Utrecht University
Faculty of Science
Department of Information and Computing Sciences

Final Exam Simulation, Tuesday April 8, 2014, 13.00-15.00 hr.

- Switch off your mobile phone, PDA and any other mobile device and put it far away.
- This exam consists of 7 questions and a bonus question, and has 4 pages.
- Answers may be provided in either Dutch or English.
- All your answers should be clearly written down and provide a clear explanation. Unreadable or unclear answers may be judged as false.
- Please write down your name and student number on every exam paper that you hand in.
- A statistical table is is distributed separately and should be returned.
- The maximum score (11 in total) is divided as follows:

Question	Score
1	2
2	1
3	1
4	1
5	1.5
6	1.5
7	2
8	1 (bonus)

Good luck, veel succes !

The home care company

We consider a home care company in the center of the Netherlands. It employs 10 nurses and 20 assistants. There are n customers requiring care from the company. The care for customers is performed in two steps. Customer i ($i = 1, 2, \ldots, n$) first requires attendance from an assistant. The time required for this equals the maximum of 10 minutes and an amount following a normal distribution with an average of p_i. The variance of the time spent by the assistant equals v_i^2. After that, the customer requires attendance from a nurse. This takes an amount of time from which equals the maximum of 5 minutes and an amount following a normal distribution with an average of q_i. The time spent by the nurse has variance w_i^2. It turns out that the average assistant service time over all customers equals 20 minutes and that for a nurse this number equals 10 minutes.

You may assume that the travelling time from one customer to the next follows a normal distribution with an average of 15 minutes and a standard deviation of 2 minutes, where the travelling time is set to 1 minute if the normal distribution yields a number smaller than 1.

Furthermore, customers are assigned to assistants in increasing order of their index i. This implies that, if an assistant has finished her/his work, she/he is going to travel to the customer with lowest index which has not been assigned to an assistant yet. Nurses start travelling to a customer upon request only. The company wants that the nurse arrives shortly after the assistant is finished. Therefore, a nurse is requested to start travelling 5 minutes after the start of the work of the assistant. If, at this point in time, there is no nurse available, the request in put into a FIFO queue.

The company wants to perform a simulation study to determine the *makespan*, i.e., the total time required to serve the complete set of the customers. Moreover, they want to determine the average service time of the customers.

(1) Which events are included in the event-scheduling model for this problem? Draw an event graph and for each arc give the corresponding time delay.

(2) Describe in words or pseudo code the event-handlers of the event(s) which schedule the work of a nurse.

(3)
(a) Suppose that because of new working rules, the assistant personally have to give an update to the nurse at each customer, i.e. an assistant is not allowed to leave before the arrival of a nurse. What is the effect on the makespan? Now assume that the working time of the assistant and travelling time are deterministic, 20 and 15 minutes respectively. What is the effect on the makespan then? Explain your answer.
(b) Suppose that the working process of the assistants is improved such that the variance v_i^2 is reduced. What is the effect on the makespan? Explain your answer.

(4) Give two difficulties that may occur in the validation of the simulation model of the home care company (part (1)), and describe how to deal with these difficulties.

(5) Suppose that the travelling times of care personnel are subject to disturbances δ (in minutes). If a nurse or assistant travels from one customer to another, the travelling time equals $15 + \delta$ minutes, where δ is generated from a probability distribution with the following density function:

$$f_\delta(x) = \begin{cases} 0.2 * 120e^{120x} & \text{for } x < 0; \\ 0.8 * 20e^{-20x} & \text{for } x \geq 0 \end{cases}$$

Explain the meaning of this density function.
How can we generate these disturbances in a program written in an imperative programming language like Java, C#, or C++ and without using **any** specific random generation libraries or functions?
Note: You do not have to give a program, but just a description or pseudo-code.

(6) The organization owns a shop where people can rent or buy specific products. For anti-allergic bath oil, which has to be bought, the demand is normally distributed with a known expected value and variance. It is known that the average demand per four weeks is 400 liters with a variance of 100 per four weeks. The weekly demands are assumed to be independent. The shop wants to use the (r, q)-model as a basis for its inventory management.

- Explain the (r, q)-model
- Given the fixed ordering cost 10 EURO and the holding cost 0.2 EURO per liter per week, determine the optimal value of q.
- Given a constant lead time of one week and a desired Stock-Out-Probability(SOP) = 0.025, compute the value of r. What is the size of the safety stock in this example? *A statistical table is attached.*

(7) The organization wants to apply an advanced planning tool to determine the assignment of nurses and assistants to customers and to determine the expected starting time of the service of customer i. They want to minimize the expected makespan.
(i) Describe the problem as a combined optimization and simulation problem, i.e. formulate (either in words or formulas) the objective function and the constraints and indicate at which point a simulation has to be performed.
(ii) Describe a solution algorithm for this problem based on local search with sampling.

Bonus

(8) In the $M|G|1$ queue the average queue length L_q is given by

$$L_q = \frac{1}{2}(1 + c_B^2)\frac{\rho^2}{1-\rho},$$

where B denotes the service time, ρ is the utilization factor and $c_B = \frac{\sigma(B)}{E(B)}$. Give an intuitive proof of this formula.