$\vec{u} = (1,0,0) \text{ and } \vec{v} = (0,0,1).$$

Using these vectors, any point on the plane can be reached:

$$P = \lambda_1 \vec{u} + \lambda_2 \vec{v}.$$

We can now use λ_1, λ_2 to define a color at P:

$$F(\lambda_1, \lambda_2) = \dots.$$



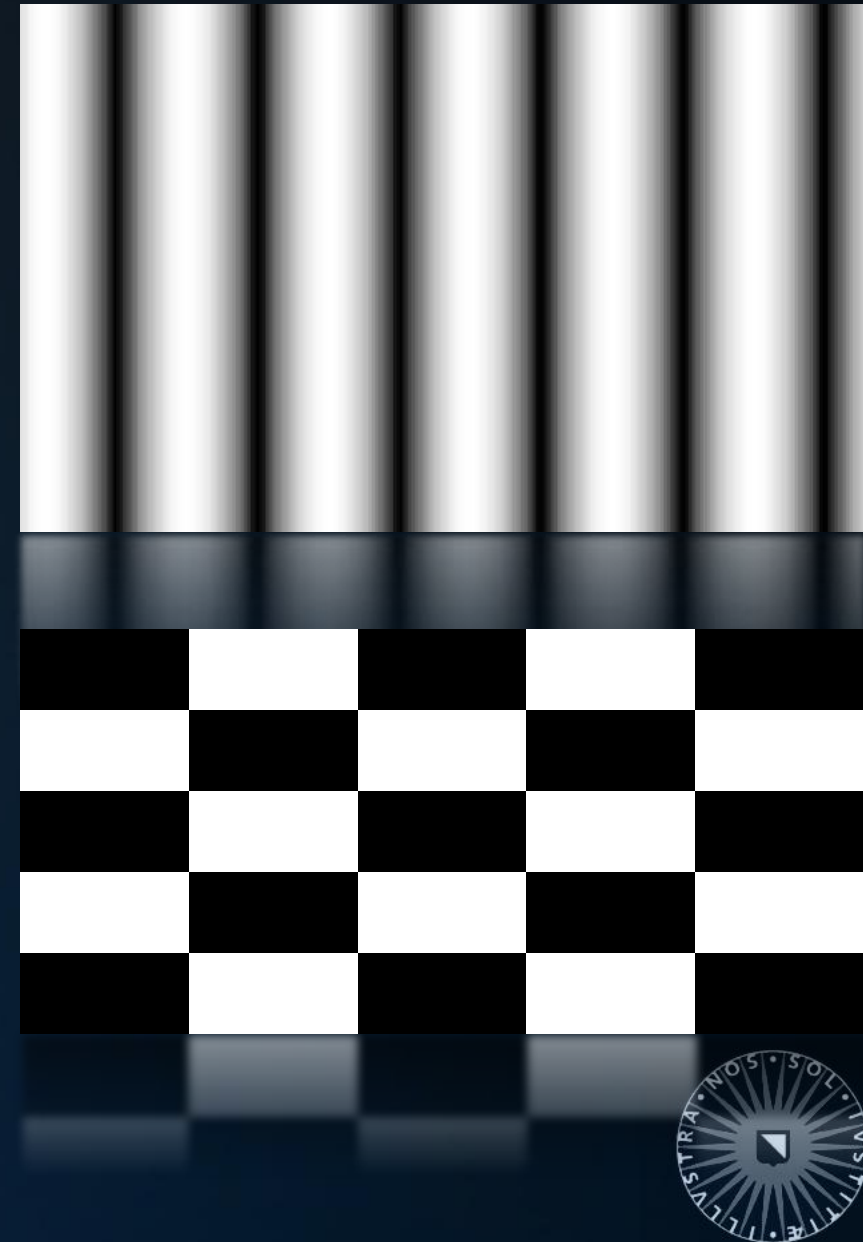
Textures

Example:

$$F(\lambda_1, \lambda_2) = \sin(\lambda_1)$$

Another example:

$$F(\lambda_1, \lambda_2) = ((\text{int})(2 * \lambda_1) + (\text{int})\lambda_2) \& 1$$



Textures

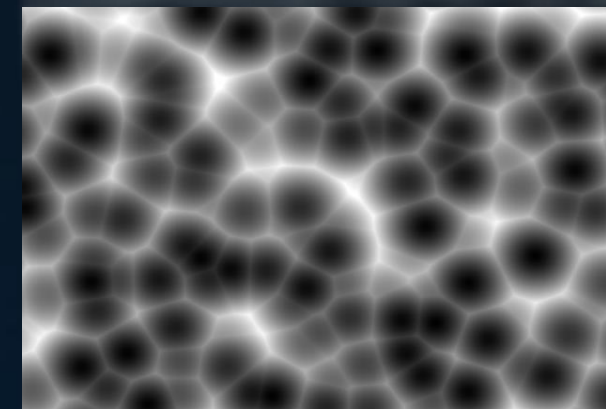
Other examples (not explained here):

Perlin noise

Details: <http://www.noisemachine.com/talk1>

Voronoi / Worley noise

Details: “A cellular texture basis function”, S. Worley, 1996.



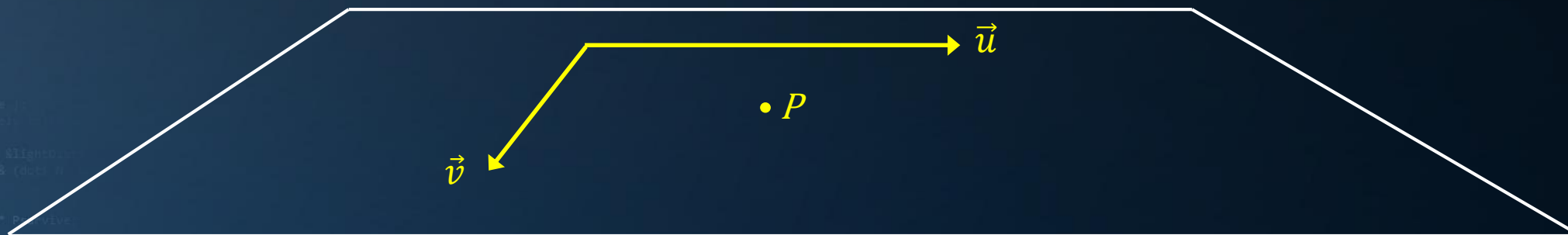
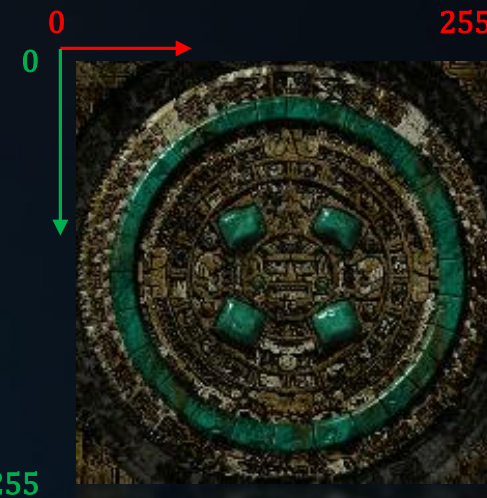
Textures

Obviously, not all textures can be generated procedurally.

Finding P using basis vectors $\vec{u}$, $\vec{v}$ and parameters λ_1, λ_2 :

$$\begin{pmatrix} P_x \\ P_y \end{pmatrix} = \lambda_1 \vec{u} + \lambda_2 \vec{v} = \begin{pmatrix} \lambda_1 \vec{u}_x + \lambda_2 \vec{v}_x \\ \lambda_1 \vec{u}_y + \lambda_2 \vec{v}_y \end{pmatrix}$$

...But what about the opposite, i.e. can we find λ_1, λ_2 given P?



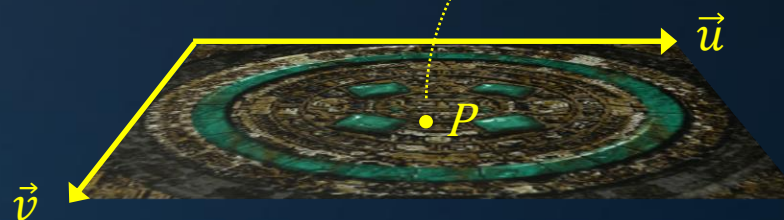
Textures

Obviously, not all textures can be generated procedurally.

For the generic case, we lookup the color value in a pixel buffer.

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} P \cdot \vec{u} \\ P \cdot \vec{v} \end{pmatrix} * \begin{pmatrix} T_{width} \\ T_{height} \end{pmatrix}$$

Note that we find the pixel to read based on P ; we don't find a ' P ' for every pixel of the texture.



Textures

Retrieving a pixel from a texture:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} P \cdot \vec{u} \\ P \cdot \vec{v} \end{pmatrix} * \begin{pmatrix} T_{width} \\ T_{height} \end{pmatrix}$$

We don't want to read outside the texture. To prevent this, we have two options:

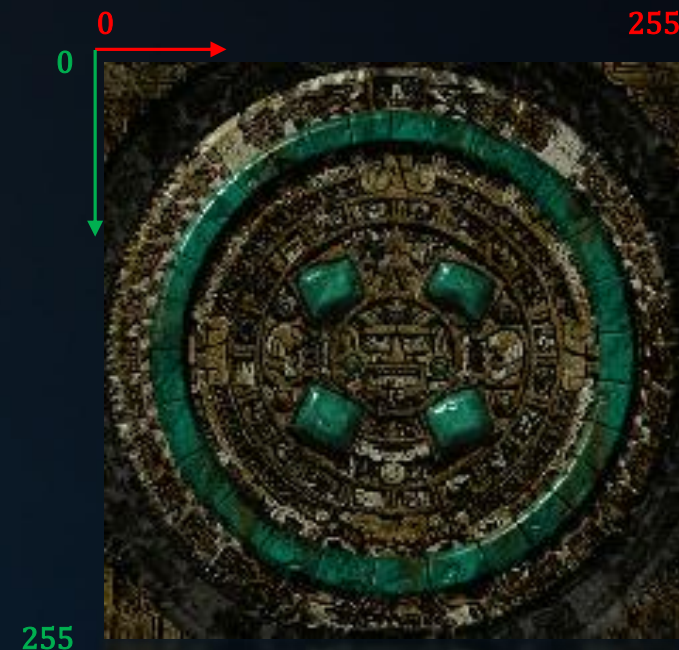
1. Clamping

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} clamp(P \cdot \vec{u}, 0, 1) \\ clamp(P \cdot \vec{v}, 0, 1) \end{pmatrix} * \begin{pmatrix} T_{width} \\ T_{height} \end{pmatrix}$$

2. Tiling

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} frac(P \cdot \vec{u}) \\ frac(P \cdot \vec{v}) \end{pmatrix} * \begin{pmatrix} T_{width} \\ T_{height} \end{pmatrix}$$

Tiling is efficiently achieved using a bitmask. This requires texture dimensions that are a power of 2.



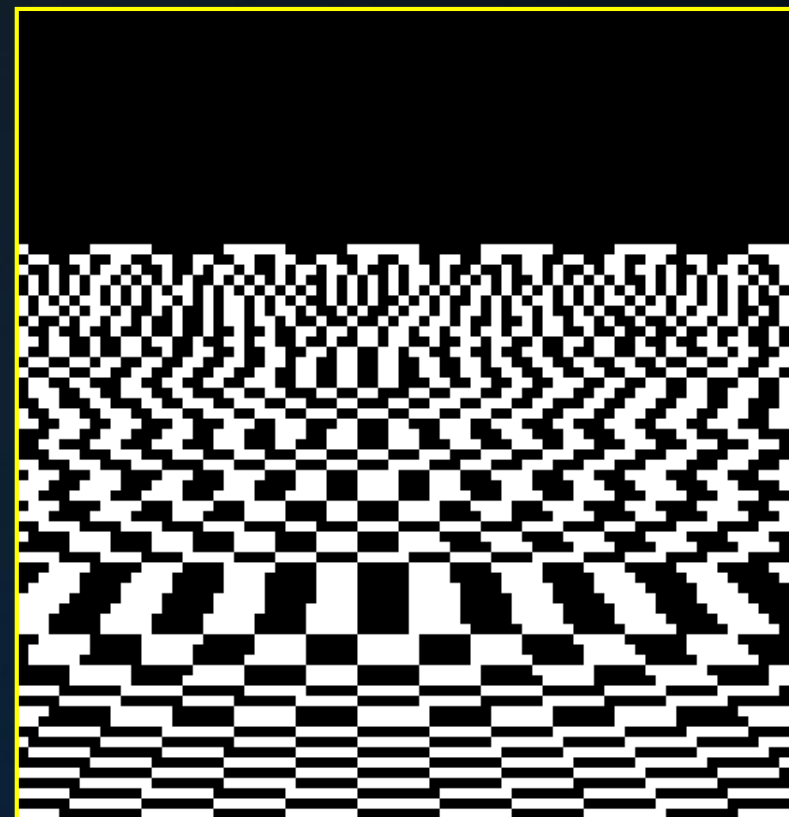
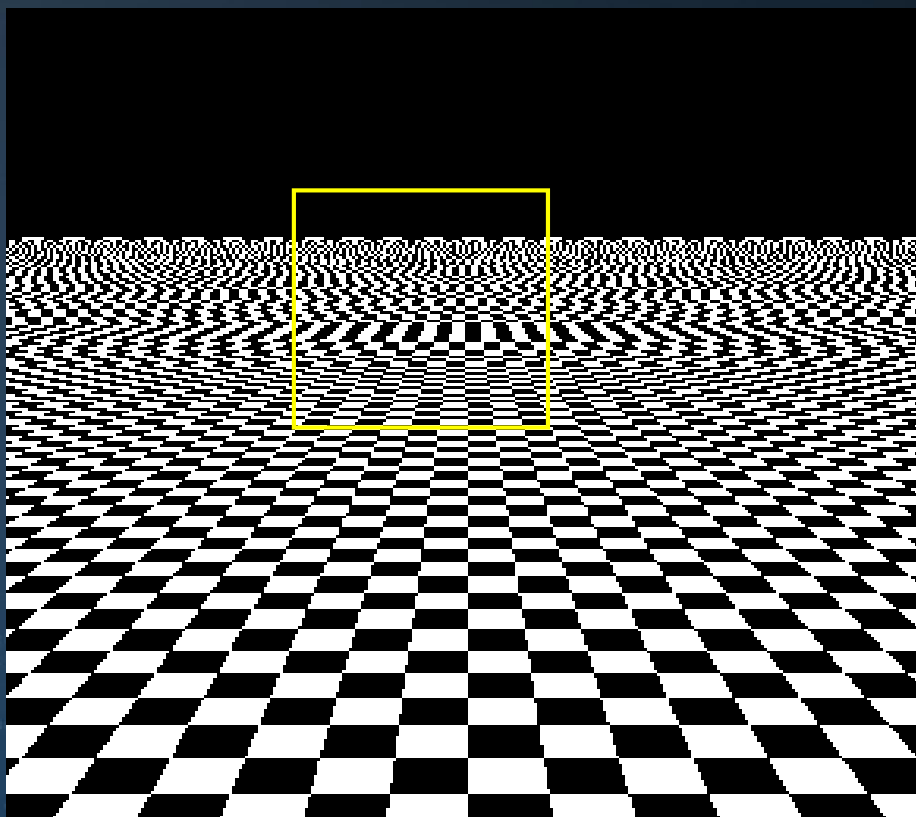
Textures

Texture mapping: undersampling

```

ics
& (depth < MAXDEPTH)
{
    if (inside & !isrefl)
    {
        nt = nt / nc;
        nnt = nnt * nc;
        nnt2t = 1.0f - nnt * nnt;
        D, N );
    }
    {
        at a = nt - nc, b = nt - nc;
        at Tr = 1 - (R0 + (1 - R0) * nnt);
        (Tr) R = (D * nnt - N * (1 - nnt));
        E * diffuse;
        = true;
        refl + refr)) && (depth < MAXDEPTH)
        D, N );
        refl * E * diffuse;
        = true;
    }
    MAXDEPTH)
    survive = SurvivalProbability( diffuse,
    estimation - doing it properly, closely following
    df;
    radiance = SampleLight( &rand, I, &L, &R, &N, &D, &N );
    e.x + radiance.y + radiance.z) > 0) && (depth < MAXDEPTH)
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Pdf;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf);
    random walk - done properly, closely following
    (survive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &N, &D, &N );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;

```



Textures

Fixing oversampling

Oversampling: reading the same pixel from a texture multiple times.
Symptoms: blocky textures.

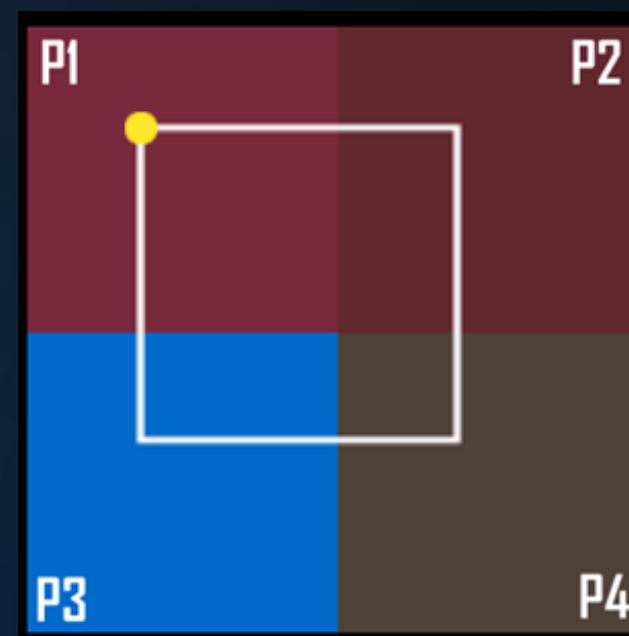
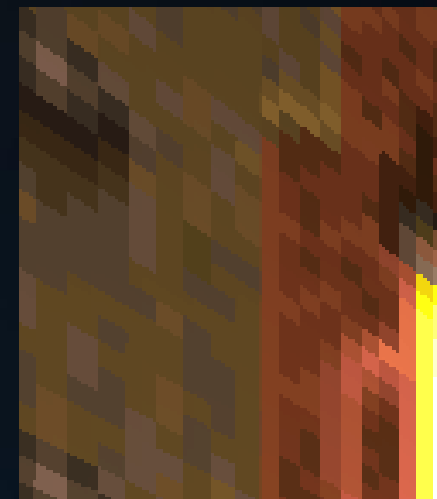
Remedy: bilinear interpolation:
Instead of clamping the pixel location to the nearest pixel, we read from four pixels.

$$w_{p1}: (1 - \text{frac}(x)) * (1 - \text{frac}(y))$$

$$w_{p2}: \text{frac}(x) * (1 - \text{frac}(y))$$

$$w_{p3}: (1 - \text{frac}(x)) * \text{frac}(y)$$

$$w_{p4}: 1 - (w_{p1} + w_{p2} + w_{p3})$$



Textures

Fixing oversampling

```
ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0)
    {
        nt = nt / nc; ddn = ddn * ddn;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    at a = nt - nc, b = nt * nc;
    at Tr = 1 - (R0 + (1 - R0) * cos2t);
    Tr) R = (D * nnt - N * (ddn * cos2t));
    E * diffuse;
    = true;
    refl + refr)) && (depth < MAXDEPTH)
    D, N );
    refl * E * diffuse;
    = true;
    MAXDEPTH)
    survive = SurvivalProbability( diffuse,
    estimation - doing it properly, closely
    df;
    radiance = SampleLight( &rand, I, &L, &lig
    e.x + radiance.y + radiance.z) > 0) && (d
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurf
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiant
    random walk - done properly, closely following 3rd
    vive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
}
```



Textures

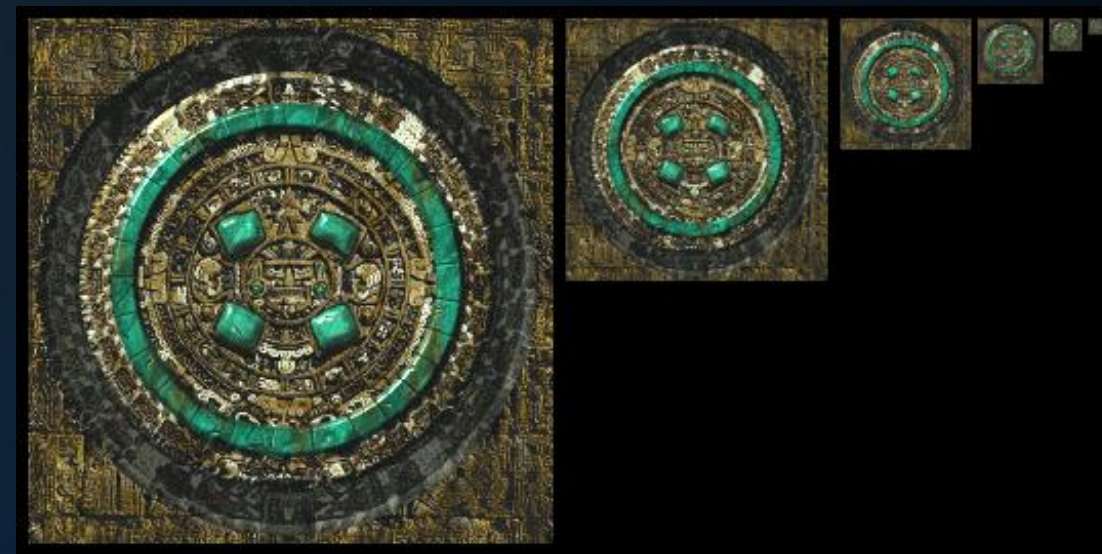
Fixing undersampling

Undersampling: skipping pixels while reading from a texture.
Symptoms: Moiré, flickering, noise.

Remedy: MIP-mapping.

The texture is reduced to 25% by averaging 2x2 pixels. This is repeated until a 1x1 image remains.

When undersampling occurs, we switch to the next MIP level.



NO MIPMAPS



WITH MIPMAPS



Textures

Trilinear interpolation: blending between MIP levels.

```
ics
& (depth < MAXDEPTH)
{
    nt = inside ? 1.0f : 0.0f;
    nt = nt / nc, ddn = dot(N, D);
    cos2t = 1.0f - nnt * ddn;
    D, N );
}

at a = nt - nc, b = nt + nc;
at Tr = 1 - (R0 + (1 - R0) * cos2t);
Tr) R = (D * nnt - N * (ddn * cos2t));

E * diffuse;
= true;

efl + refr)) && (depth < MAXDEPTH)
{
    D, N );
    refl * E * diffuse;
    = true;

MAXDEPTH)

survive = SurvivalProbability( diffuse, I,
estimation - doing it properly, closely
df;
radiance = SampleLight( &rand, I, &L, &lig
e.x + radiance.y + radiance.z) > 0) && (o

w = true;
at brdfPdf = EvaluateDiffuse( L, N ) * Psurf;
at3 factor = diffuse * INVPI;
at weight = Mis2( directPdf, brdfPdf );
at cosThetaOut = dot( N, L );
E * ((weight * cosThetaOut) / directPdf) * (radiance

random walk - done properly, closely following 3rd order Markov chain
vive)

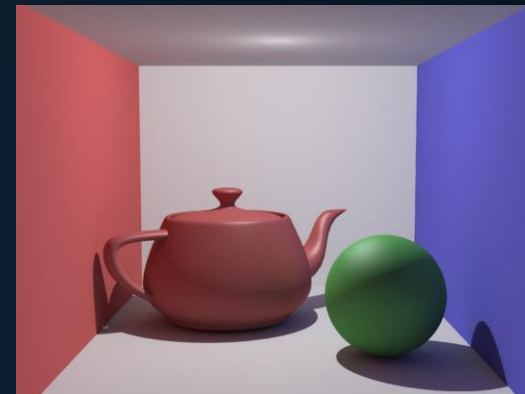
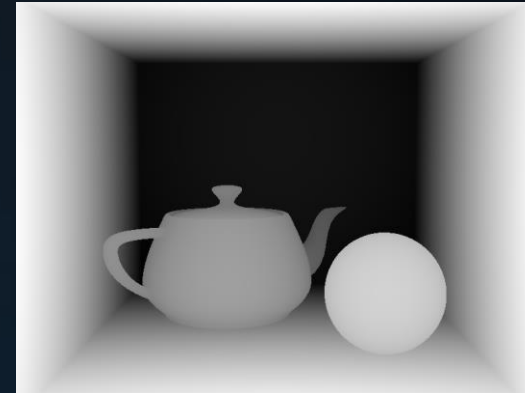
;
at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
survive;
pdf;
n = E * brdf * (dot( N, R ) / pdf);
sion = true;
```



```
ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 1.01)
    {
        nt = nt / nc; ddn = ddn * ddn;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    at a = nt - nc, b = nt + nc;
    at Tr = 1 - (R0 + (1 - R0) * ddn);
    Tr) R = (D * nnt - N * (ddn * cos2t));
    E * diffuse;
    = true;
    -
    refl + refr)) && (depth < MAXDEPTH)
    {
        D, N );
        refl * E * diffuse;
        = true;
    }
    MAXDEPTH)
    survive = SurvivalProbability( diffuse );
    estimation - doing it properly, closely following Small's
    if;
    radiance = SampleLight( &rand, I, &L, &lightPdf );
    e.x + radiance.y + radiance.z) > 0) && (dot( N, L ) > 0)
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiance
    random walk - done properly, closely following Small's
    vive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
}
```

Today's Agenda:

- Ray Tracing Recap
- Shading
- Textures



INFOGR – Computer Graphics

Jacco Bikker & Debabrata Panja - April-July 2019

END OF Lecture 6: “Ray Tracing”

Next lecture: “Ray Tracing (2)”

