Utrecht University
Faculty of Science
Department of Information and Computing Sciences

Final Exam Simulation, Monday April 18, 2016, 13.30-16.30 hr.

- Switch off your mobile phone, PDA and any other mobile device and put it far away.
- This exam consists of **9** questions
- Answers may be provided in either Dutch or English.
- All your answers should be clearly written down and provide a clear explanation. Unreadable or unclear answers may be judged as false.
- Please write down your name and student number on every exam paper that you hand in. Hand in this exam completely together with your answers on exam papers.
- A statistical table is handed out separately and should be returned at the end.
- The maximum score (10 in total) is divided as follows:

Question	Score
1	1
2	1
3	1
4	1.5
5	1
6	0.5
7	1
8	1.5
9	1.5

Good luck, veel succes !

The emergency department

Modelling

We consider the emergency department of a medium-sized hospital. Patients arrive according to a Poisson process with an average of 5 per hour. When a patient arrives, he/she is immediately assigned to one of the categories A, B, or C (this is called triage). Category A consists of patients in a life threatening situation who most urgently need medical help. Morever, patients of category B need medical help more urgently than patients of category C. In the department, there is one team of a docter and a nurse, denoted by DN_1, for the treatment of patients of category A, and one team of a doctor and a nurse, denoted by DN_2, for the treatment of the other patients. If a patient arrives and the team for his/her category is free, treatment is started immediately after triage. DN_1 treats patients in order of arrival. DN_2 gives priority to patients of category B which means the following:

- When treatment of a patient is finished and there is at least one patient of category B waiting, the next patient is the patient of category B with the earliest arrival time.
- When treatment of a patient is finished and there are no patients of category B waiting but there is at least one waiting patient of category C, the next patient is the patient of category C with the earliest arrival time.

On average, the fraction of patients of category A,B, and C, equals 0.25, 0.25 and 0.5 respectively. The time required for the treatment of a patient from category A, B, or C is assumed to follow a Gamma distribution with $\alpha = 4$ and $\beta = \beta_A, \beta_B, \beta_C$, respectively.

The follow-up of the treatment is outside the scope of this simulation study. The hospital wants to perform a simulation study of the emergency department to find the maximum waiting time of a patient.

(1) Give a set of events that provide an event-scheduling model for this problem. For each event, give a name and describe in a few words to which point in the process it corresponds. Draw an event graph and give the (stochastic) time delay on each of the arcs.

(2) Give two other (than the maximum waiting time) appropriate performance measures for this simulation study and describe how to compute them in the simulation.

(3) Describe three *different* possible actions to validate the simulation model of the emergency department. The descriptions of the actions should be as specific as possible and include which aspects of the model are adressed. A general remark like: "discussion with an expert" will not result in any credit points. Different for example means that "action X for category A" and "same action X for category B" do not count as two possible actions.

Since DN_2 turned out to be quite busy, the department considers to extend the team DN_2 with one nurse. This results in a required treatment time which equals 0.8 times the original treatment time. The time required for the treatment of a patient from category B or C now follows a Gamma distribution with $\alpha = 4$ and $\beta = 0.8\beta_B, 0.8\beta_C$, respectively. However, the additional nurse can be called away if a patient from category A has arrived who cannot be treated at once. While this patient is waiting the nurse has to stay with him/her and possibly already give some medicines. As soon as there are no waiting patients of category A, the nurse joins DN_2 again. We assume, that if the nurse has been away during the treatment of the patient at DN_2 the complete treatment takes **exactly** 25 % longer, for example a treatment that takes 12 minutes for DN_2 with the additional nurse present all the time, takes 15 minutes if the addional nurse has been away part of the time (regardless of the time length of the absence of the additional nurse).

(4) Give a set of events that provide an event-scheduling model for this problem. For each event, give a name and describe in a few words to which point in the process it corresponds. Draw the corresponding event graph.

Stochastic aspects

(5) A careful study of the treatment times at the emergency department revealed that $\beta_A = 10$, i.e. the treatment time of a patient of category A follows a Gamma$(4, 10)$ distribution. Describe how can we generate these treatment times in a program written in an imperative programming language like Java or C♯ without using *any* specific random generation libraries or functions.
Note: You do not have to give a program, but just a description or pseudo-code.

(6) Describe a set-up for the production runs of the simulation model of question (1) with the maximum waiting time of a patient as performance measure. The set-up should use the fact that the emergency department is working continuously (24 hours per day, 7 days a week).

(7) The hospital also has an information desk with one employee. Customers arrive according to a Poisson process with an average of 30 per hour. The service time follows an exponential distribution with an average of 90 seconds. What is the utilization factor of the service employee? Compute the average length of the queue in front of the information desk.

(8) The pharmacy of the hospital keeps inventory of different types of medicines. For some type of medicine, the demand is normally distributed with a known expected value and variance. It is known that the average demand per week is 150 boxes with a variance of 100. The weekly demands are assumed to be independent and normally distributed. The pharmacy wants to use the (r, q)-model as a basis for its inventory management.

(a) Explain the (r, q)-model.

(b) Given the fixed ordering cost 30 and the holding cost 0.1 Euro per box per week, determine the optimal value of q.

(c) Given a constant lead time of two weeks and a backlogging cost of 10 Euro per item, compute the value of r. What is the size of the safety stock in this example?

Simulation and optimization

We study the planning of the surgery department. We consider a day on which there are n planned operations. The duration of operation j is $p_j + \epsilon_j$, where ϵ_j follows an exponential distribution with average E_j. There are m doctors; each will be working in one of the operation rooms. We assume that each operation can be performed by each doctor. Because of the necessary preparations, the planning always has to be made the day before. At 12.00 the hospital makes the planning for the next day. On the day of operation, all operations will be performed and an operation cannot be moved to another doctor and room. The planning has to be such that the expected workload, i.e. total time needed to perform the operations, is divided as equally as possible over the doctors.

(9) We want to solve this planning problem as a combined optimization and simulation problem.

(a) What are the decision variables?

(b) Define an appropriate objective function.

(c) Describe an algorithm to solve the above problem based on combined optimization and simulation and clearly explain when you have to perform a simulation.