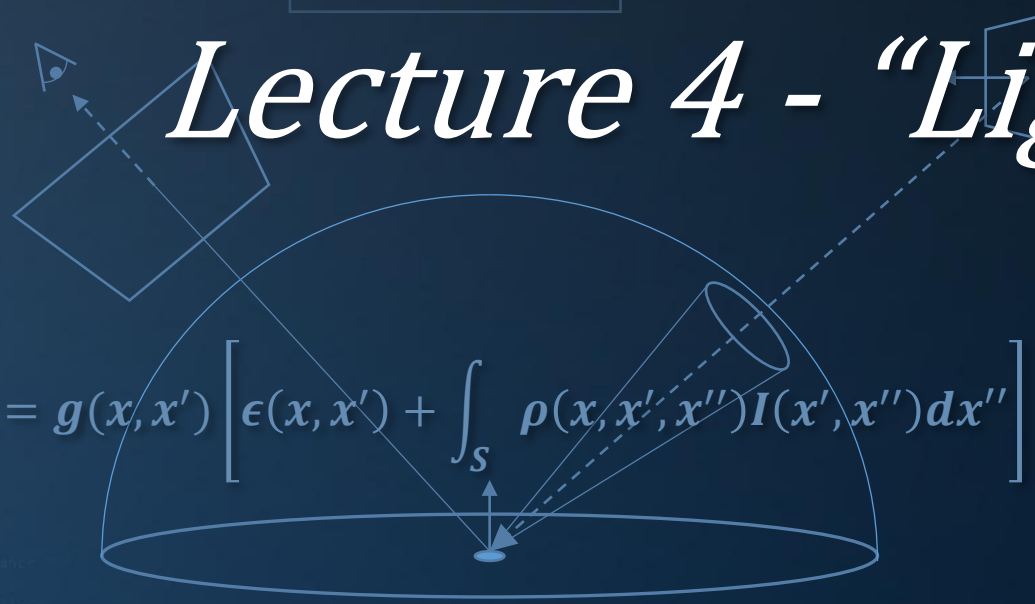
INFOMAGR – Advanced Graphics

Jacco Bikker - November 2021 - February 2022

Lecture 4 - "Light Transport"



$$I(x,x') = g(x,x')\left[\epsilon(x,x') + \int_S \rho(x,x',x'')I(x',x'')dx''\right]$$

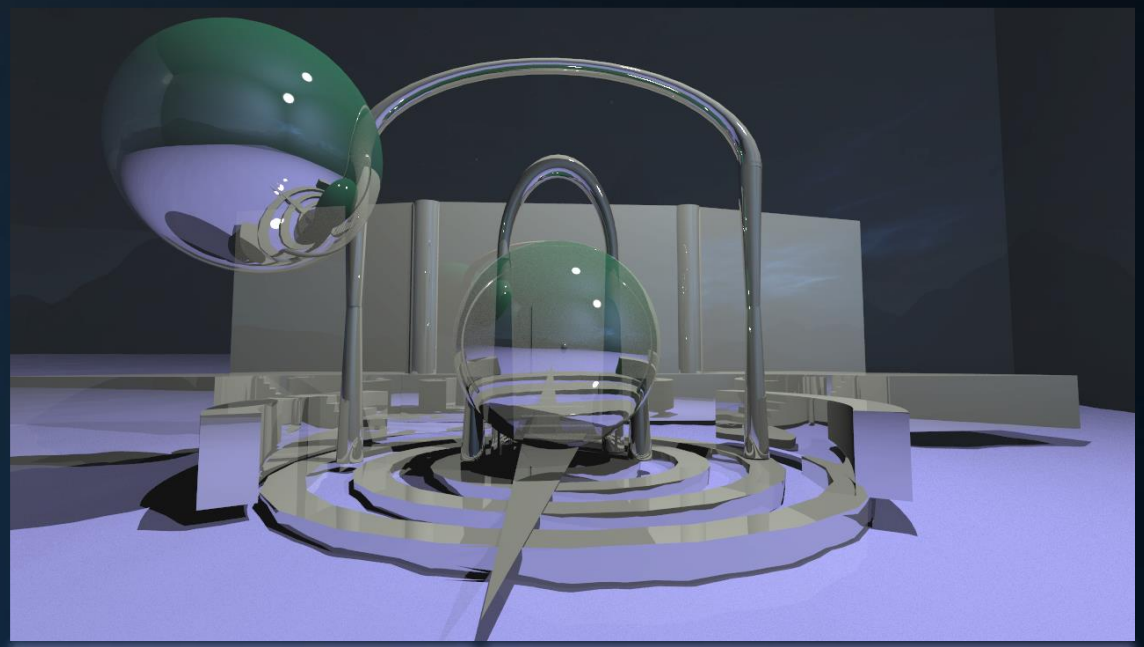
Welcome!

Today’s Agenda:

- Introduction
- The Rendering Equation
- Light Transport

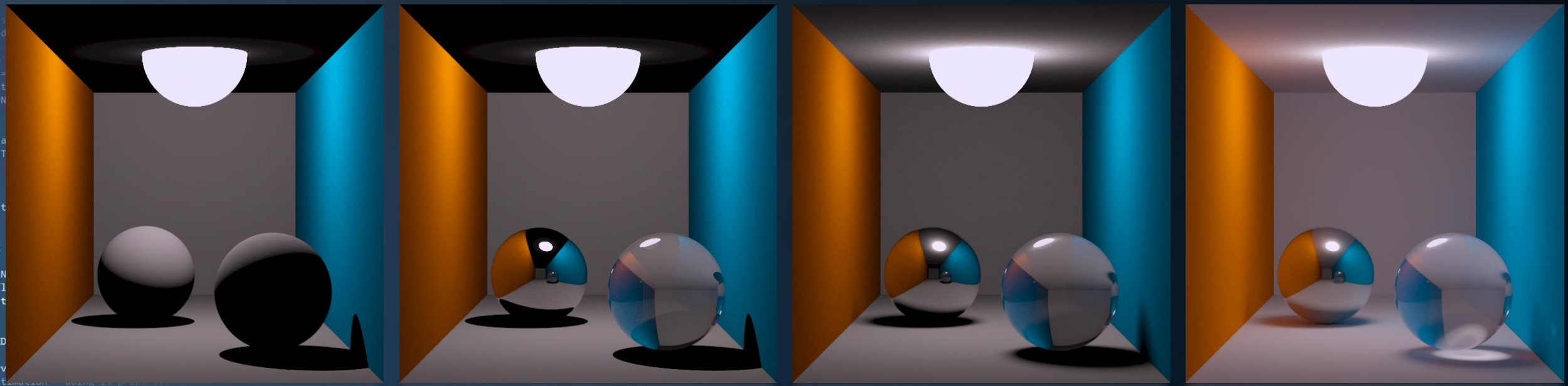
Introduction

Whitted



Introduction

Whitted

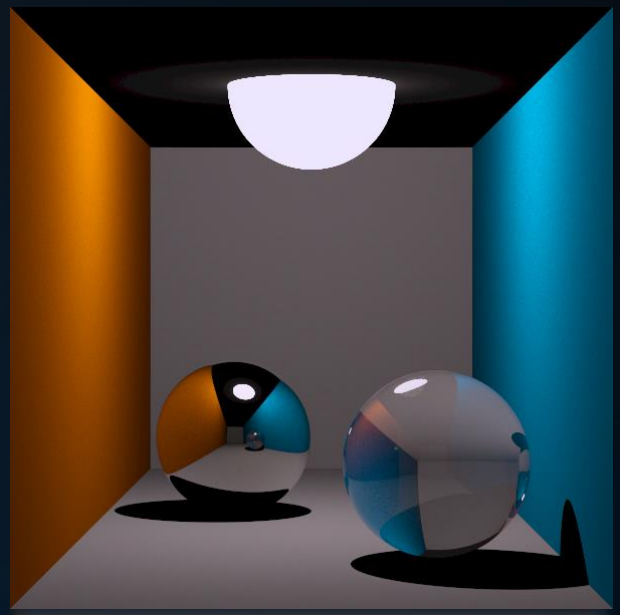


Introduction

Whitted

Missing:

- Area lights
- Glossy reflections
- Caustics
- Diffuse interreflections
- Diffraction
- Polarization
- Phosphorescence
- Temporal effects
- Motion blur
- Depth of field
- Anti-aliasing



Introduction

Anti-aliasing

Adding anti-aliasing to a Whitted-style ray tracer:

Send multiple primary rays through each pixel, and average their result.

Problem:

- How do we aim those rays?
- What if all rays return the same color?

Introduction

Anti-aliasing – Sampling Patterns

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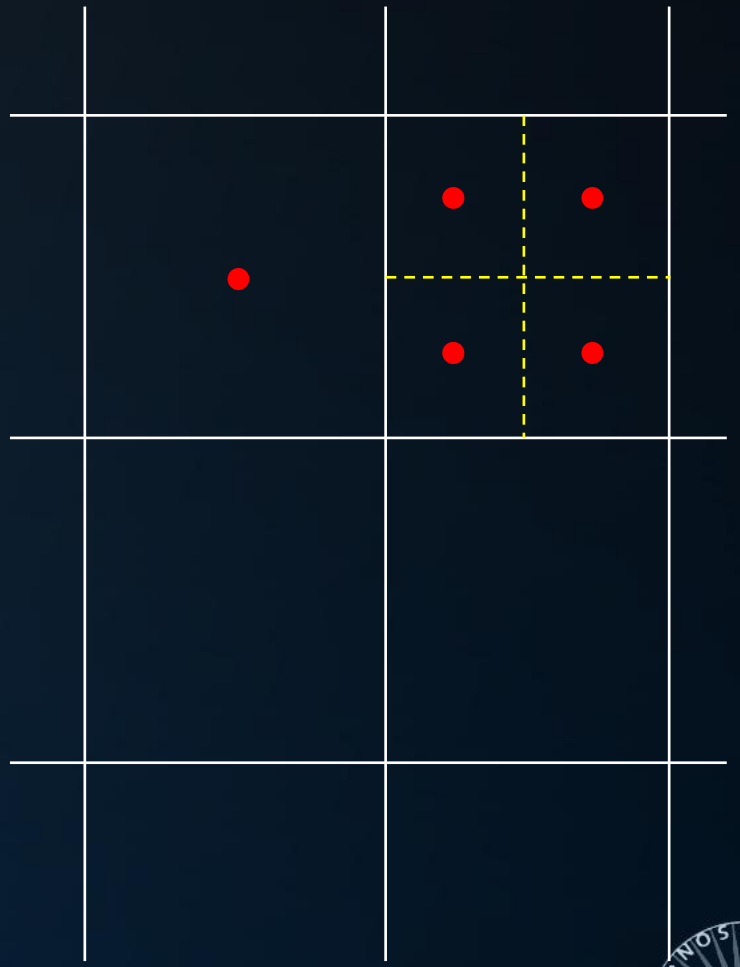
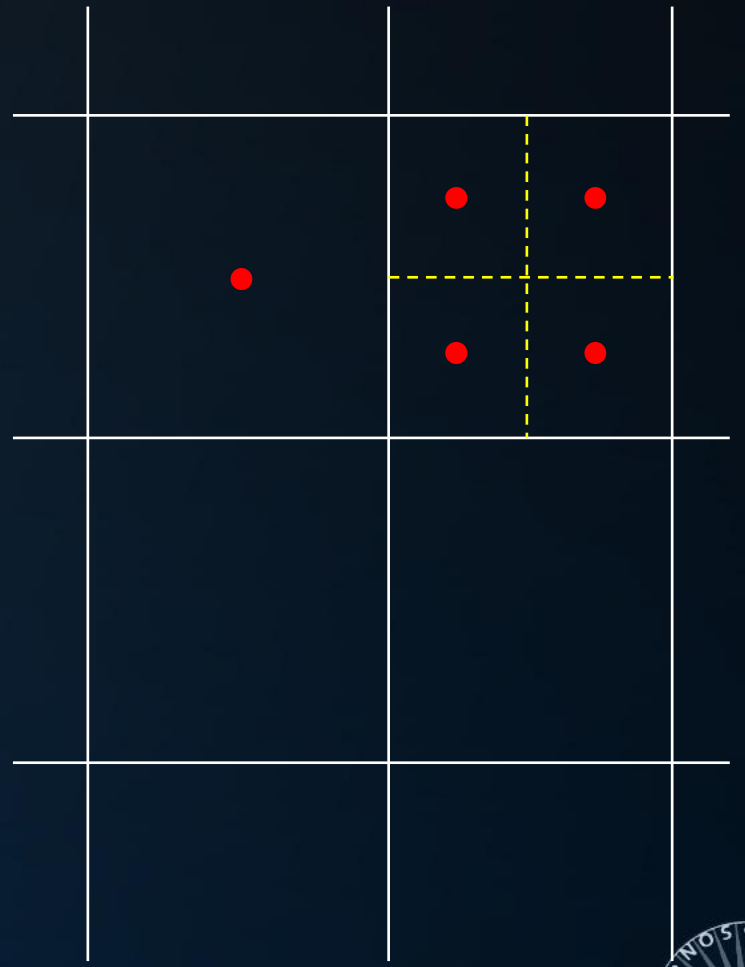


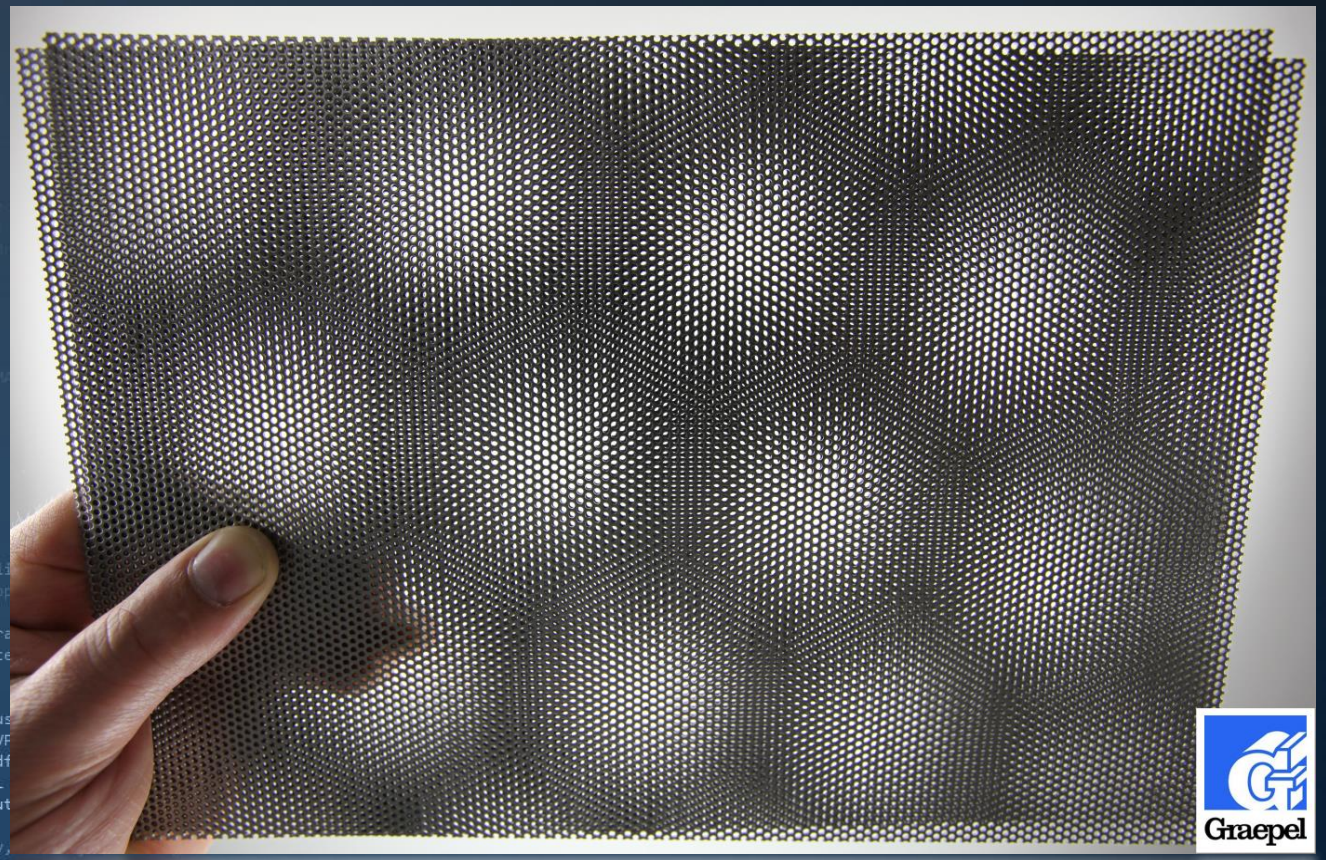
Introduction

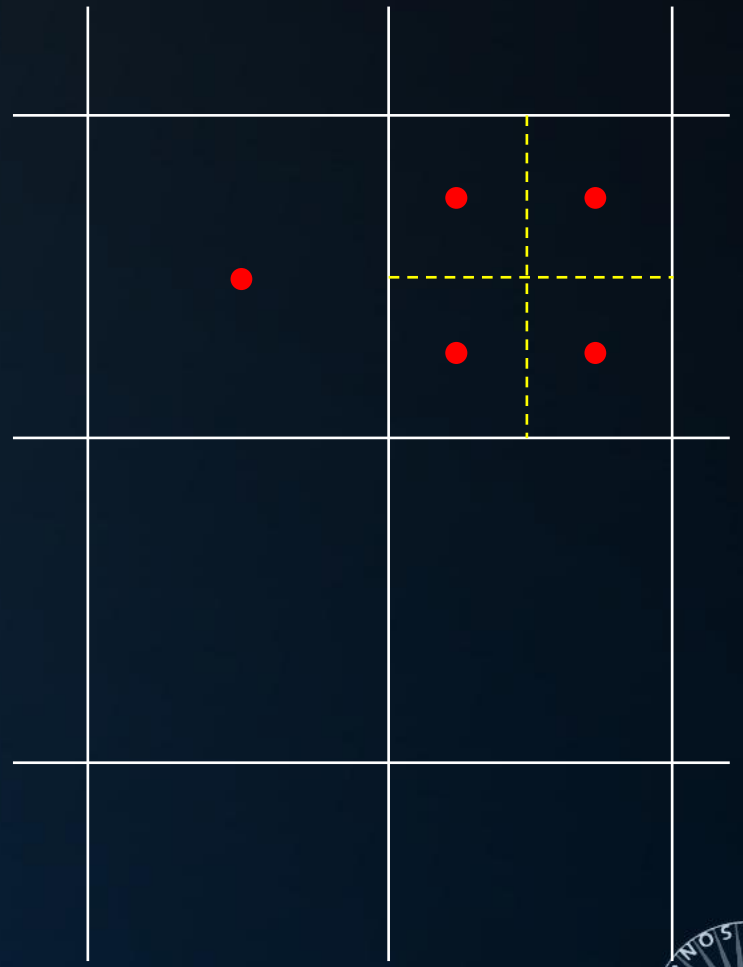
Anti-aliasing – Sampling Patterns



Introduction

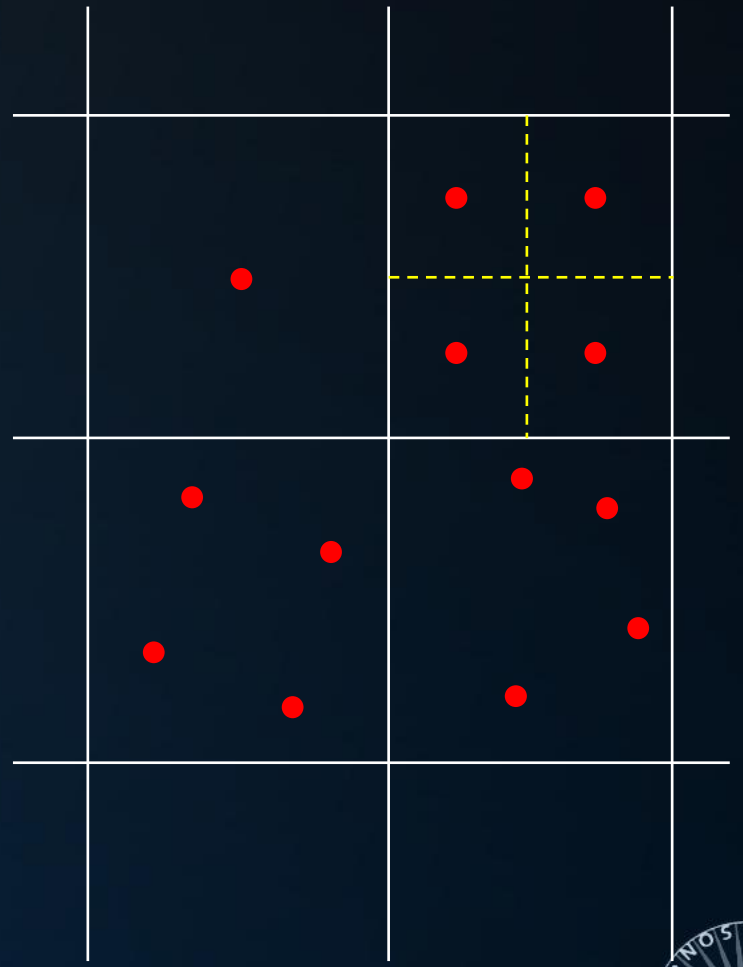
Anti-aliasing – Sampling Patterns





Introduction

Anti-aliasing – Sampling Patterns



Introduction

Anti-aliasing – Sampling Patterns

Adding anti-aliasing to a Whitted-style ray tracer:

Send multiple primary rays through each pixel, and average their result.

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- What if all rays return the same color?

More info: https://mynameismjp.wordpress.com/2012/10/24/msaa-overview

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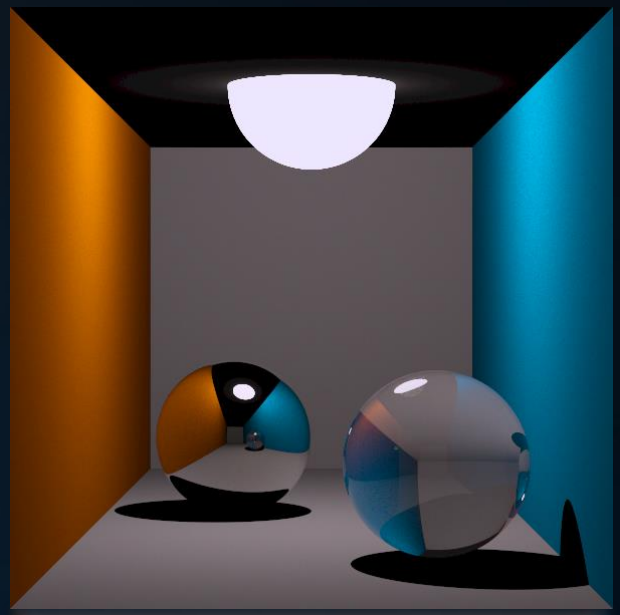
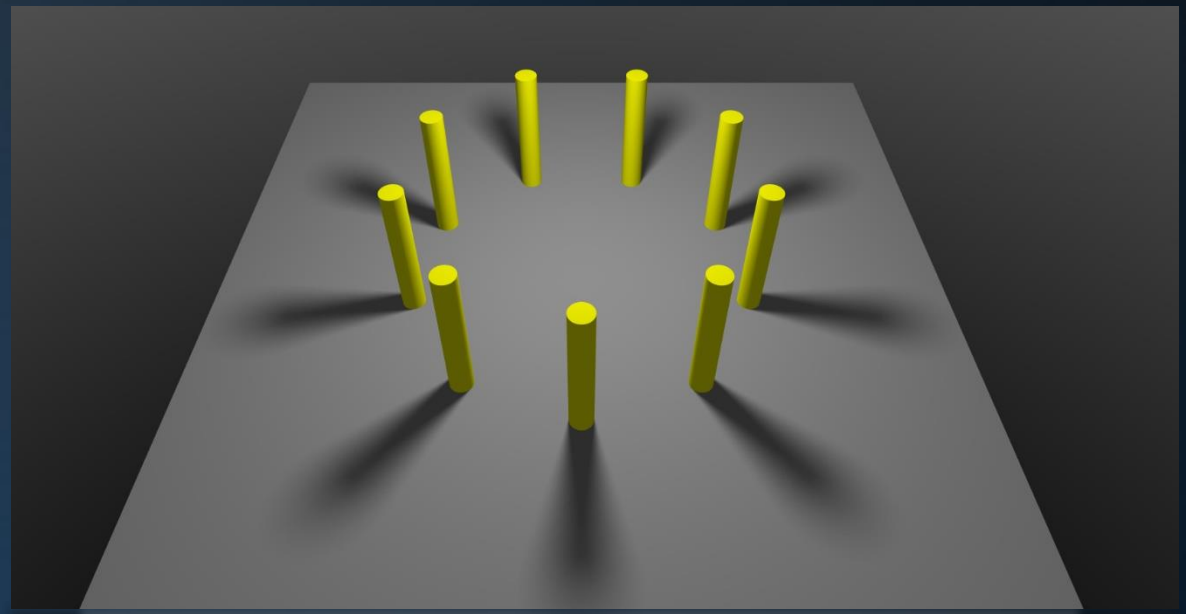
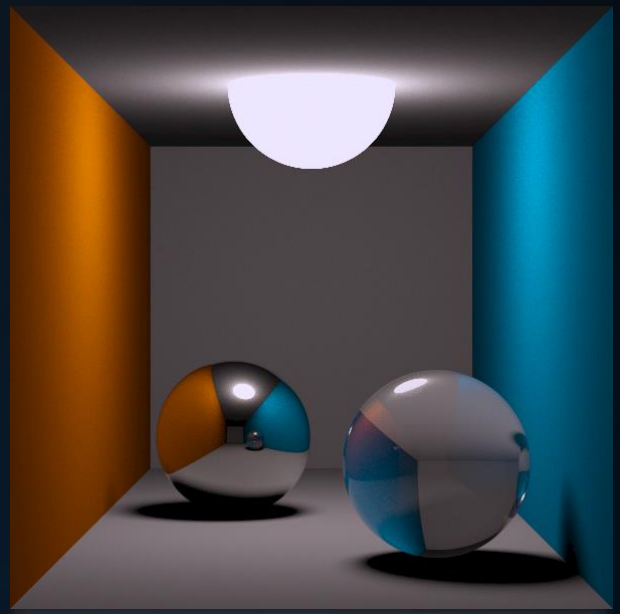


Introduction

Distribution Ray Tracing*



Soft shadows

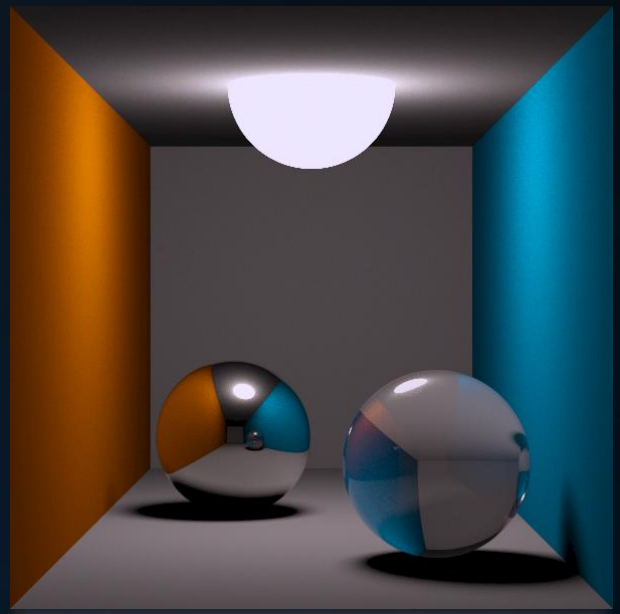


*: Distributed Ray Tracing, Cook et al., 1984

Introduction

Distribution Ray Tracing*

Glossy reflections



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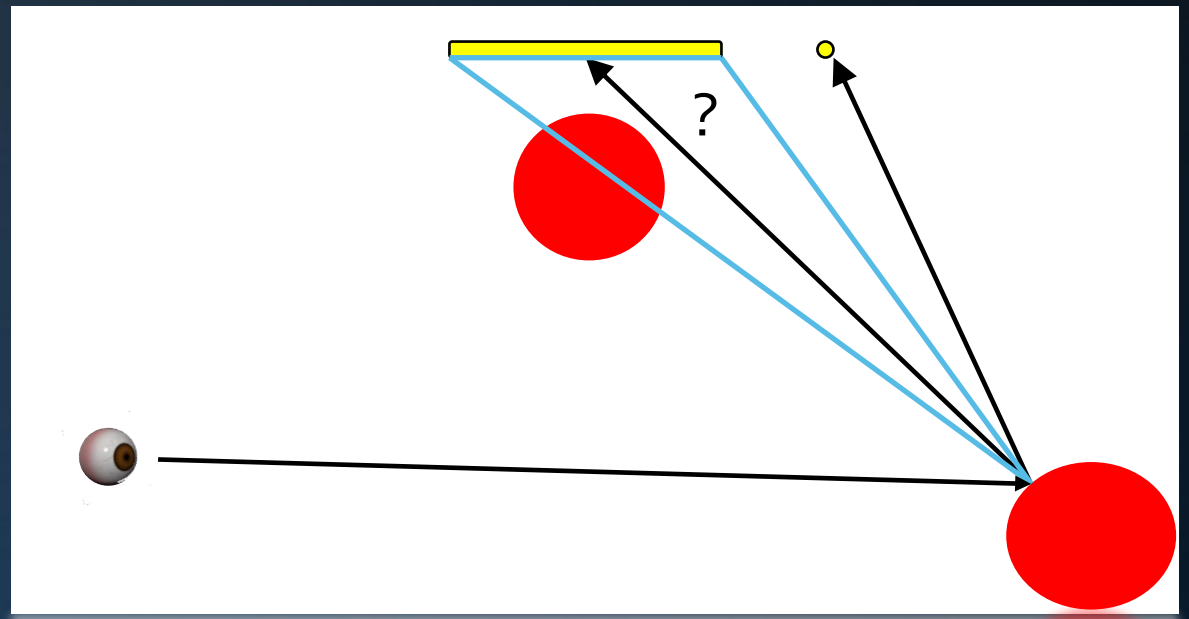

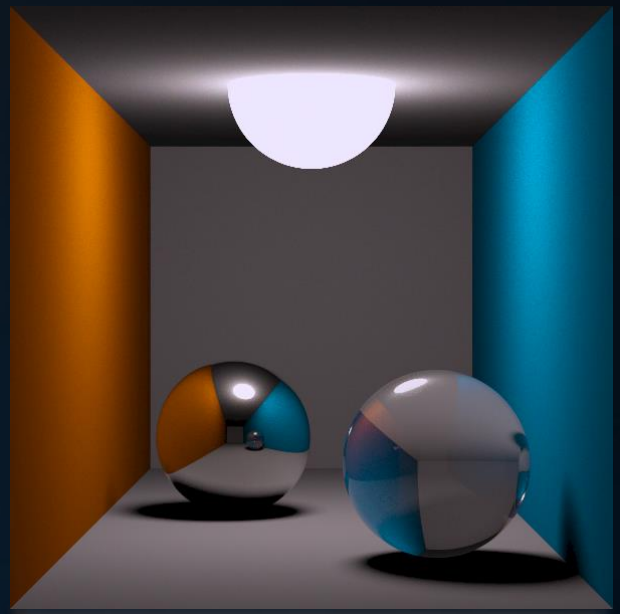

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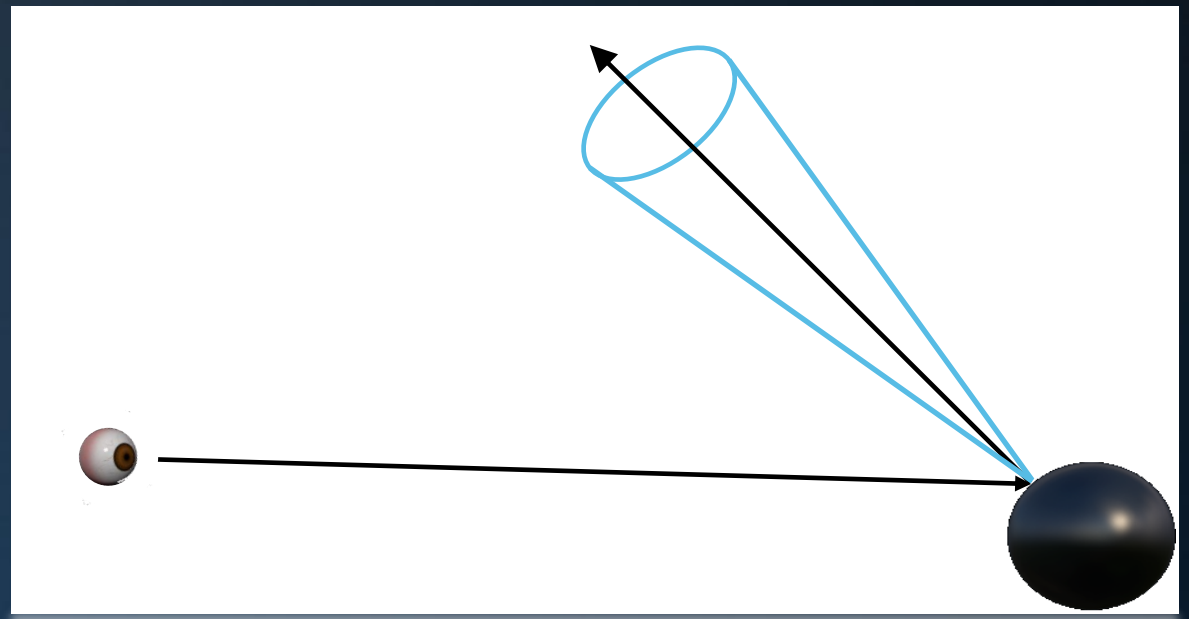

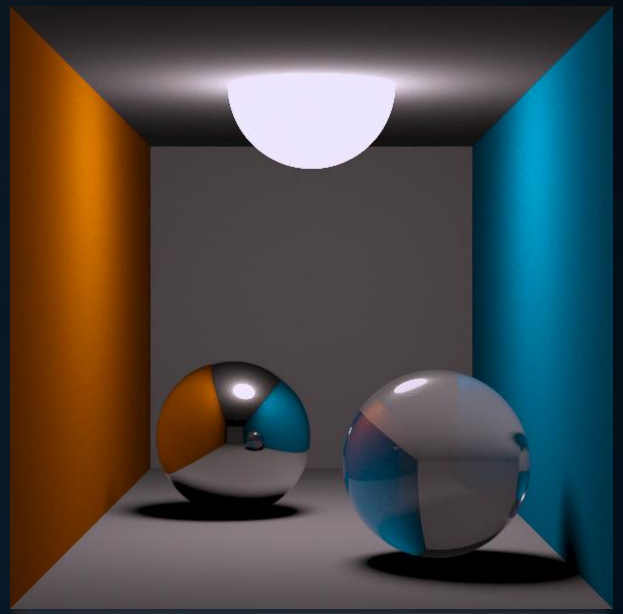

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Introduction

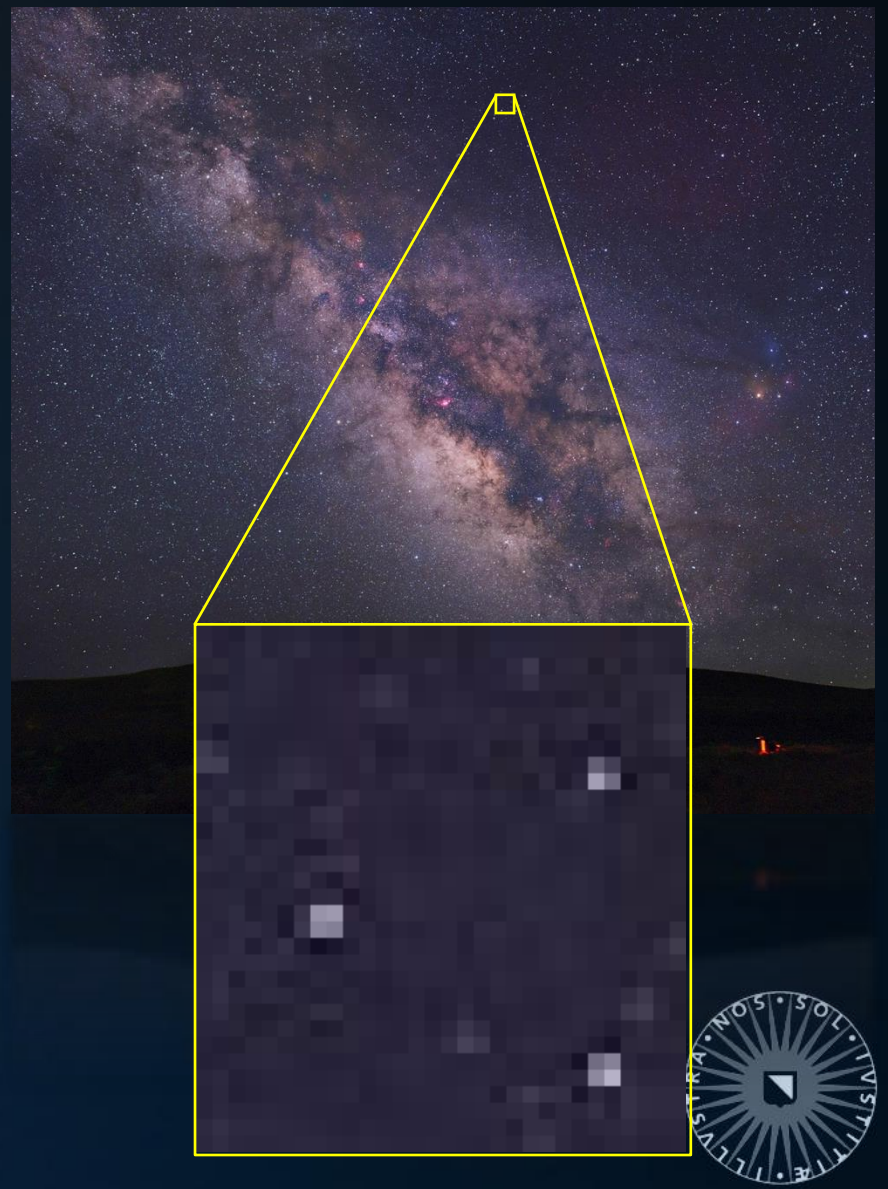
Distribution Ray Tracing

Whitted-style ray tracing is a *point sampling* algorithm:

- We may miss small features
- *We cannot sample areas*

Area sampling:

- Anti-aliasing: one pixel
- Soft shadows: one area light source
- Glossy reflection: directions in a cone
- Diffuse reflection: directions on the hemisphere



Introduction

Area Lights

Visibility of an area light source:

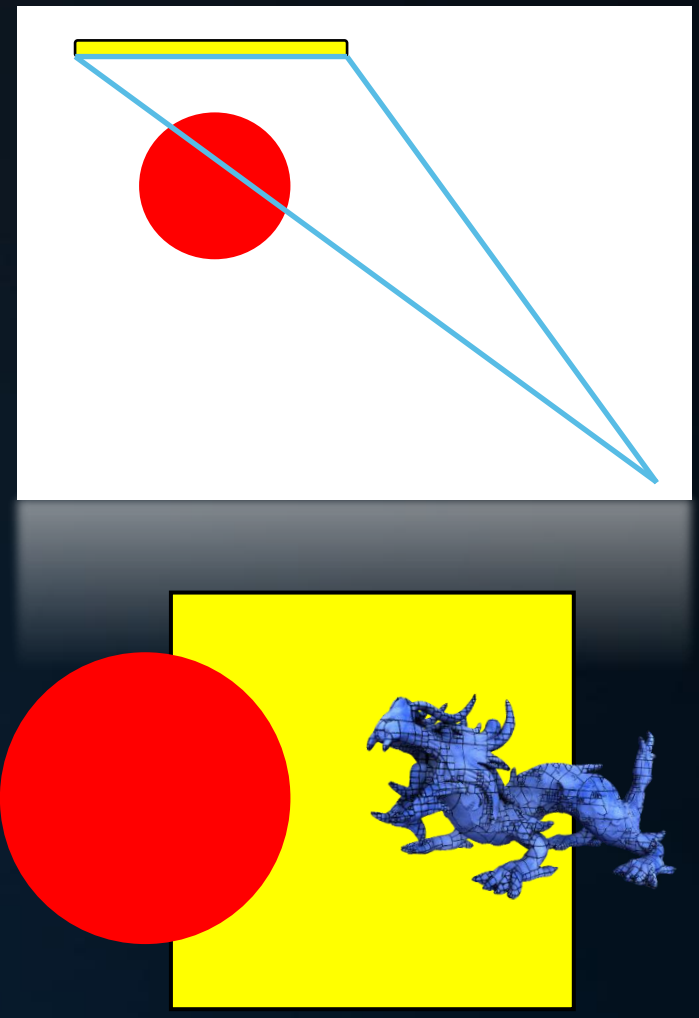
$$V_A = \int_A V(x, \omega_i)\, d\omega_i$$

Analytical solution case 1:

$$V_A = A_{light} - A_{light \cap sphere}$$

Analytical solution case 2:

$$V_A = \ ?$$



Introduction

Approximating Integrals

An integral can be approximated as a Riemann sum:

$$V_A = \int_A^B f(x)\,dx \approx \sum_{i=1}^{N} f(t_i)\,\Delta_i\,, \text{where} \sum_{i=1}^{N} \Delta_i = B - A$$

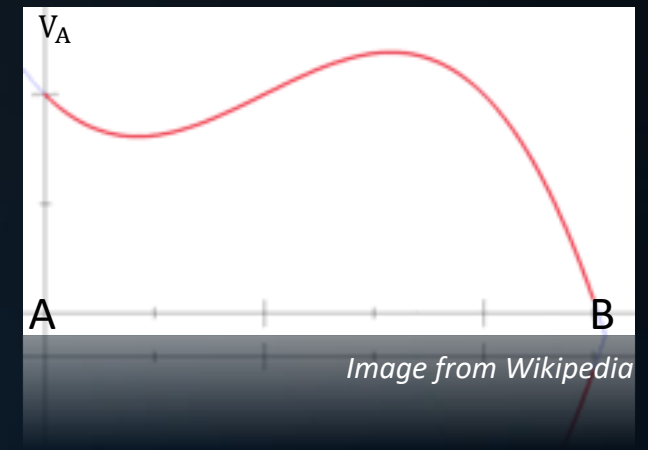
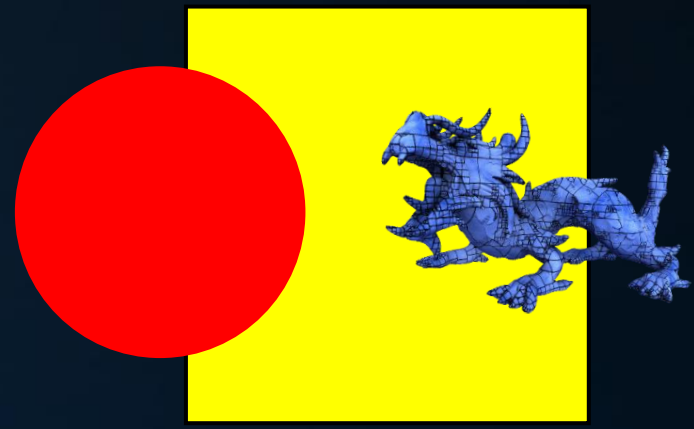


Image from Wikipedia

Note that the intervals do not need to be uniform, as long as we sample the full interval. If the intervals are uniform, then

$$\sum_{i=1}^{N} f(t_i)\,\Delta_i = \Delta_i \sum_{i=1}^{N} f(t_i) = \frac{B-A}{N} \sum_{i=1}^{N} f(t_i)\,.$$



Regardless of uniformity, restrictions apply to N when sampling multi-dimensional functions (ideally, $N = M^d, M \in \mathbb{N}$). Also note that aliasing may occur if the intervals are uniform.

Introduction

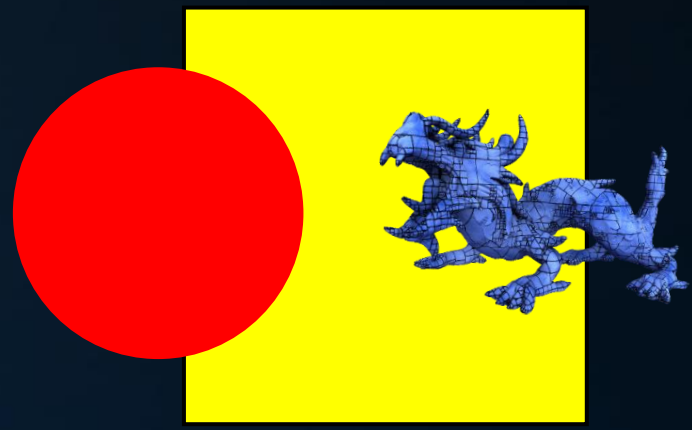
Monte Carlo Integration

Alternatively, we can approximate an integral by taking random samples:

$$V_A = \int_A^B f(x)\,dx \approx \frac{B-A}{N}\sum_{i=1}^{N} f(X_i)$$

Here, $X_1 .. X_N \in [A, B]$.
As N approaches infinity, V_A approaches the *expected value* of f (X).

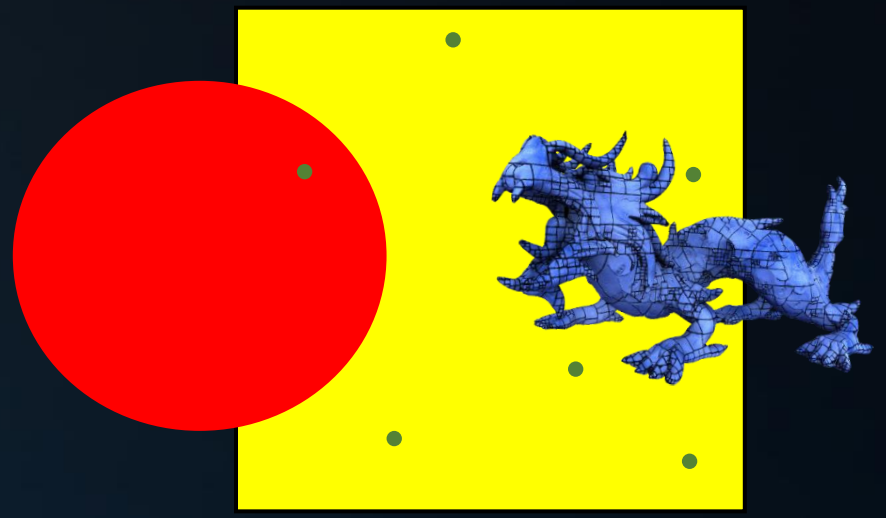
Unlike in Riemann sums, we can use arbitrary N for Monte Carlo integration, regardless of dimension.



Introduction

Monte Carlo Integration of Area Light Visibility

To estimate the visibility of an area light source, we take N random point samples.



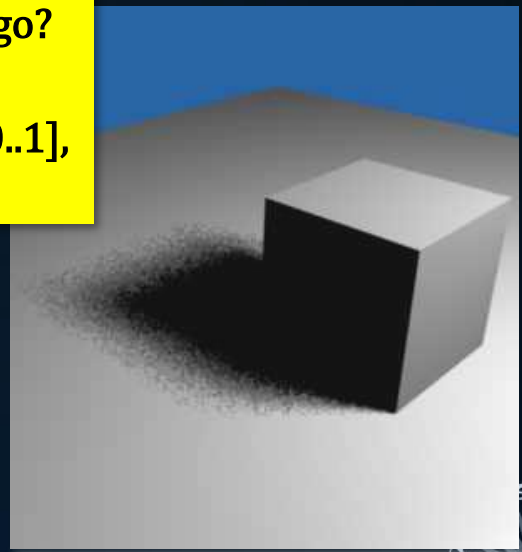
In this case, 5 out of 6 samples are unoccluded:

$$V \approx \frac{1}{6}(1+1+1+0+1+1) = \frac{5}{6}$$

Properly formulated using a MC integrator:

$$V = \int_{S^2} V(p)\,dp \approx \frac{1}{N}\sum_{i=1}^{N} V(P)$$

Q: Where did the $\frac{B-A}{N}$ go?
A: the domain of the visibility function is [0..1], so $B - A = 1$.



With a small number of samples, the variance in the estimate shows up as noise in the image.

Introduction

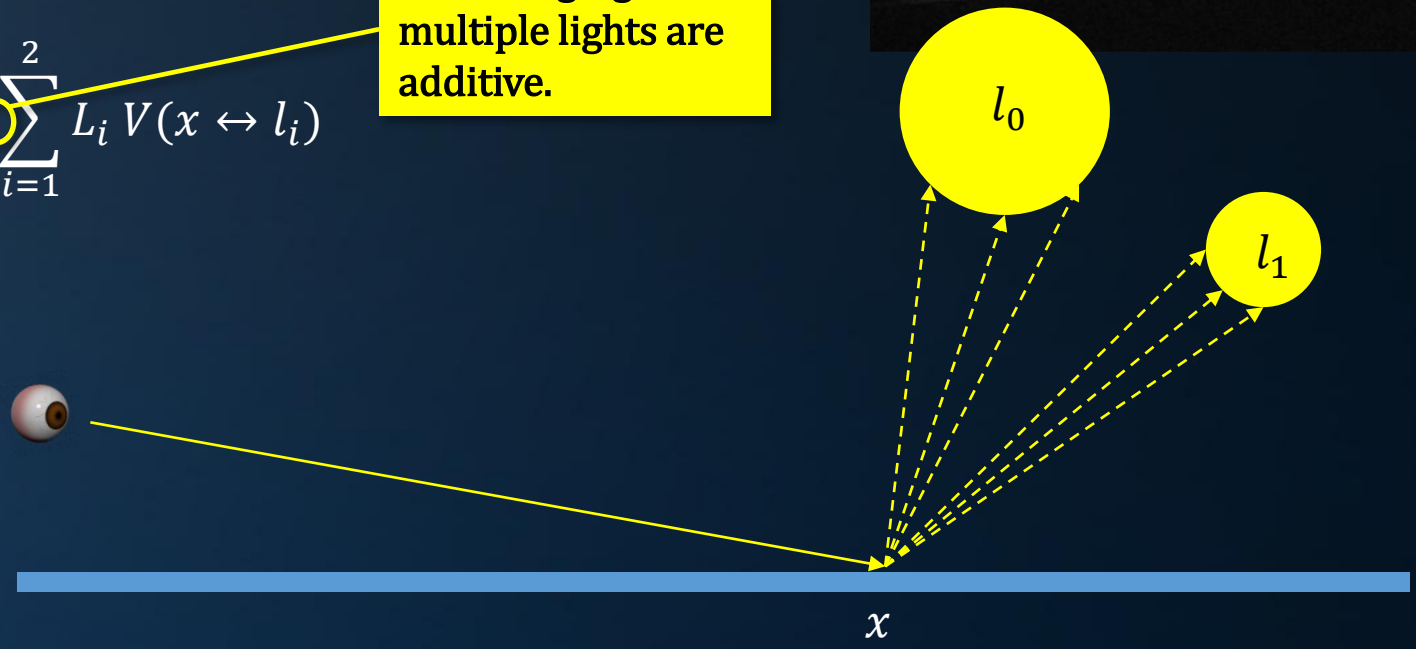
Monte Carlo Integration of Area Light Visibility

We can also use Monte Carlo to estimate the contribution of multiple lights:

1. Take the average of N samples from each light source;
2. Sum the averages.

$$E(x \leftarrow) = \frac{1}{N} \sum_{j=0}^{N} \sum_{i=1}^{2} L_i \, V(x \leftrightarrow l_i)$$

No averaging here: multiple lights are additive.



Introduction

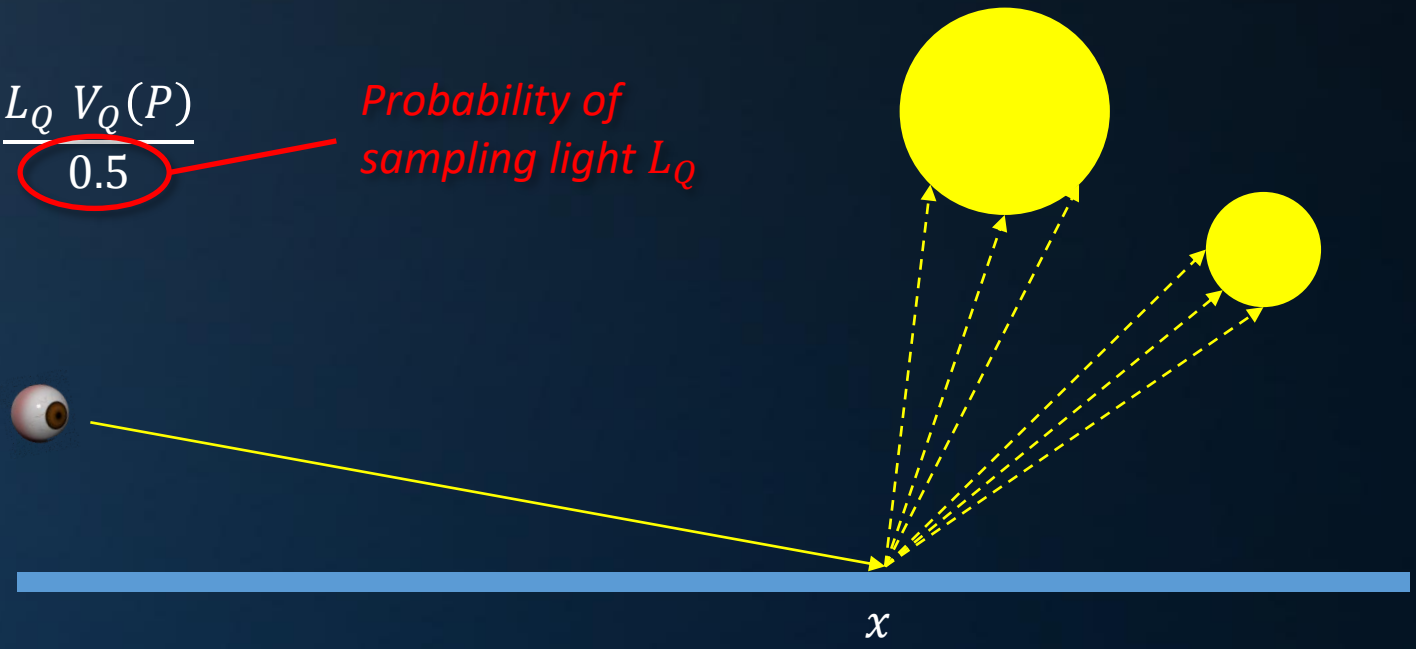
Monte Carlo Integration of Area Light Visibility

Alternatively, we can just take N samples, and pick a random light source for each sample.

$$E(x \leftarrow) = \frac{2}{N} \sum_{i=1}^{N} L_Q \, V_Q(P), \qquad Q \in \{1,2\}$$

$$= \frac{1}{N} \sum_{i=1}^{N} \frac{L_Q \, V_Q(P)}{0.5}$$

Probability of sampling light L_Q



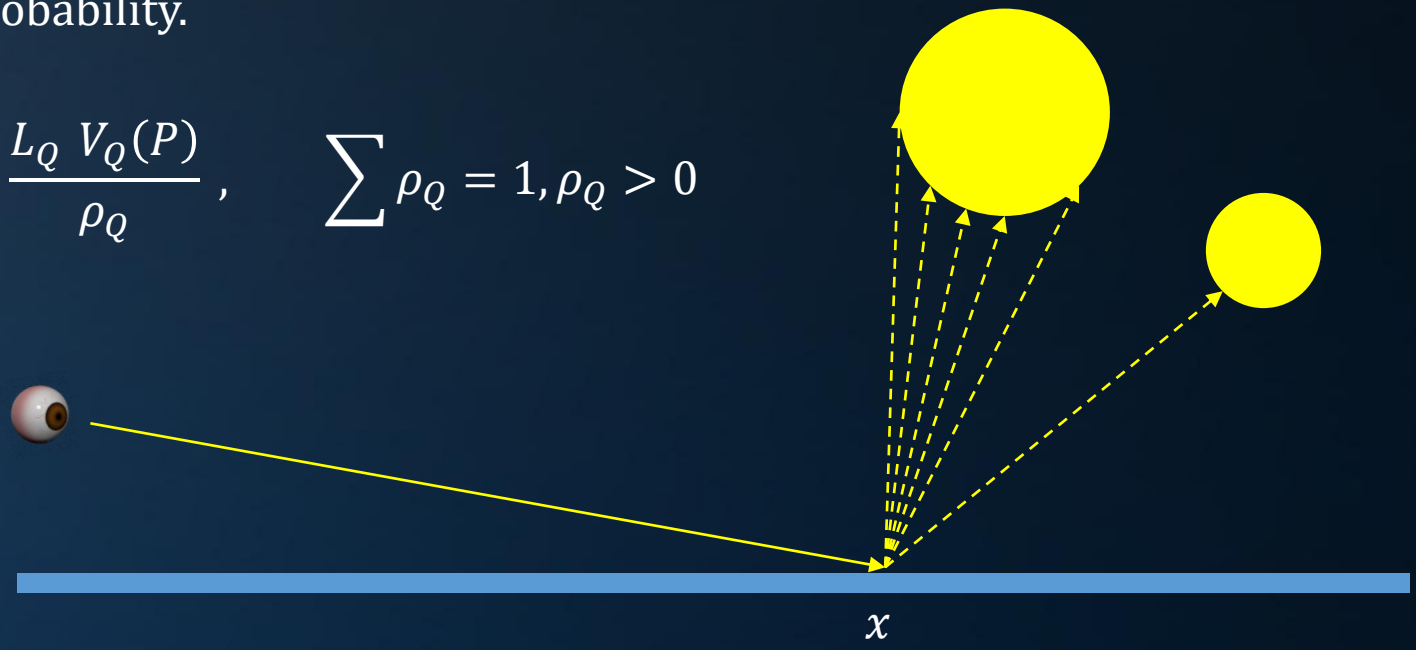
Introduction

Monte Carlo Integration of Area Light Visibility

We obtain a better estimate with fewer samples if we do not treat each light equally.

In the previous example, each light had a 50% probability of being sampled. We can use an arbitrary probability, by dividing the sample by this probability.

$$E(x \leftarrow) = \frac{1}{N} \sum_{i=1}^{N} \frac{L_Q \; V_Q(P)}{\rho_Q}, \qquad \sum \rho_Q = 1, \rho_Q > 0$$



Introduction

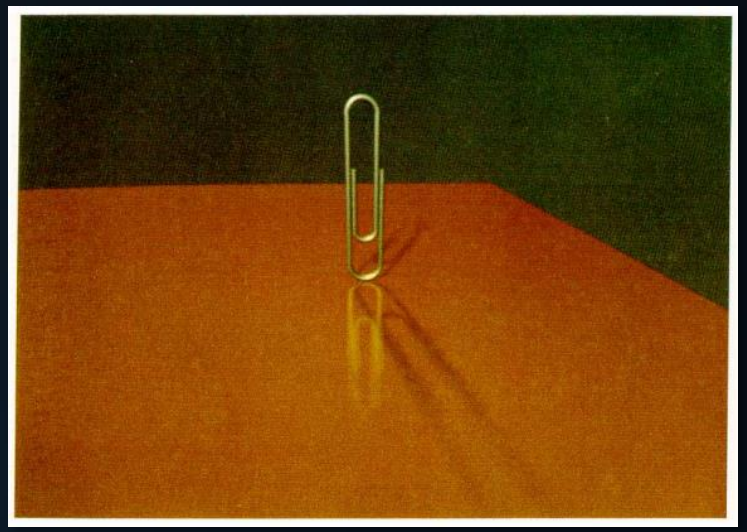
Distribution Ray Tracing

Key concept of distribution ray tracing:

We estimate integrals using Monte Carlo integration.

Integrals in rendering:

- Area of a pixel
- Lens area (aperture)
- Frame time
- Light source area
- Cones for glossy reflections
- Wavelengths
- …



Introduction

Open Issues

Remaining issues:

- Energy distribution in the ray tree / efficiency
- Diffuse interreflections

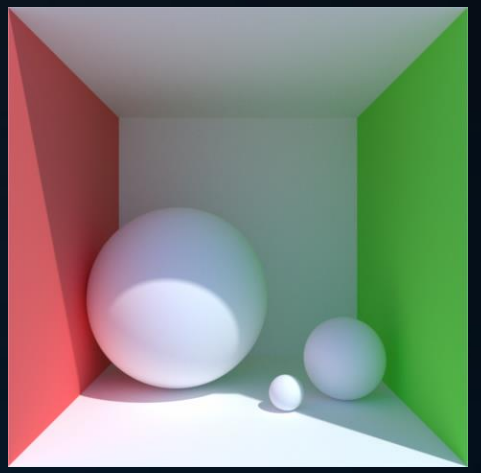
Today’s Agenda:

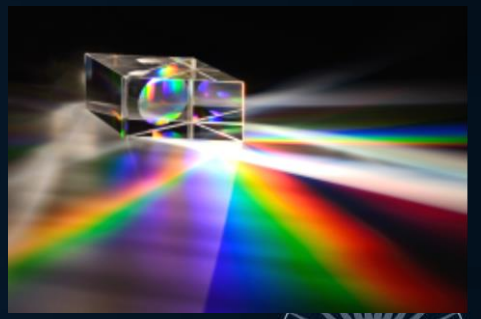
- Introduction
- The Rendering Equation
- Light Transport

Rendering Equation

Whitted, Cook & Beyond

Missing in Whitted:	Cook:
▪ Area lights	✓ Area lights
▪ Glossy reflections	✓ Glossy reflections
▪ Caustics	✗ Caustics
▪ Diffuse interreflections	✗ Diffuse interreflections
▪ Diffraction	✗ Diffraction
▪ Polarization	✗ Polarization
▪ Phosphorescence	✗ Phosphorescence
▪ Temporal effects	✗ Temporal effects
▪ Motion blur	✓ Motion blur
▪ Depth of field	✓ Depth of field
▪ Anti-aliasing	✓ Anti-aliasing





Rendering Equation

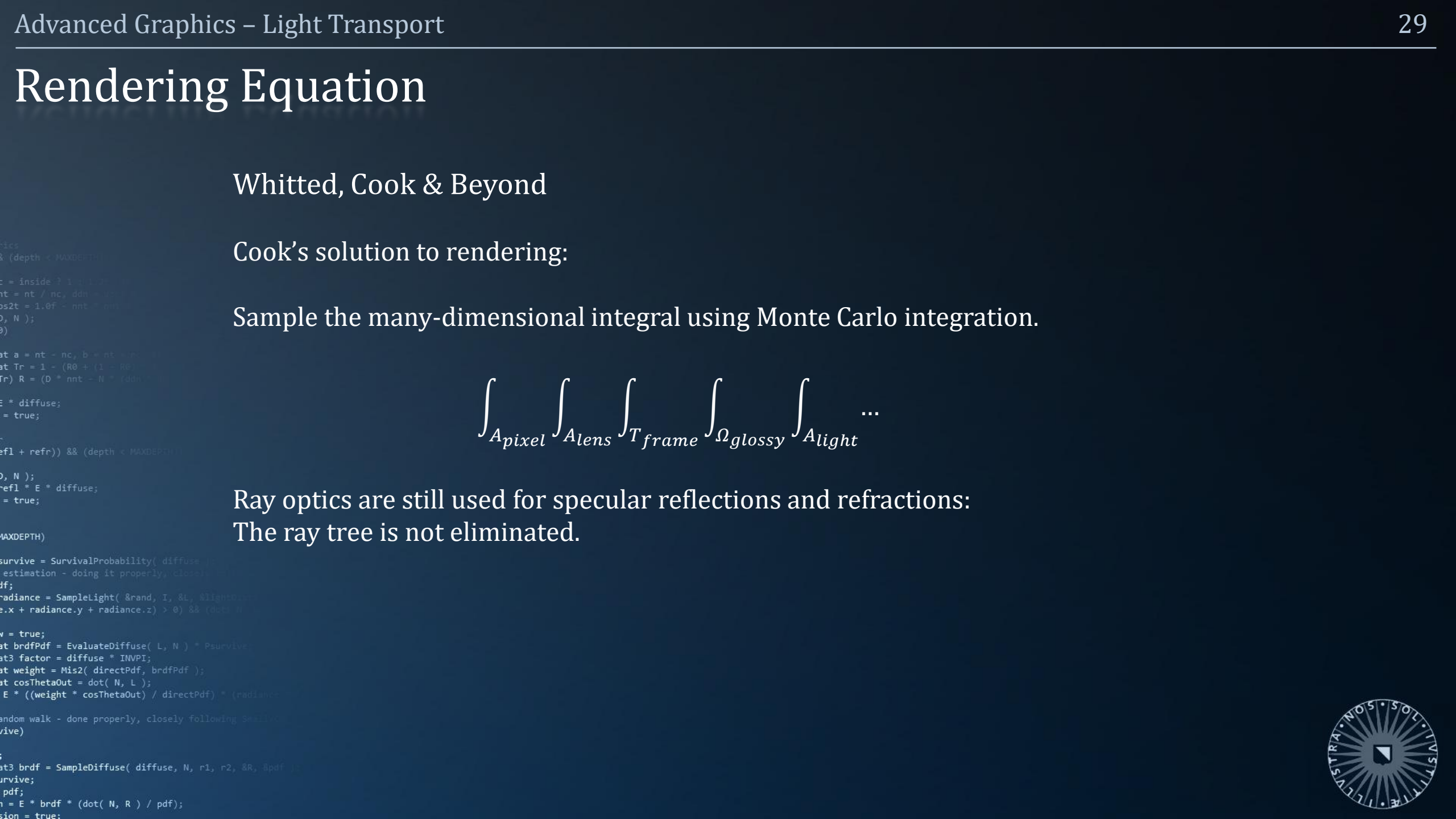
Whitted, Cook & Beyond

Cook's solution to rendering:

Sample the many-dimensional integral using Monte Carlo integration.

$$\int_{A_{pixel}} \int_{A_{lens}} \int_{T_{frame}} \int_{\Omega_{glossy}} \int_{A_{light}} \dots$$

Ray optics are still used for specular reflections and refractions:
The ray tree is not eliminated.



Rendering Equation

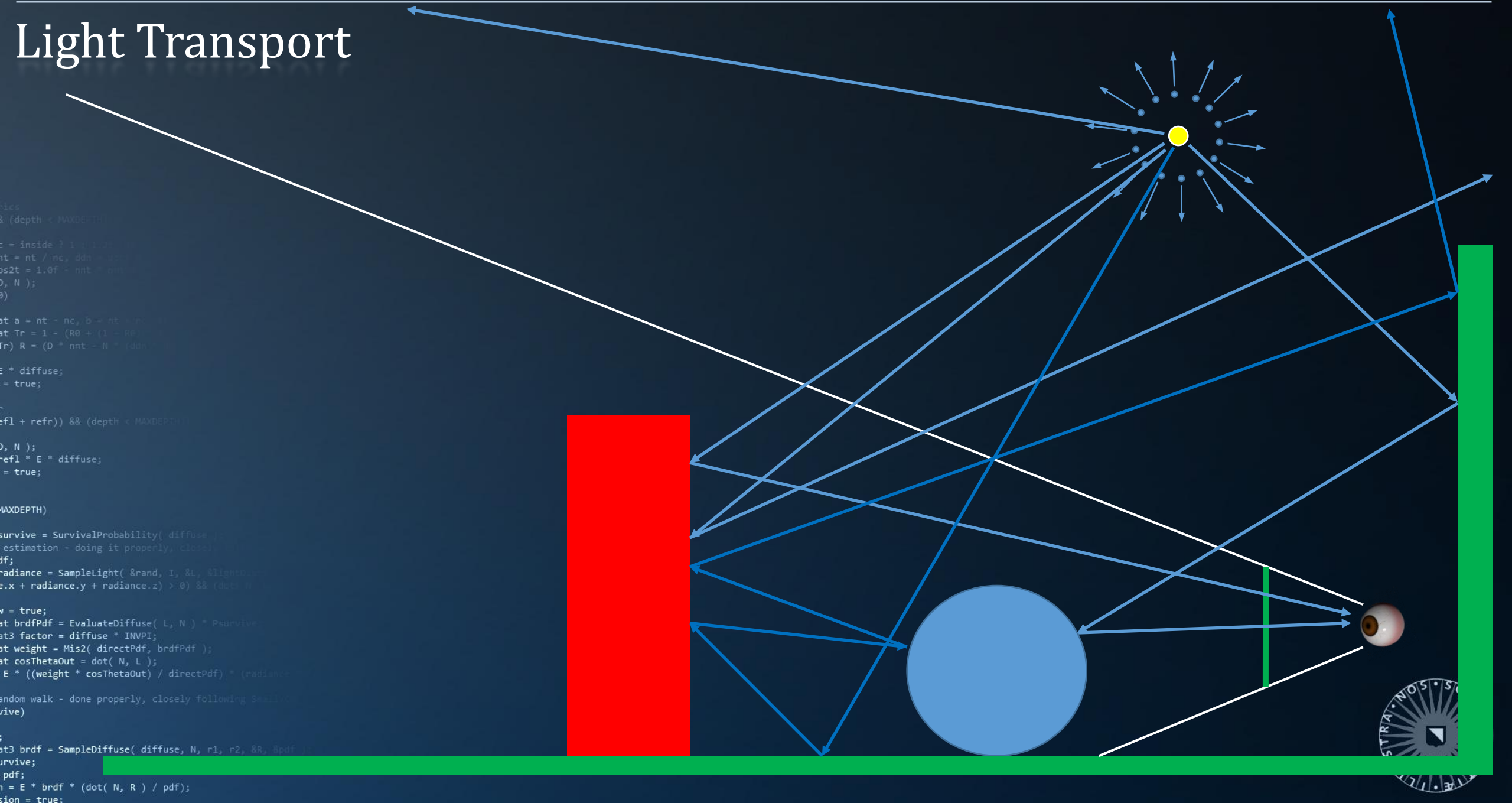
God's Algorithm

1 room
1 bulb
100 watts
10^{20} photons per second

Photon behavior:

- Travel in straight lines
- Get absorbed, or change direction:
  - Bounce (random / deterministic)
  - Get transmitted
- Leave into the void
- Get detected

Light Transport



Rendering Equation

God's Algorithm - Mathematically

A photon may arrive at a sensor after travelling in a straight line from a light source to the sensor:

$$L(s \leftarrow x) = L_E(s \leftarrow x)$$

Or, it may be reflected by a surface towards the sensor:

$$L(s \leftarrow x) = \int_A f_r(s \leftarrow x \leftarrow x')\, L(x \leftarrow x')\, G(x \leftrightarrow x')\, dA(x')$$

Those are the options.
Adding direct and indirect illumination together:

$$L(s \leftarrow x) = L_E(s \leftarrow x) + \int_A f_r(s \leftarrow x \leftarrow x')\, L(x \leftarrow x')\, G(x \leftrightarrow x')\, dA(x')$$

Rendering Equation

God's Algorithm - Mathematically

$$L(s \leftarrow x) = L_E(s \leftarrow x) + \int_A f_r(s \leftarrow x \leftarrow x')\, L(x \leftarrow x')\, G(x \leftrightarrow x')\, dA(x')$$

Emission

Hemisphere

Reflection

Indirect

Geometry factor

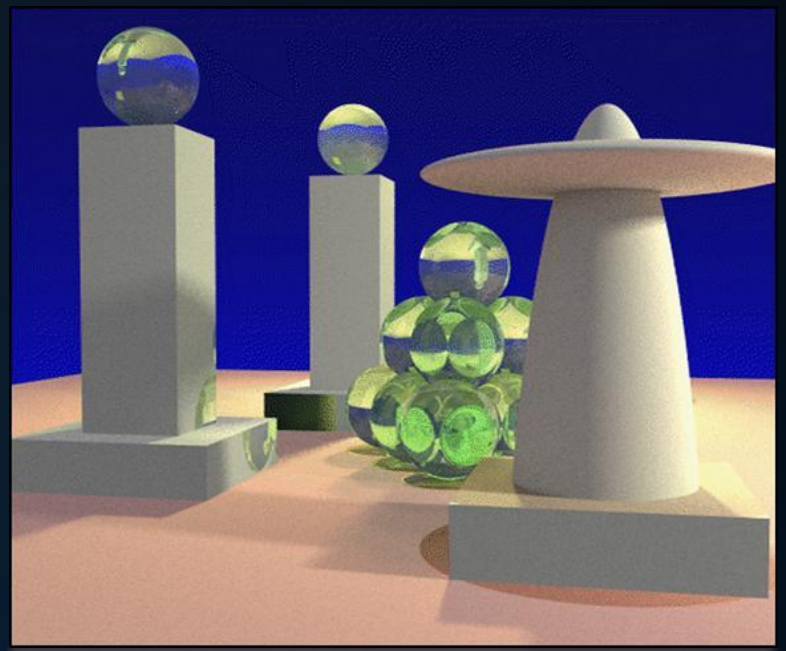
Rendering Equation

$$L(s \leftarrow x) = L_E(s \leftarrow x) + \int_A f_r(s \leftarrow x \leftarrow x')\, L(x \leftarrow x')\, G(x \leftrightarrow x')\, dA(x')$$

The Rendering Equation*:

- Light transport from lights to sensor
- Recursive
- Physically based

The equation allows us to determine to which extend rendering algorithms approximate real-world light transport.



*: The Rendering Equation, Kajiya, 1986

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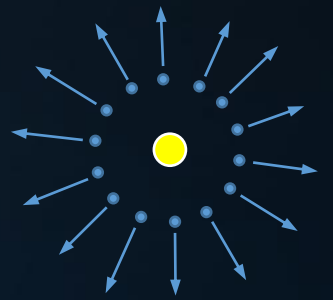
Light Transport

Light Transport Quantities

Radiant flux - Φ :

"Radiant energy emitted, reflected, transmitted or received, per unit time."



Units: watts = joules per second

$$W = J\,s^{-1}\,.$$

Simplified particle analogy:
number of photons.

Note: photon energy depends on electromagnetic wavelength: $\mathrm{E} = \frac{\mathrm{hc}}{\lambda}$ *, where h is Planck's constant, c is the speed of light, and* λ *is wavelength. At* $\lambda = 550\mathrm{nm}$ *(yellow), a single photon carries* $3.6 * 10^{-19}$ *joules.*

Light Transport

Light Transport Quantities

In a vacuum, radiant flux emitted by a point light source remains constant over distance:

A point light emitting 100W delivers 100W to the surface of a sphere of radius r around the light. This sphere has an area of $4\pi r^2$; energy per surface area thus decreases by $1/r^2$.

In terms of photons: the density of the photon distribution decreases by $1/r^2$.

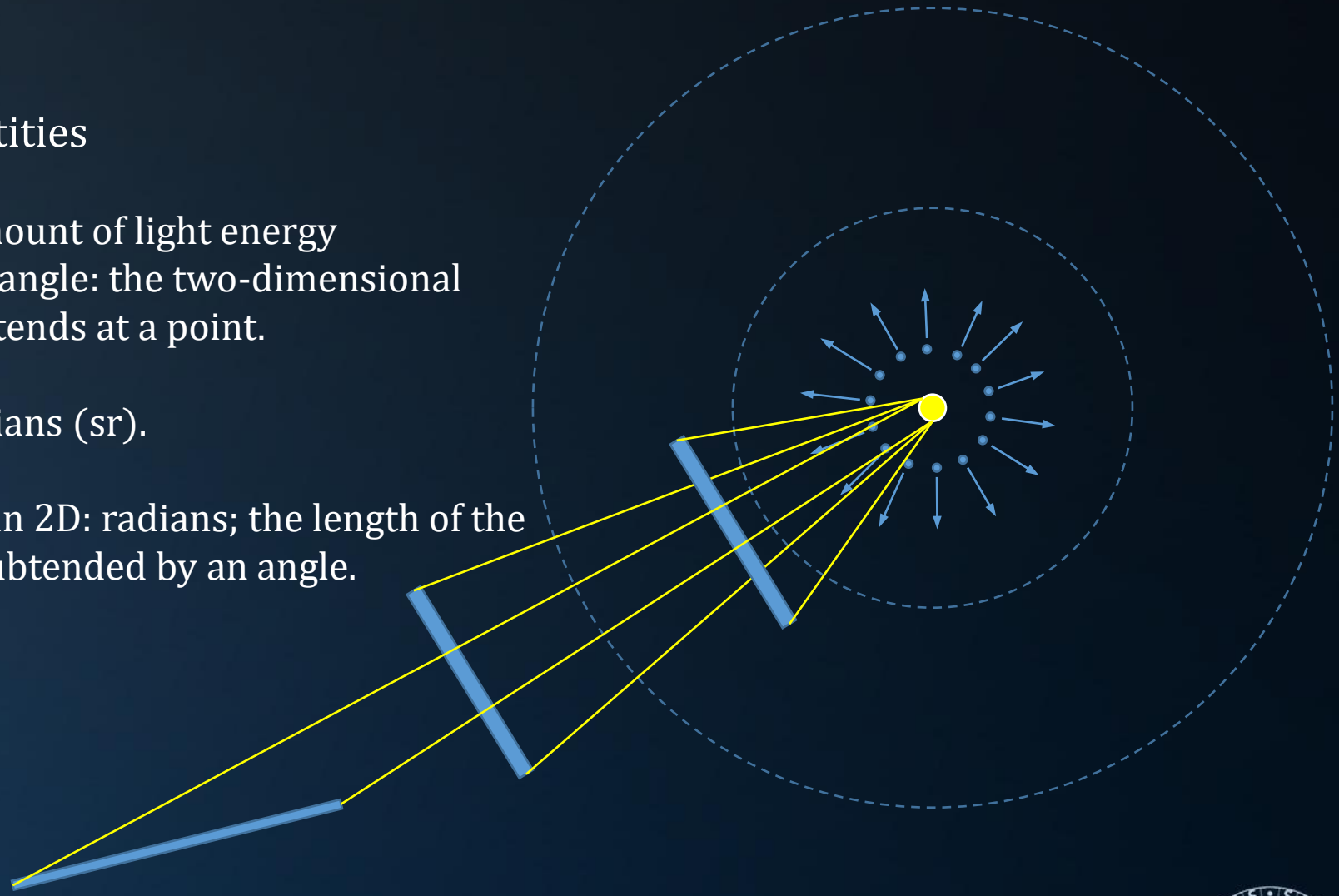
Light Transport

Light Transport Quantities

A surface receives an amount of light energy proportional to its solid angle: the two-dimensional space that an object subtends at a point.

Solid angle units: steradians (sr).

Corresponding concept in 2D: radians; the length of the arc on the unit sphere subtended by an angle.



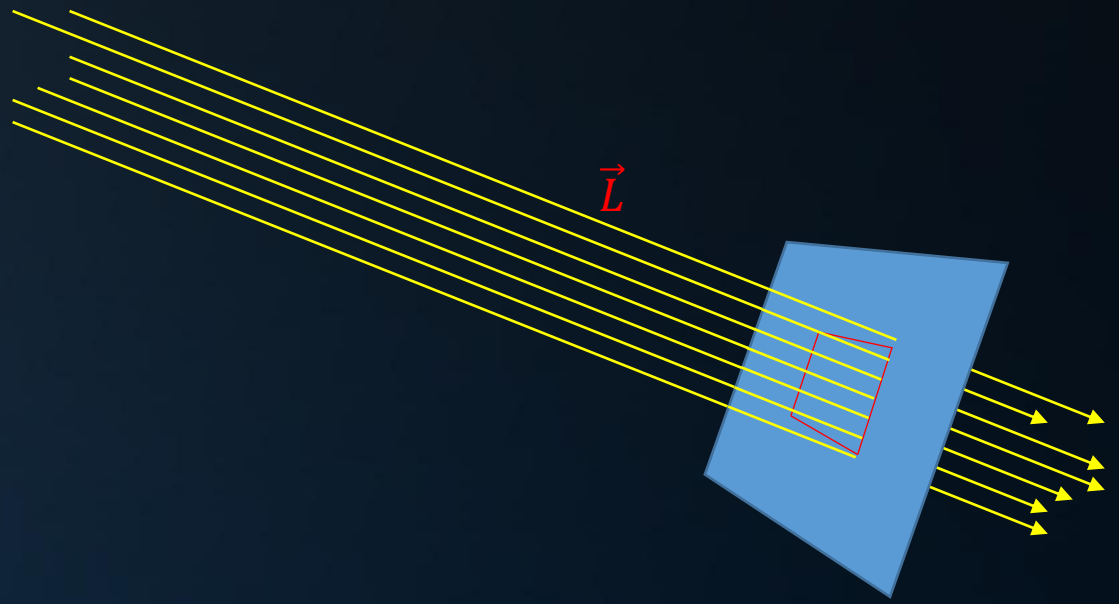
Light Transport

Light Transport Quantities

Radiance - L :

"The power of electromagnetic radiation emitted, reflected, transmitted or received per unit projected area per unit solid angle."

Units: $Wsr^{-1}m^{-2}$



Simplified particle analogy:
Amount of particles passing through a pipe with unit diameter, per unit time.

Note: radiance is a continuous value:
while flux at a point is 0 (since both area and solid angle are 0), we can still define flux per area per solid angle for that point.

Light Transport

Light Transport Quantities

Irradiance - E :

"The power of electromagnetic radiation per unit area incident on a surface."

Units: Watts per m^2 = joules per second per m^2

$$Wm^{-2} = Jm^{-2}s^{-1} .$$

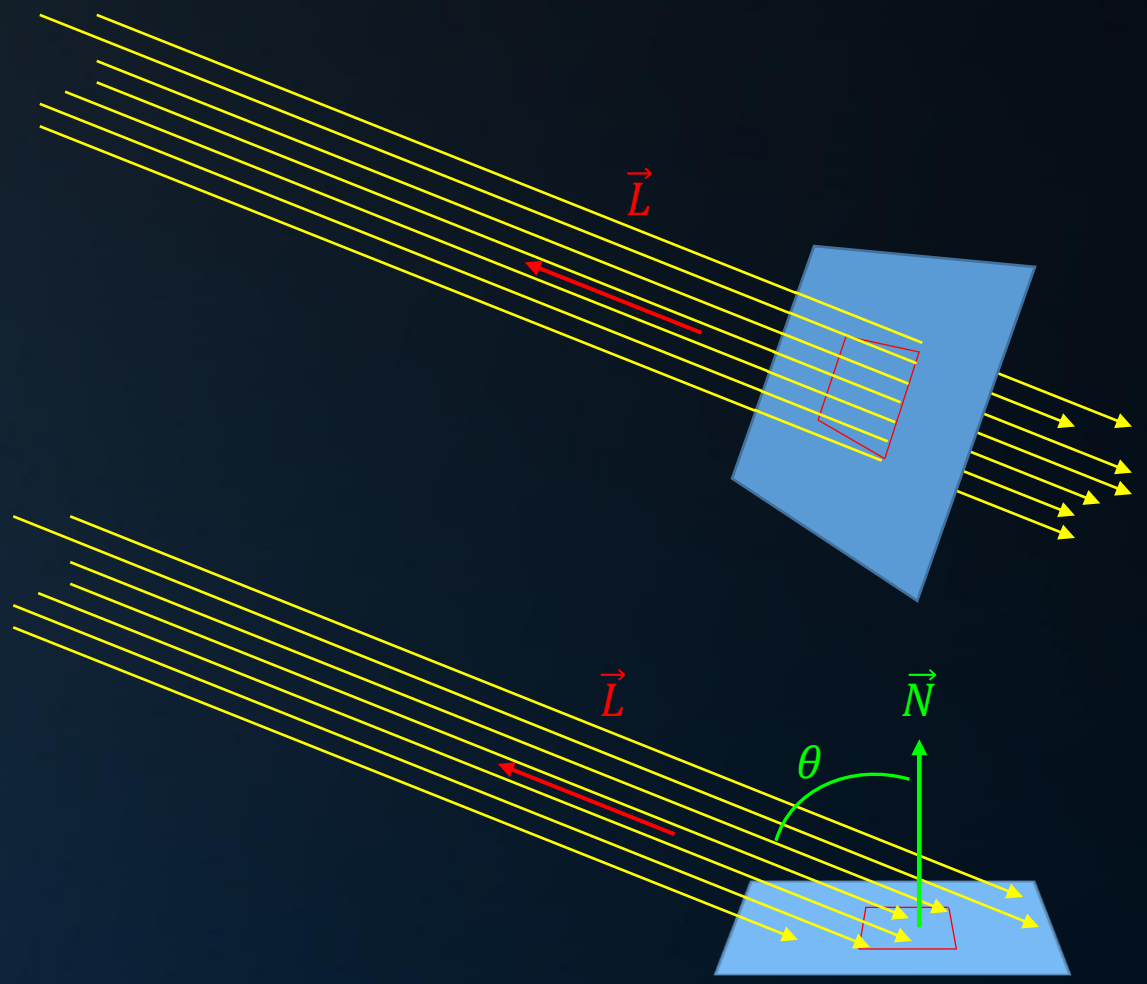
Simplified particle analogy:
number of photons arriving per unit area per unit time, from all directions.

Light Transport

Light Transport Quantities

Converting radiance to irradiance:

$$E = L \cos\theta$$



Light Transport

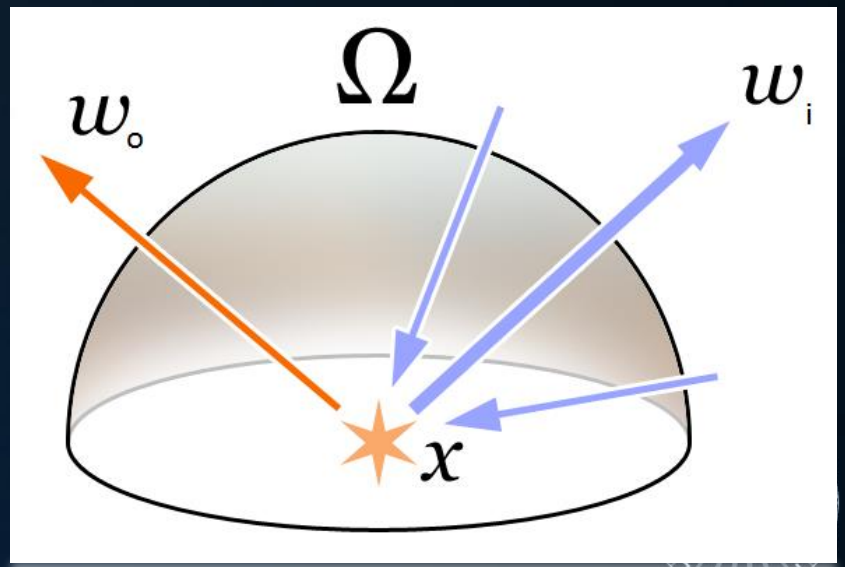
Light Transport

$$L(s \leftarrow x) = L_E(s \leftarrow x) + \int_A f_r(s \leftarrow x \leftarrow x')\, L(x \leftarrow x')\, G(x \leftrightarrow x')\, dA(x')$$

$$L_o(x, \omega_o) = L_E(x, \omega_o) + \int_\Omega f_r(x, \omega_o, \omega_i)\, L_i(x, \omega_i) \cos\theta_i \; d\omega_i$$

Radiance Radiance Radiance / Irradiance

BRDF



Light Transport

Bidirectional Reflectance Distribution Function

BRDF: function describing the relation between radiance emitted in direction ω_o and irradiance arriving from direction ω_i:

$$f_r(\omega_o, \omega_i) = \frac{L_o(\omega_o)}{E_i(\omega_i)} = \frac{L_o(\omega_o)}{L_i(\omega_i)\cos\theta_i} = \frac{outgoing\ radiance}{incoming\ irradiance}$$

Or, if spatially variant:

$$f_r(x, \omega_o, \omega_i) = \frac{L_o(x, \omega_o)}{E_i(x, \omega_i)} = \frac{L_o(x, \omega_o)}{L_i(x, \omega_i)\cos\theta_i}$$

Properties:

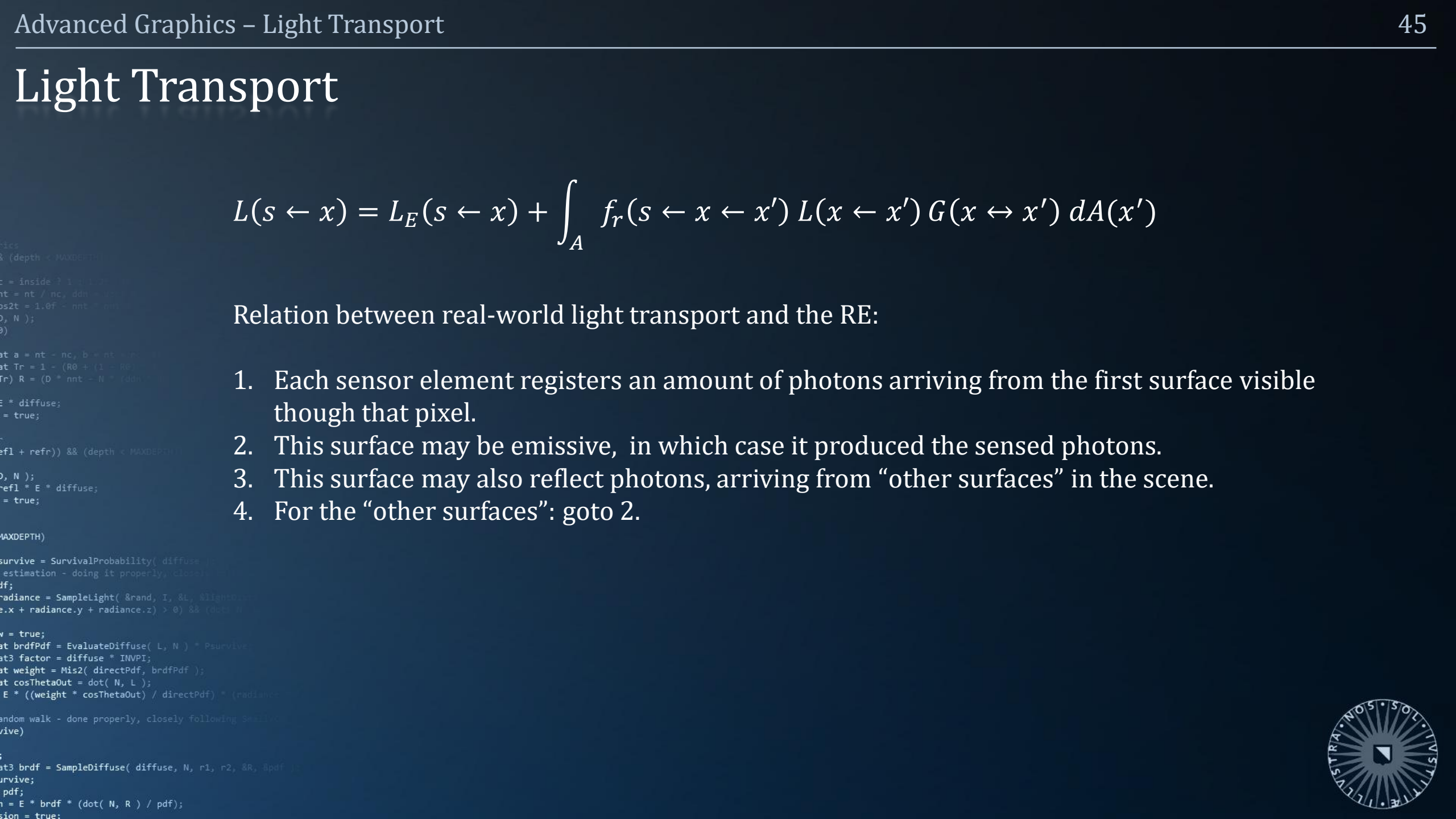
- Should be positive: $f_r(\omega_o, \omega_i) \geq 0$
- Helmholtz reciprocity should be obeyed: $f_r(\omega_o, \omega_i) = f_r(\omega_i, \omega_o)$
- Energy should be conserved: $\int_\Omega f_r(\omega_o, \omega_i)\cos\theta_o\, d\omega_o \leq 1$

Light Transport

$$L(s \leftarrow x) = L_E(s \leftarrow x) + \int_A f_r(s \leftarrow x \leftarrow x')\, L(x \leftarrow x')\, G(x \leftrightarrow x')\, dA(x')$$

Relation between real-world light transport and the RE:

1. Each sensor element registers an amount of photons arriving from the first surface visible though that pixel.
2. This surface may be emissive, in which case it produced the sensed photons.
3. This surface may also reflect photons, arriving from “other surfaces” in the scene.
4. For the “other surfaces”: goto 2.



Today’s Agenda:

- Introduction
- The Rendering Equation
- Light Transport

INFOMAGR – Advanced Graphics

Jacco Bikker - November 2021 - February 2022

END of “Light Transport”

next lecture: “The Perfect BVH”