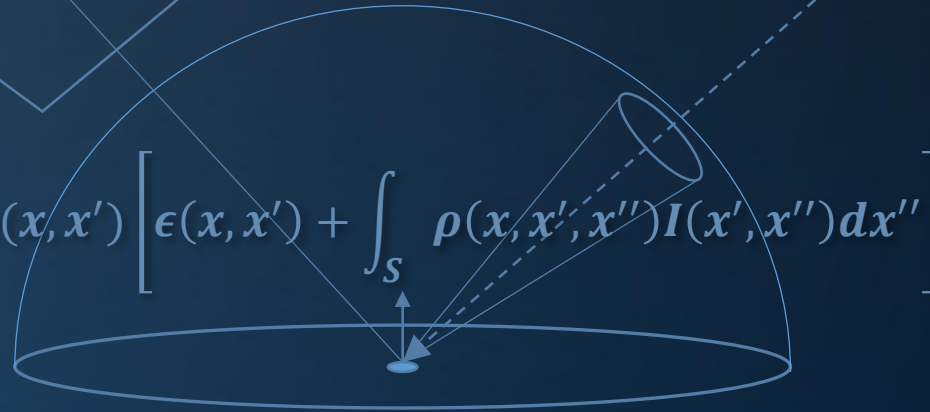


# INFOMAGR – Advanced Graphics

Jacco Bikker - November 2021 - February 2022

## Lecture 5 - “The Perfect BVH”

Welcome!



$$I(x, x') = g(x, x') \left[ \epsilon(x, x') + \int_S \rho(x, x', x'') I(x', x'') dx'' \right]$$



```

ics
& (depth < MAXDEPTH) {
    // Inside?
    if (inside ? 1 : 0) {
        nt = nt / nc; ddn = ddn * nc;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    // Outside?
    at a = nt - nc, b = nt + nc;
    at Tr = 1 - (R0 + (1 - R0) * ddn);
    (Tr) R = (D * nnt - N * (ddn > 0 ? 1 : -1));
    // Diffuse
    E * diffuse;
    = true;
    // Refractive
    refl + refr)) && (depth < MAXDEPTH) {
        D, N );
        refl * E * diffuse;
        = true;
    }
    // Survival
    MAXDEPTH)
    survive = SurvivalProbability( diffuse );
    // Estimation
    estimation - doing it properly, closely following Small's
    if;
    radiance = SampleLight( &rand, I, &L, &lightPdf );
    e.x + radiance.y + radiance.z) > 0) && (depth < MAXDEPTH) {
        w = true;
        at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
        at3 factor = diffuse * INVPI;
        at weight = Mis2( directPdf, brdfPdf );
        at cosThetaOut = dot( N, L );
        E * ((weight * cosThetaOut) / directPdf) * (radiance.x + radiance.y + radiance.z);
    }
    // Random walk
    random walk - done properly, closely following Small's
    (survive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    ion = true;
}

```

# Today's Agenda:

- Building Better BVHs
- Refitting
- Fast BVH Construction
- The Top-level BVH

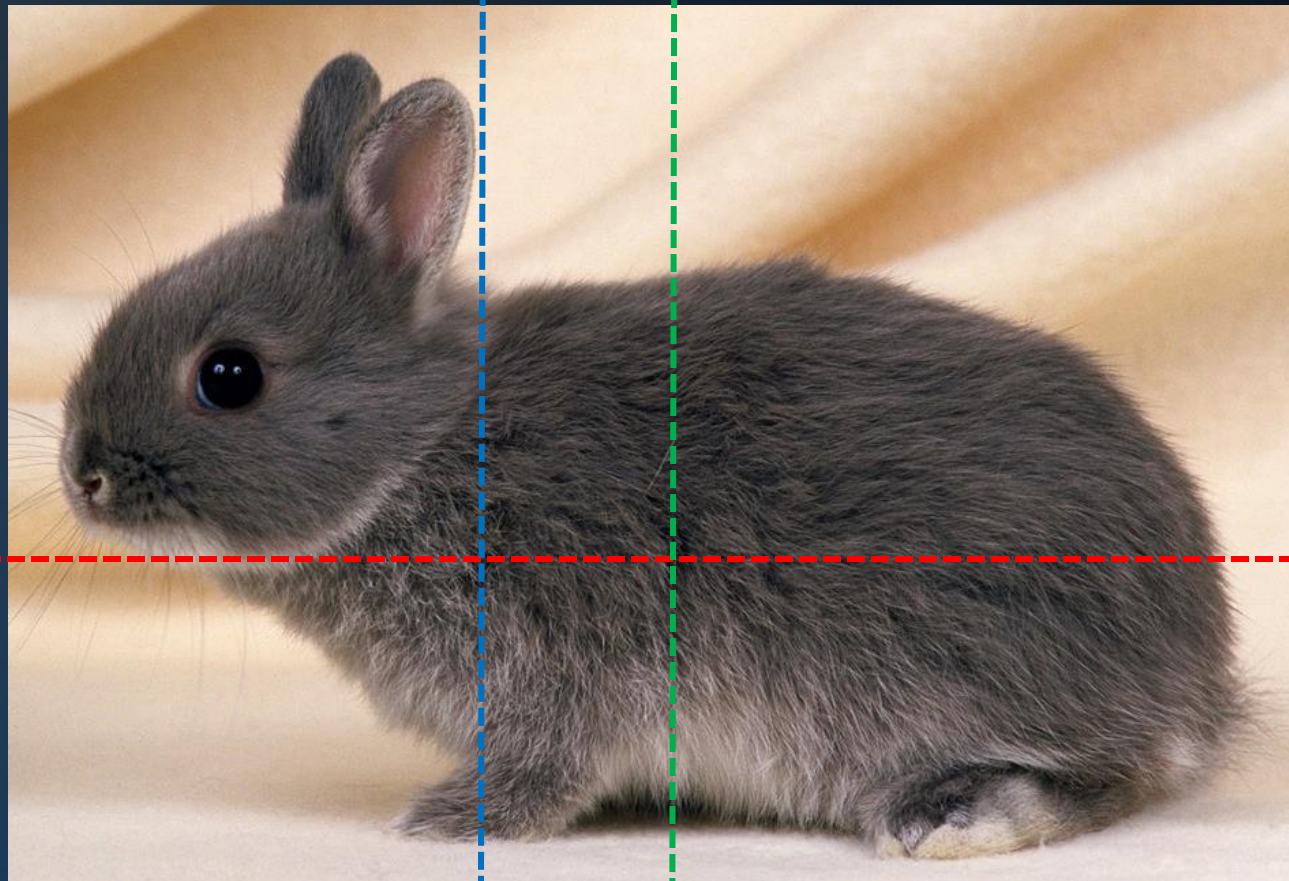


# Better BVHs

```

ics
& (depth < MAXDEPTH)
{
    if (nt < nc)
    {
        nt = nt / nc; ddn = ddn / nc;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    else
    {
        at a = nt - nc, b = nt + nc;
        at Tr = 1 - (R0 + (1 - R0) * a);
        Tr) R = (D * nnt - N * (ddn * a));
    }
    E * diffuse;
    = true;
    refl + refr)) && (depth < MAXDEPTH)
    {
        D, N );
        refl * E * diffuse;
        = true;
    }
    MAXDEPTH)
    survive = SurvivalProbability( diffuse );
    estimation - doing it properly, closely following
    df;
    radiance = SampleLight( &rand, I, &L, &light,
    e.x + radiance.y + radiance.z) > 0) && (depth <
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiance
    random walk - done properly, closely following Small's
    vive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;

```

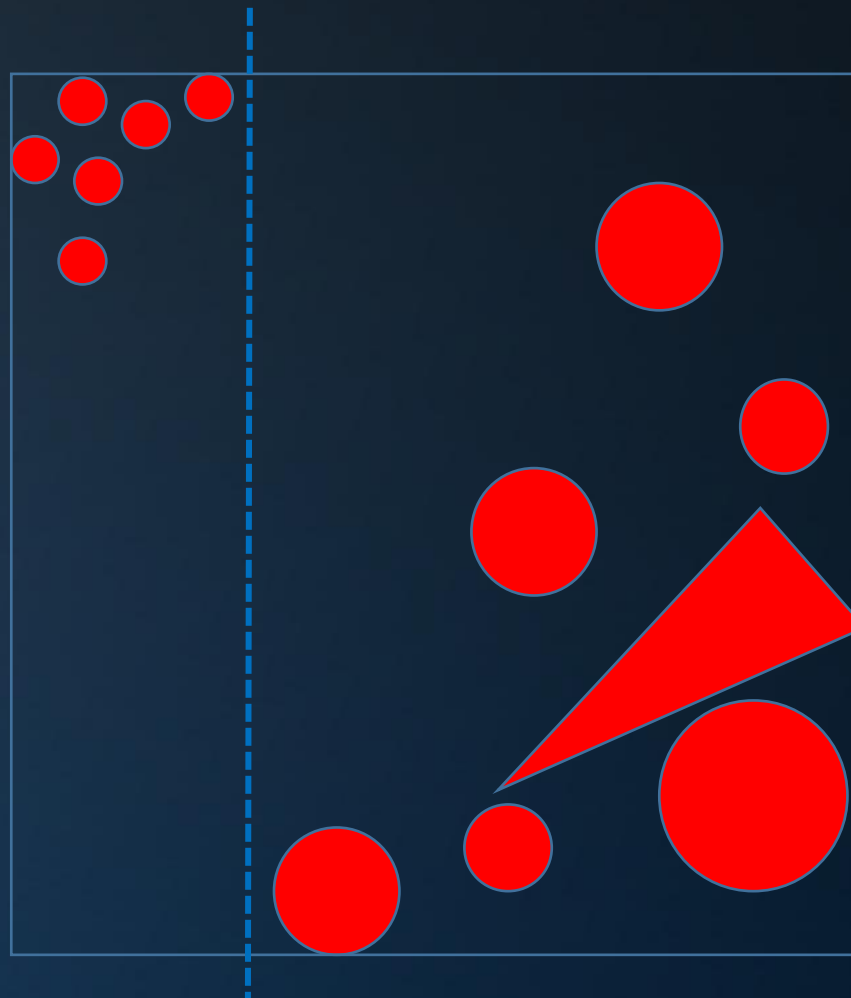


# Better BVHs

```

ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 1.0f - 0.5f * nnt)
    {
        nt = nt / nc, ddn = ddn * nc;
        cos2t = 1.0f - nnt * nnt;
        D, N );
    }
    if (a = nt - nc, b = nt * nc, c = nt * nc)
    {
        at Tr = 1 - (R0 + (1 - R0) * cos2t);
        (Tr) R = (D * nnt - N * (ddn * cos2t));
    }
    E * diffuse;
    = true;
    -
    refl + refr)) && (depth < MAXDEPTH)
    {
        D, N );
        refl * E * diffuse;
        = true;
    }
    MAXDEPTH)
    survive = SurvivalProbability( diffuse );
    estimation - doing it properly, closely following Small's
    if;
    radiance = SampleLight( &rand, I, &L, &light, &N );
    e.x + radiance.y + radiance.z) > 0) && (depth < MAXDEPTH)
    {
        w = true;
        at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
        at3 factor = diffuse * INVPI;
        at weight = Mis2( directPdf, brdfPdf );
        at cosThetaOut = dot( N, L );
        E * ((weight * cosThetaOut) / directPdf) * (radiance
    }
    random walk - done properly, closely following Small's
    vive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;

```



# Better BVHs

## What Are We Trying To Solve?

A BVH is used to reduce the number of ray/primitive intersections.

But: it introduces new intersections.

The ideal BVH minimizes:

- # of ray / primitive intersections
- # of ray / node intersections.



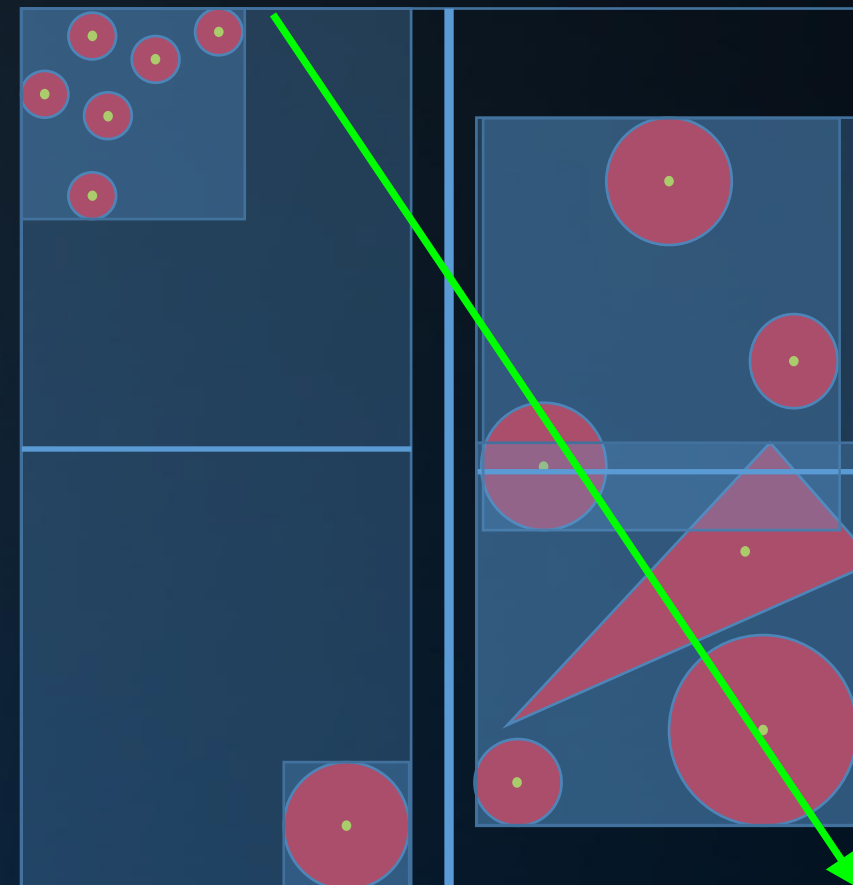
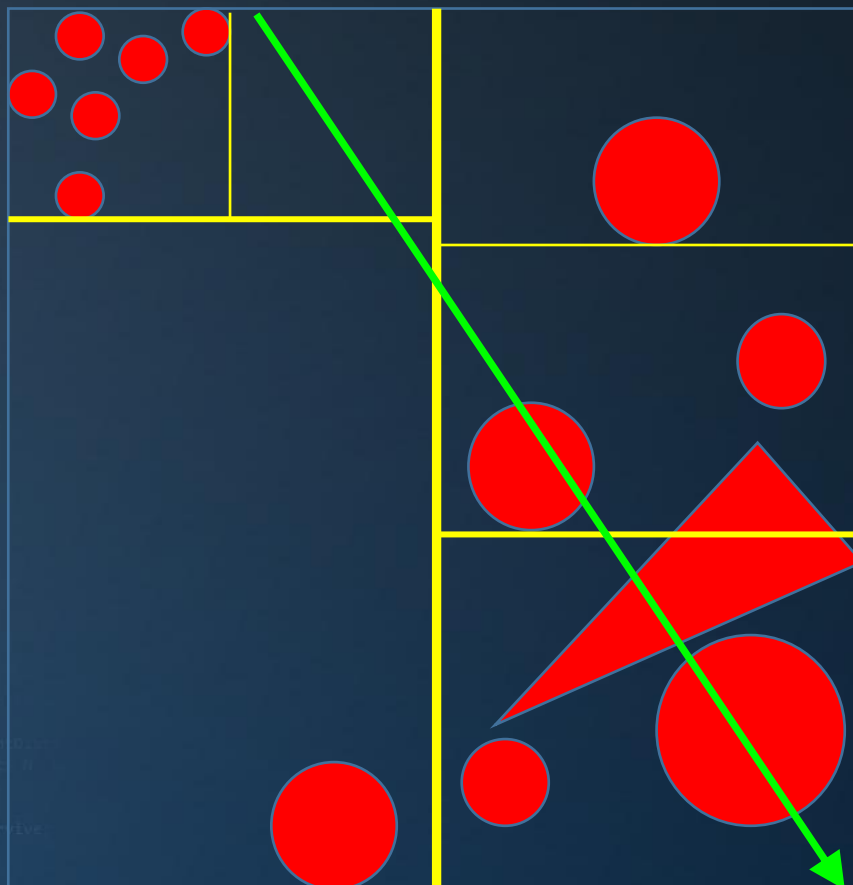


# Better BVHs

```

ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0.25)
    {
        nt = nt / nc; ddn = ddn * nc;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    {
        at a = nt - nc, b = nt * nc;
        at Tr = 1 - (R0 + (1 - R0) * a);
        (Tr) R = (D * nnt - N * (ddn * a));
    }
    E * diffuse;
    = true;
    {
        refl + refr)) && (depth < MAXDEPTH)
    {
        D, N );
        refl * E * diffuse;
        = true;
    }
    MAXDEPTH)
    survive = SurvivalProbability( diffuse );
    estimation - doing it properly, closely
    if;
    radiance = SampleLight( &rand, I, &L, &align );
    e.x + radiance.y + radiance.z) > 0) && (depth < MAXDEPTH)
    {
        w = true;
        at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
        at3 factor = diffuse * INVPI;
        at weight = Mis2( directPdf, brdfPdf );
        at cosThetaOut = dot( N, L );
        E * ((weight * cosThetaOut) / directPdf) * (radiance
    }
    random walk - done properly, closely following Small's
    (survive)
    {
        at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
        survive;
        pdf;
        n = E * brdf * (dot( N, R ) / pdf);
        ion = true;
    }
}

```



# Better BVHs

## BVH versus kD-tree

The BVH better encapsulates geometry.

➔ This reduces the chance of a ray hitting a node.

➔ This is all about probabilities!

*What is the probability of a ray hitting a random triangle?*

*What is the probability of a ray hitting a random node?*

This probability is proportional to surface area.





# Better BVHs



Route 1: 10% up-time, \$1000 fine



Route 2: 100% up-time, \$100 fine





# Better BVHs

## Optimal Split Plane Position

The ideal split minimizes the *expected cost* of a ray intersecting the resulting nodes.

This expected cost is based on:

- Number of primitives that will have to be intersected
- Probability of this happening

The cost of a split is thus:

$$A_{left} * N_{left} + A_{right} * N_{right}$$



# Better BVHs

## Optimal Split Plane Position

The ideal split minimizes the *expected cost* of a ray intersecting the resulting nodes.

This expected cost is based on:

- Number of primitives that will have to be intersected
- Probability of this happening

The cost of a split is thus:

$$A_{left} * N_{left} + A_{right} * N_{right}$$



# Better BVHs

## Optimal Split Plane Position

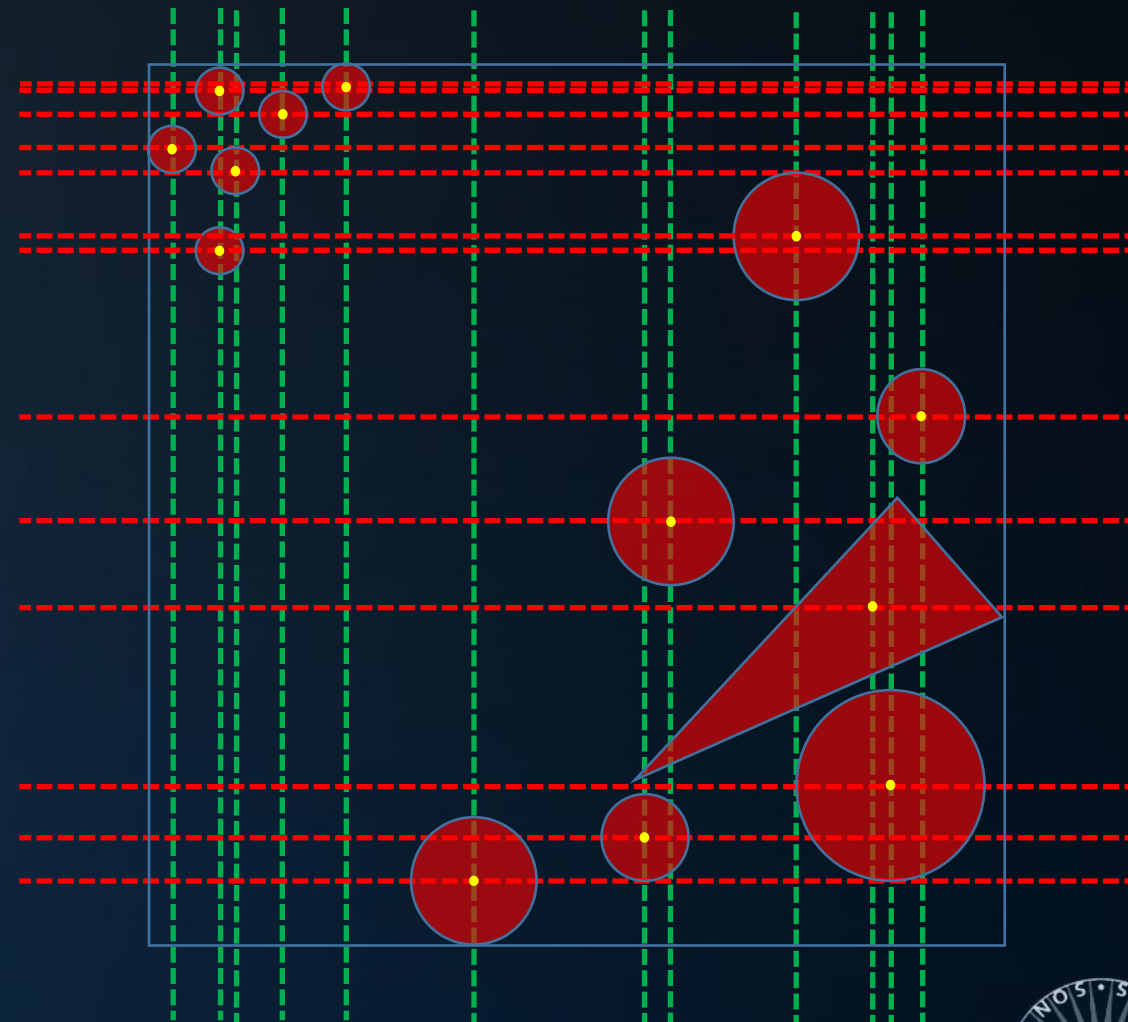
Which positions do we consider?

*Object subdivision may happen over  $x$ ,  $y$  or  $z$  axis.*

*The cost function is constant between primitive centroids.*

➔ For  $N$  primitives:  $3(N - 1)$  possible locations

➔ For a 2-level tree:  $(3(N - 1))^2$  configurations





# Better BVHs

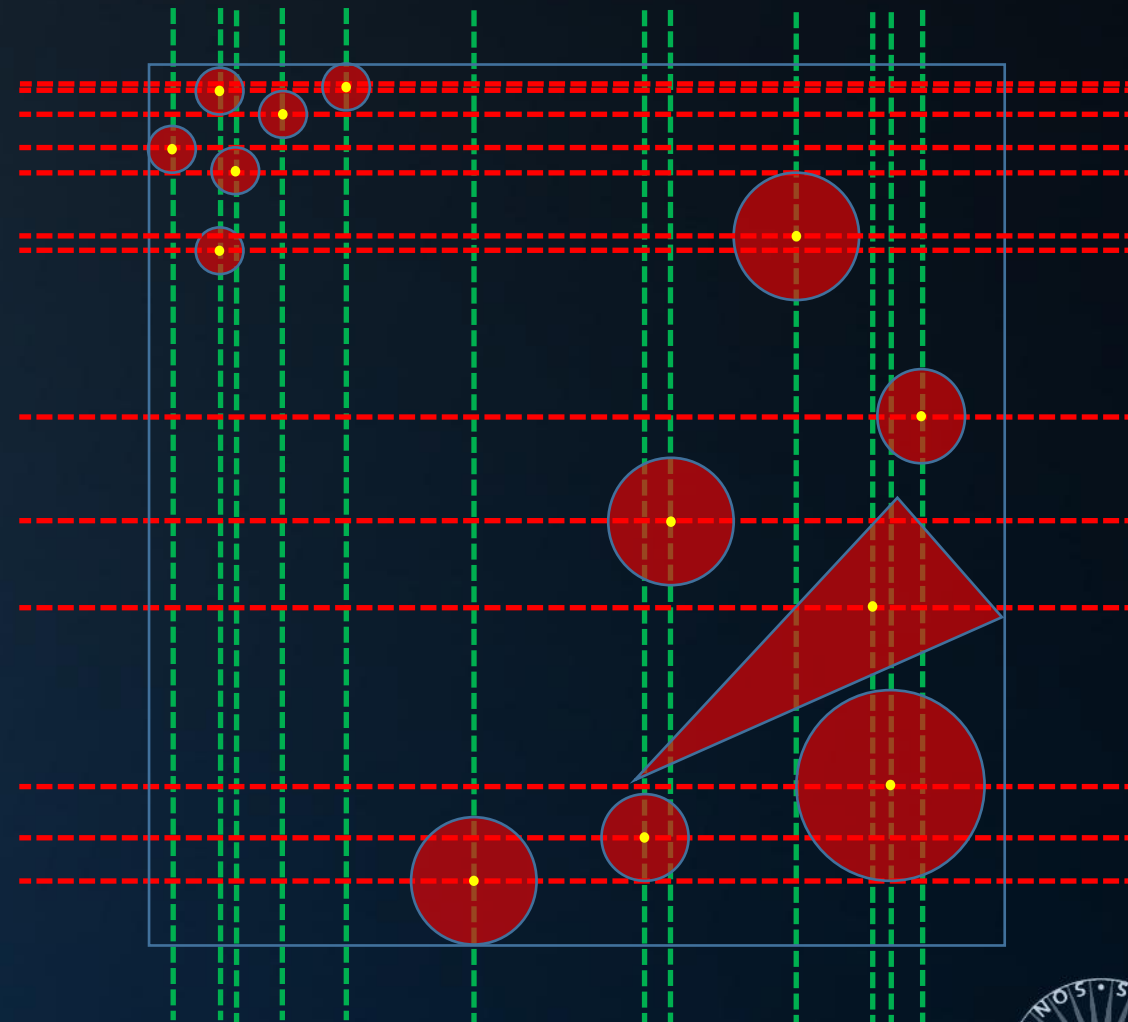
## SAH and Termination

A split is ‘not worth it’ if it doesn’t yield a cost lower than the cost of the parent node, i.e.:

$$A_{left} * N_{left} + A_{right} * N_{right} \geq A * N$$

This provides us with a natural and optimal termination criterion.

(and it solves the problem of the Bad Artist)



# Better BVHs

## Optimal Split Plane Position

The *surface area heuristic* (SAH) is applied in a greedy manner\*.

\*. Heuristics for Ray Tracing using Space Subdivision, MacDonald & Booth, 1990.



# Better BVHs

## Optimal Split Plane Position

## Comparing naïve versus SAH:

- SAH will cut #intersections in half;
- expect ~2x better performance.

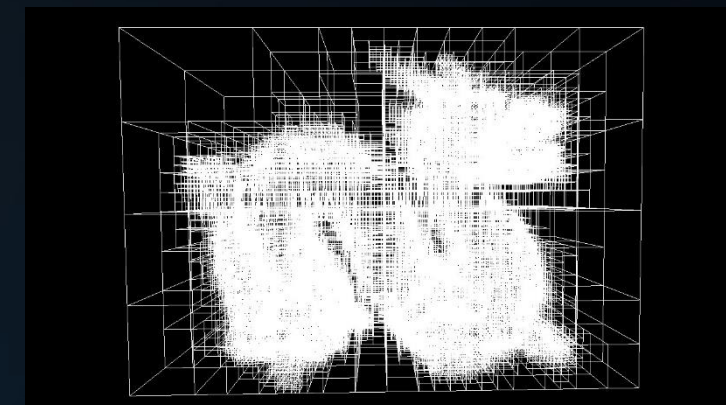
## SAH & kD-trees:

- Same scheme applies.

```

ics
& (depth < MAXDEPTH)
{
    // Inside?
    if (inside ? 1 : 0)
    {
        // Inside
        nt = nt / nc; ddn = ddn * nc;
        // Inside
        cos2t = 1.0f - nnt * nnt;
        // Inside
        D, N );
    }
    // Inside
    {
        // Inside
        at a = nt - nc, b = nt + nc;
        // Inside
        at Tr = 1 - (R0 + (1 - R0) * nc);
        // Inside
        Tr) R = (D * nnt - N * (ddn * nc));
    }
    // Inside
    {
        // Inside
        E * diffuse;
        // Inside
        = true;
    }
    // Inside
    {
        // Inside
        refl + refr)) && (depth < MAXDEPTH)
    {
        // Inside
        D, N );
        // Inside
        refl * E * diffuse;
        // Inside
        = true;
    }
    // Inside
    {
        // Inside
        MAXDEPTH)
    {
        // Inside
        survive = SurvivalProbability( diffuse );
        // Inside
        estimation - doing it properly, closely following
        // Inside
        if;
        // Inside
        radiance = SampleLight( &rand, I, &L, &light );
        // Inside
        e.x + radiance.y + radiance.z) > 0) && (depth < MAXDEPTH)
    {
        // Inside
        w = true;
        // Inside
        at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
        // Inside
        at3 factor = diffuse * INVPI;
        // Inside
        at weight = Mis2( directPdf, brdfPdf );
        // Inside
        at cosThetaOut = dot( N, L );
        // Inside
        E * ((weight * cosThetaOut) / directPdf) * (radiance
    }
    // Inside
    {
        // Inside
        random walk - done properly, closely following
        // Inside
        survive)
    }
    // Inside
    {
        // Inside
        at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
        // Inside
        survive;
        // Inside
        pdf;
        // Inside
        n = E * brdf * (dot( N, R ) / pdf);
        // Inside
        sion = true;
    }
}

```



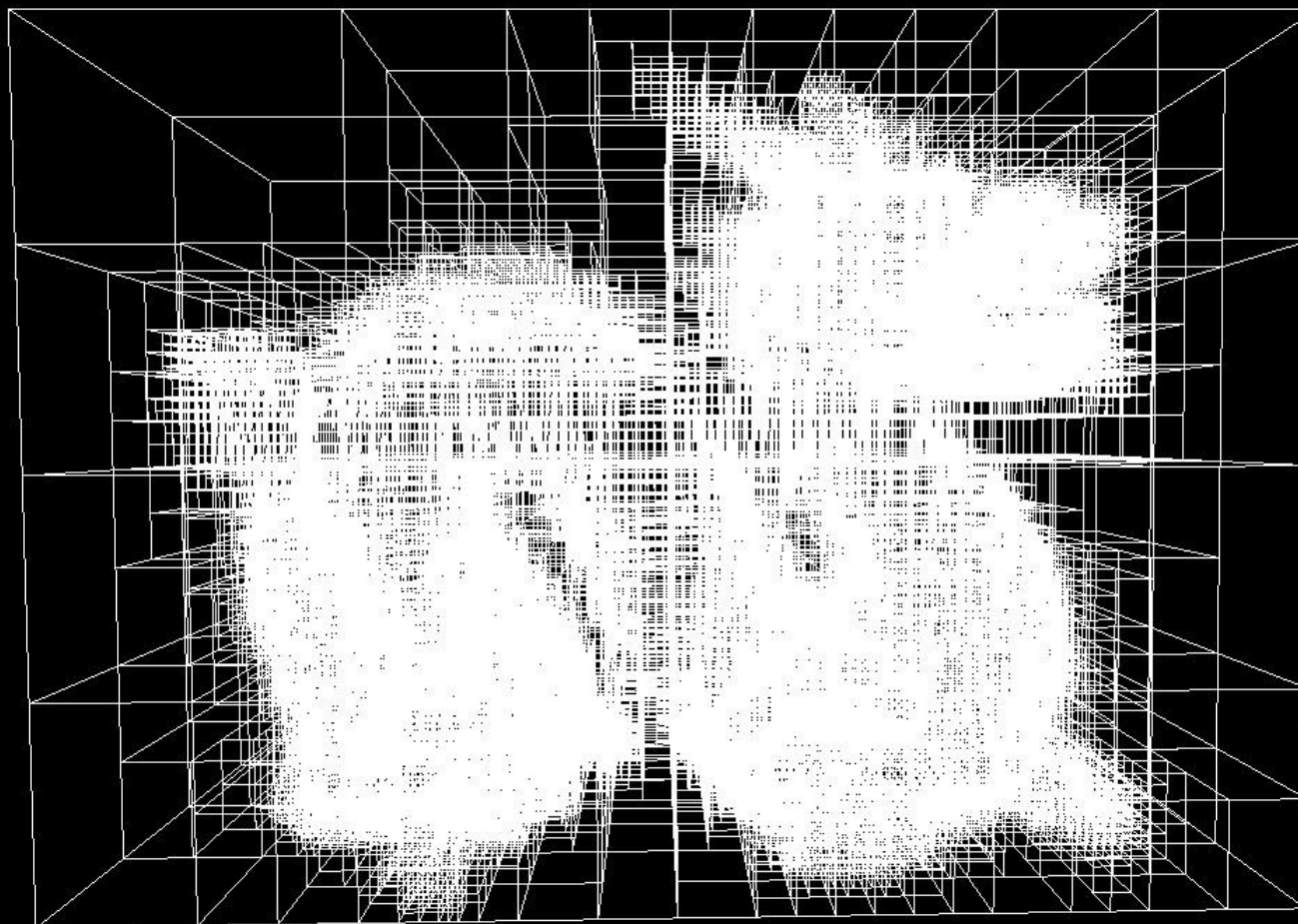


# Better BVHs

```

ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0)
    {
        nt = nt / nc; ddn = ddn / nc;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    {
        at a = nt - nc, b = nt + nc;
        at Tr = 1 - (R0 + (1 - R0) * a);
        Tr) R = (D * nnt - N * (ddn *
    }
    {
        E * diffuse;
        = true;
    }
    {
        refl + refr)) && (depth < MAXDEPTH)
    {
        D, N );
        refl * E * diffuse;
        = true;
    }
    {
        MAXDEPTH)
    {
        survive = SurvivalProbability( diffuse
        estimation - doing it properly, close
        df;
        radiance = SampleLight( &rand, I, &L,
        e.x + radiance.y + radiance.z) > 0) &&
    {
        w = true;
        at brdfPdf = EvaluateDiffuse( L, N ) *
        at3 factor = diffuse * INVPI;
        at weight = Mis2( directPdf, brdfPdf );
        at cosThetaOut = dot( N, L );
        E * ((weight * cosThetaOut) / directPdf
    }
    {
        random walk - done properly, closely following
        (survive)
    }
    {
        at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf
        survive;
        pdf;
        n = E * brdf * (dot( N, R ) / pdf);
        sion = true;
    }

```



## Median Split



# Better BVHs

```

ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0)
    {
        nt = nt / nc; ddn = ddn * ddn;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    {
        at a = nt - nc, b = nt + nc;
        at Tr = 1 - (R0 + (1 - R0) *
        Tr) R = (D * nnt - N * (ddn
    }
    E * diffuse;
    = true;
    {
        defl + refr)) && (depth < MAXDEPTH)
    {
        D, N );
        refl * E * diffuse;
        = true;
    }
    MAXDEPTH)
    survive = SurvivalProbability( diffuse
    estimation - doing it properly, close
    df;
    radiance = SampleLight( &rand, I, &L,
    e.x + radiance.y + radiance.z) > 0) &&
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) *
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPe

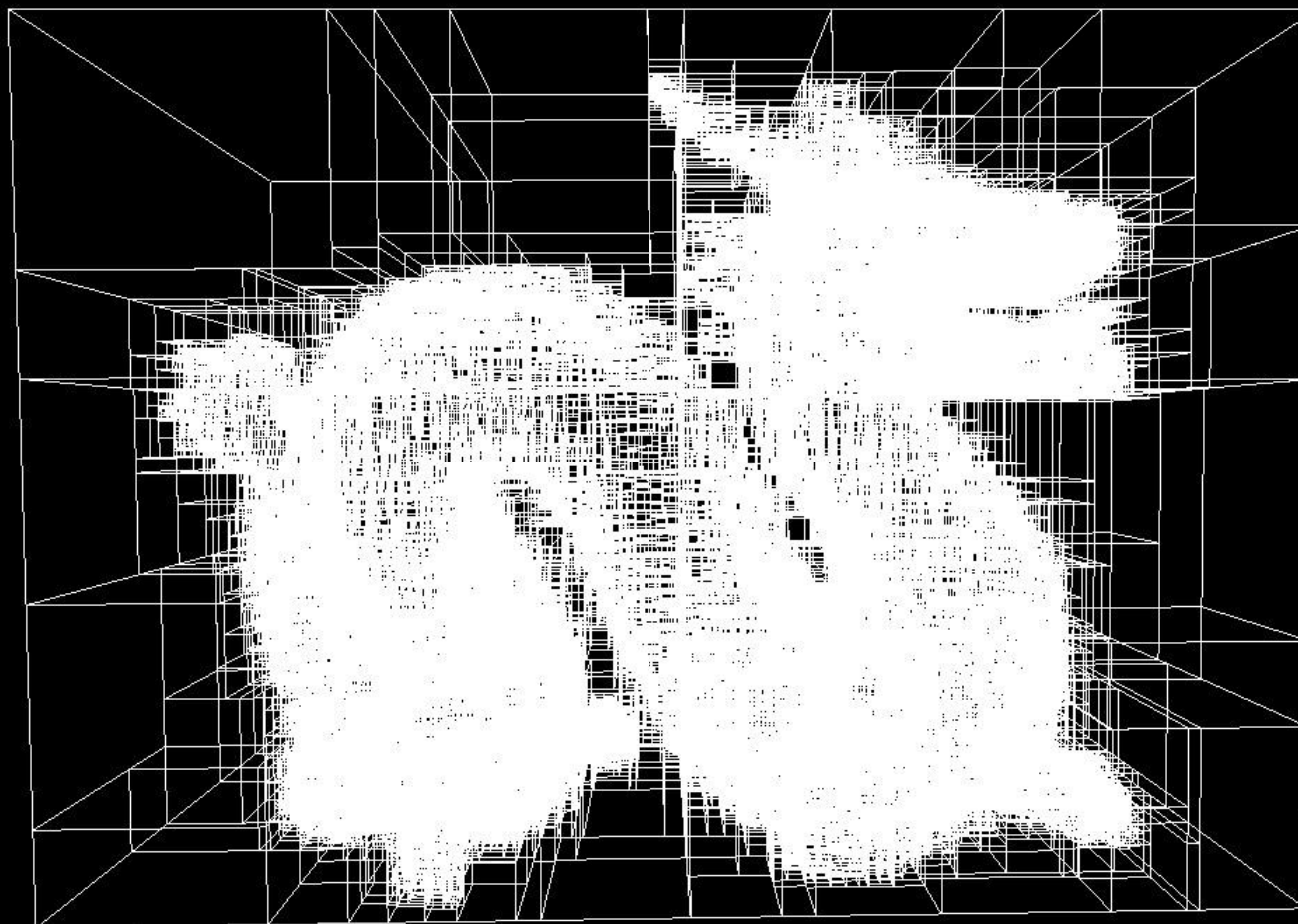
```

random walk - done properly, closely following the path  
 (survive)

```

{
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
}

```

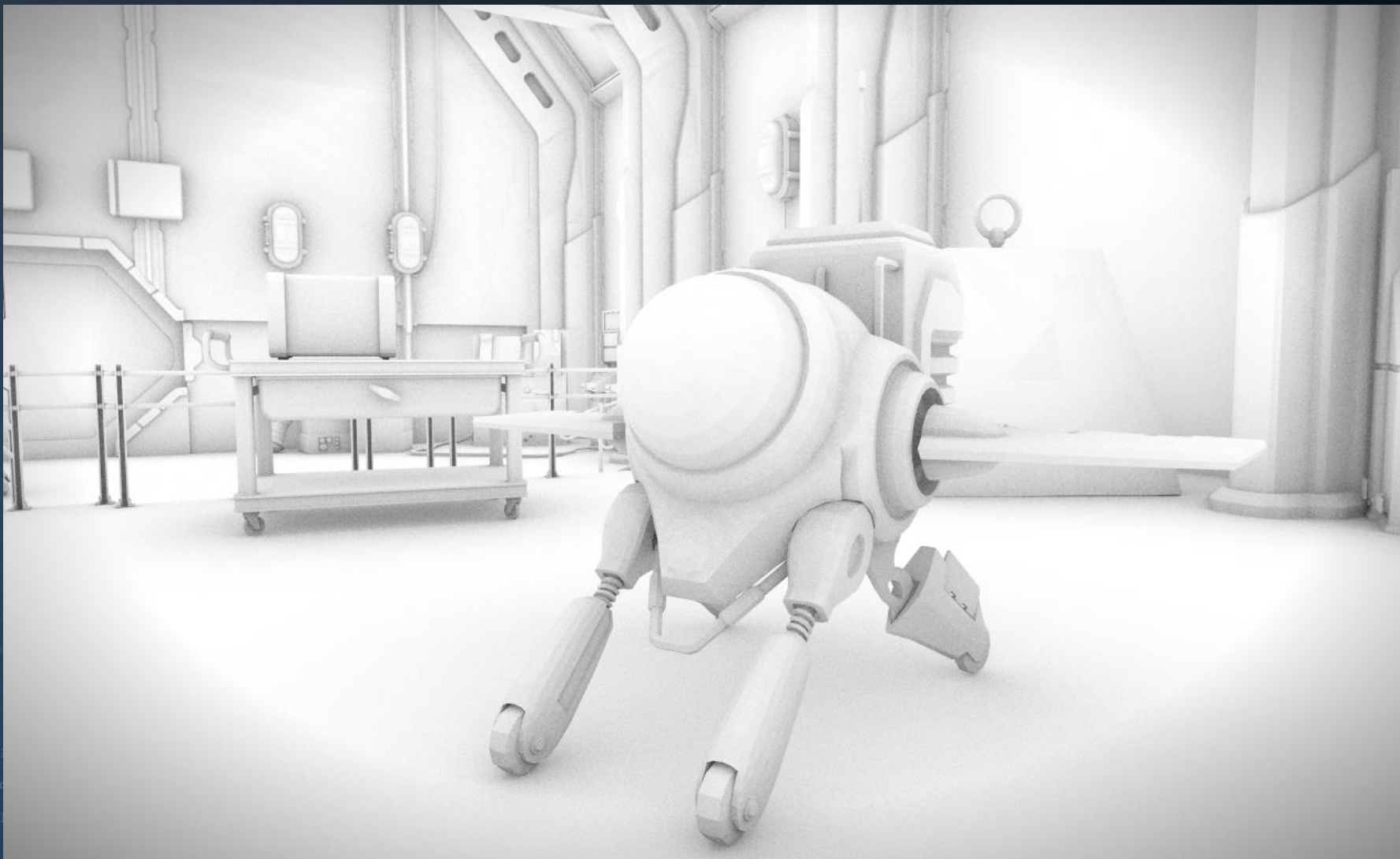


## Surface Area Heuristic



# Better BVHs

```
ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0)
    {
        nt = nt / nc; ddn = ddn * ddn;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    at a = nt - nc, b = nt + nc;
    at Tr = 1 - (R0 + (1 - R0) * ddn);
    Tr) R = (D * nnt - N * (ddn *
    E * diffuse;
    = true;
    refl + refr)) && (depth < MAXDEPTH)
    D, N );
    refl * E * diffuse;
    = true;
    MAXDEPTH)
    survive = SurvivalProbability( diffuse
    estimation - doing it properly, close
    df;
    radiance = SampleLight( &rand, I, &L,
    e.x + radiance.y + radiance.z) > 0) &&
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) *
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPe
    random walk - done properly, closely fo
    vive)
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
```



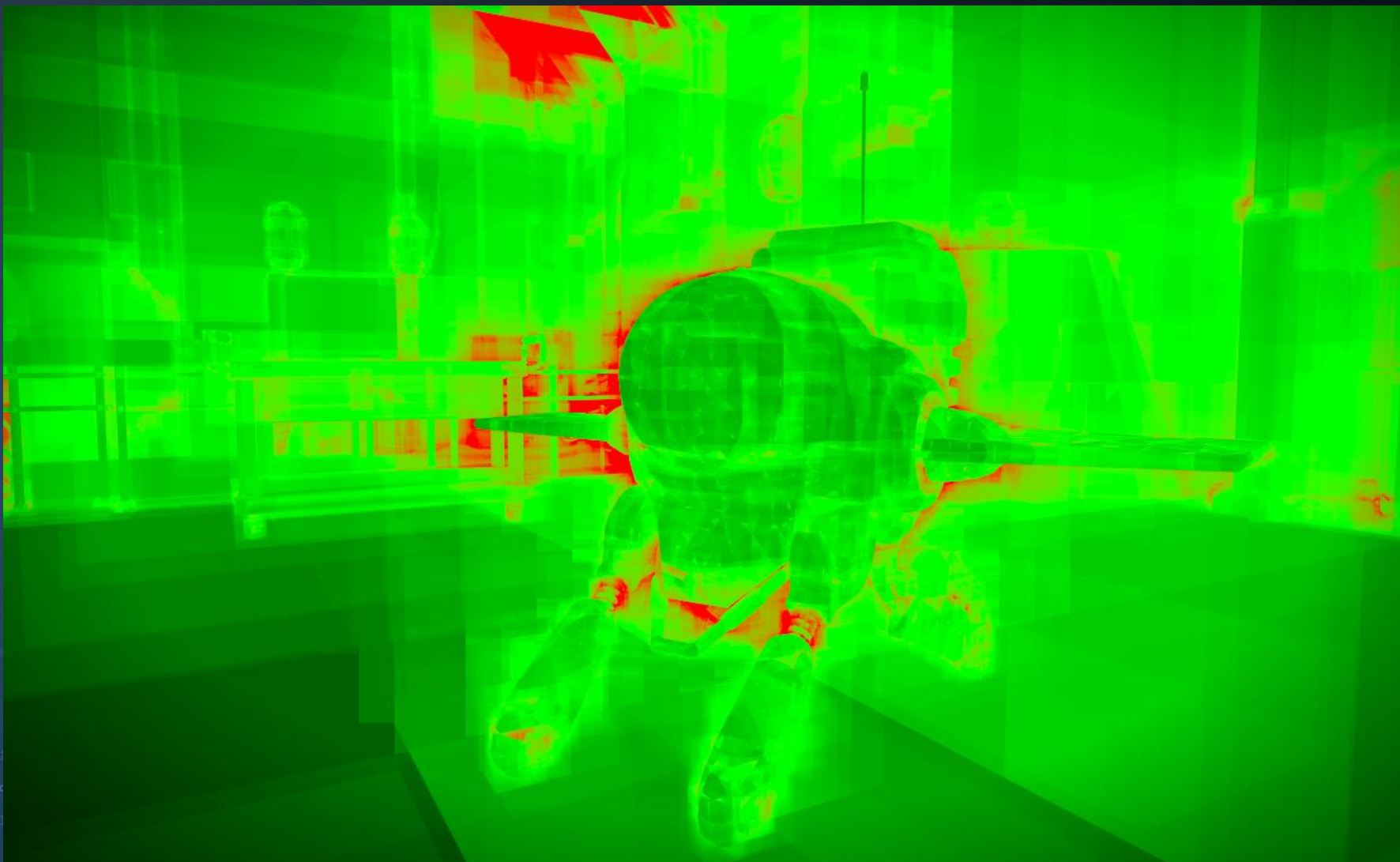


# Better BVHs

```

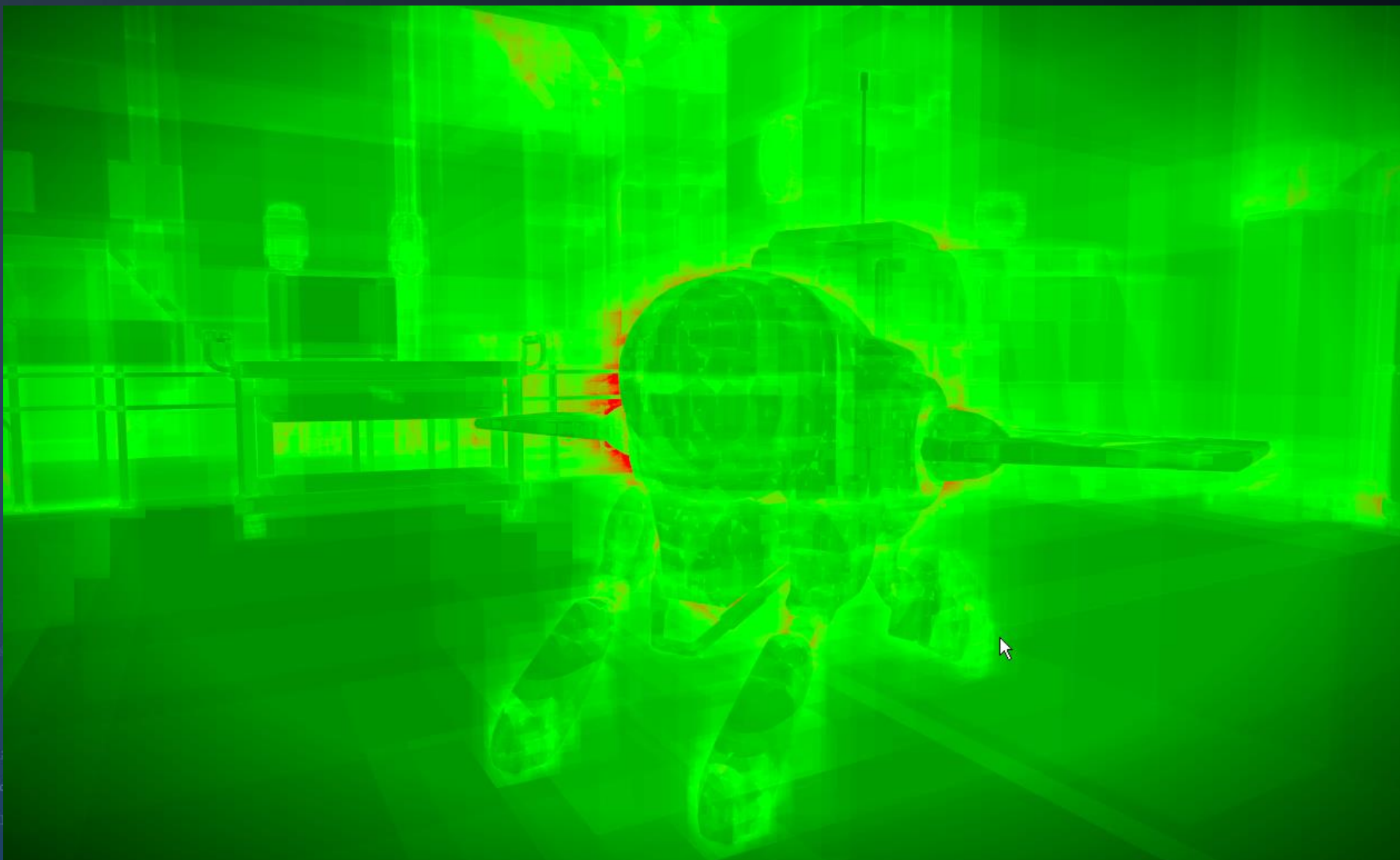
rics
& (depth < MAXDEPTH)
{
    if (nt < inside ? 1.0f : 0.5f)
    {
        nt = nt / nc; ddn = dot(N, L);
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    else
    {
        at a = nt - nc, b = nt + nc;
        at Tr = 1 - (R0 + (1 - R0) * ddn);
        Tr) R = (D * nnt - N * (ddn > 0) ? a : b);
    }
    E * diffuse;
    = true;
    = refl + refr) && (depth < MAXDEPTH)
    {
        D, N );
        refl * E * diffuse;
        = true;
    }
    MAXDEPTH)
    survive = SurvivalProbability( diffuse,
    estimation - doing it properly, close
    if;
    radiance = SampleLight( &rand, I, &L,
    e.x + radiance.y + radiance.z) > 0) &&
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) *
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPe
    random walk - done properly, closely fo
    vive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;

```



# Better BVHs

```
rics
& (depth < MAXDEPTH)
{
    if (nt < nc) {
        nt = nt / nc; ddn = ddn * nc;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    else {
        at a = nt - nc, b = nt + nc;
        at Tr = 1 - (R0 + (1 - R0) * a);
        (Tr) R = (D * nnt - N * (ddn *
    )
    E * diffuse;
    = true;
    -
    refl + refr)) && (depth < MAXDEPTH)
    D, N );
    refl * E * diffuse;
    = true;
    MAXDEPTH)
    survive = SurvivalProbability( diffuse
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    df;
    radiance = SampleLight( &rand, I, &L,
    e.x + radiance.y + radiance.z) > 0) &&
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    at3 factor = diffuse * INVPI;
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    random walk - done properly, closely fo
    vive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
}
```



```

ics
& (depth < MAXDEPTH) {
    // Inside?
    if (inside ? 1 : 0) {
        nt = nt / nc; ddn = ddn * nc;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    // Outside?
    at a = nt - nc, b = nt + nc;
    at Tr = 1 - (R0 + (1 - R0) * ddn);
    (Tr) R = (D * nnt - N * (ddn > 0 ? 1 : -1));
    // Diffuse
    E * diffuse;
    = true;
    // Refractive
    refl + refr)) && (depth < MAXDEPTH) {
        D, N );
        refl * E * diffuse;
        = true;
    }
    // Survival
    MAXDEPTH)
    survive = SurvivalProbability( diffuse );
    // Estimation
    estimation - doing it properly, closely following
    if;
    radiance = SampleLight( &rand, I, &L, &lightPdf );
    e.x + radiance.y + radiance.z) > 0) && (depth < MAXDEPTH) {
        w = true;
        at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
        at3 factor = diffuse * INVPI;
        at weight = Mis2( directPdf, brdfPdf );
        at cosThetaOut = dot( N, L );
        E * ((weight * cosThetaOut) / directPdf) * (radiance.x + radiance.y + radiance.z);
    }
    // Random walk
    random walk - done properly, closely following Small's
    (survive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    ion = true;
}

```

# Today's Agenda:

- Building Better BVHs
- Refitting
- Fast BVH Construction
- The Top-level BVH



# Refitting

## Summary of BVH Characteristics

A BVH provides significant freedom compared to e.g. a kD-tree:

- No need for a 1-to-1 relation between bounding boxes and primitives
- Bounding boxes may overlap
- Bounding boxes can be altered, as long as they fit in their parent box
- A BVH can be very bad but still valid

Some consequences / opportunities:

- We can rebuild part of a BVH
- We can combine two BVHs into one
- We can *refit* a BVH





# Refitting

## Refitting

Q: What happens to the BVH of a tree model, if we make it bend in the wind?

A: Likely, only bounds will change; the topology of the BVH will be the same (or at least similar) in each frame.

Refitting:

*Updating the bounding boxes stored in a BVH to match changed primitive coordinates.*



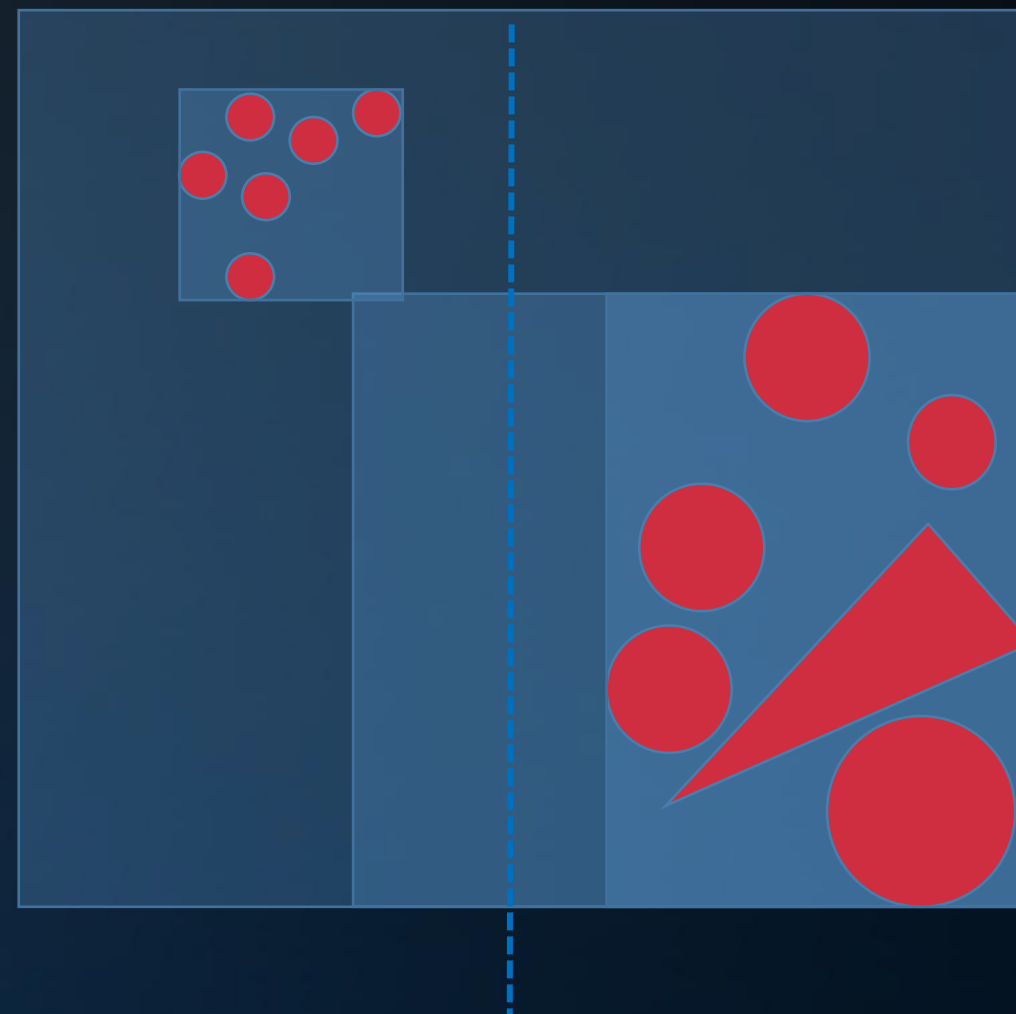
# Refitting

## Refitting

*Updating the bounding boxes stored in a BVH to match changed primitive coordinates.*

### Algorithm:

1. For each leaf, calculate the bounds over the primitives it represents
2. Update parent bounds



```

ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0)
    {
        nt = nt / nc; ddn = ddn / nc;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
}

at a = nt - nc, b = nt * nc;
at Tr = 1 - (R0 + (1 - R0) *
Tr) R = (D * nnt - N * (ddn
E * diffuse;
= true;

efl + refr)) && (depth < MAXDEPTH)
D, N );
refl * E * diffuse;
= true;

MAXDEPTH)
survive = SurvivalProbability( diffuse );
estimation - doing it properly, closely
if;
radiance = SampleLight( &rand, I, &L, &light;
e.x + radiance.y + radiance.z) > 0) && (depth <
w = true;
at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
at3 factor = diffuse * INVPI;
at weight = Mis2( directPdf, brdfPdf );
at cosThetaOut = dot( N, L );
E * ((weight * cosThetaOut) / directPdf) * (radiance
random walk - done properly, closely following Small's
vive)
;
at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
survive;
pdf;
n = E * brdf * (dot( N, R ) / pdf);
sion = true;

```

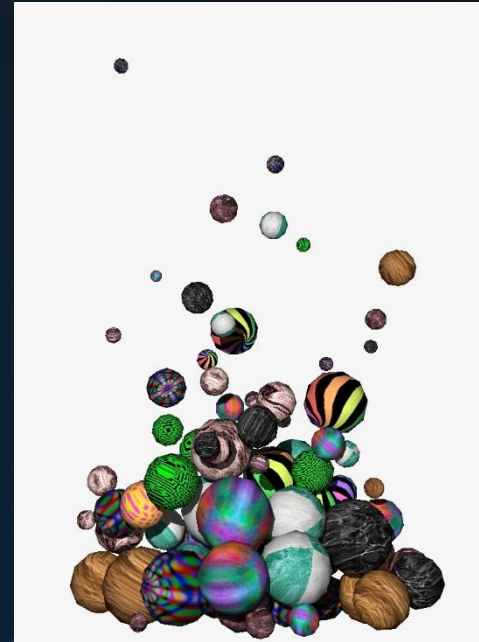
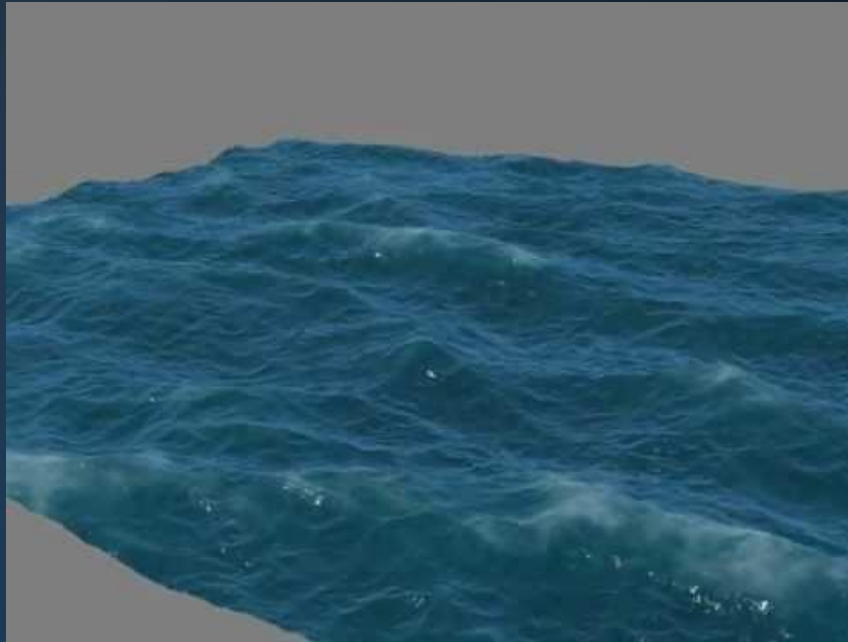


# Refitting

## Refitting - Suitability

```
ics  
& (depth < MAXDEPTH)
```

```
t = inside  
nt = nt  
os2t = 1  
D, N );  
)  
at a = nt  
at Tr = a  
Tr) R = (E * diffuse  
= true;  
efl + ref  
D, N );  
refl * E  
= true;  
MAXDEPTH)  
survive =  
estimat  
df;  
radiance  
e.x + rad  
w = true;  
at brdfPdf = EvaluateDiffuse( L, N ) * PdfDiff  
at3 factor = diffuse * INVPI;  
at weight = Mis2( directPdf, brdfPdf );  
at cosThetaOut = dot( N, L );  
E * ((weight * cosThetaOut) / directPdf) * (radiance  
random walk - done properly, closely following Saito's  
ive)  
;  
at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );  
survive;  
pdf;  
n = E * brdf * (dot( N, R ) / pdf);  
sion = true;
```



# Refitting

## Refitting – Practical

```

ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0)
    {
        nt = nt / nc; ddn = ddn * nc;
        pos2t = 1.0f - nnt * ddn;
        D, N );
    }
    at a = nt - nc; b = nt * nc;
    at Tr = 1 - (R0 + (1 - R0) *
    Tr) R = (D * nnt - N * (ddn
    E * diffuse;
    = true;
    -
    efl + refr)) && (depth < MAXDEPTH)
    D, N );
    refl * E * diffuse;
    = true;
    MAXDEPTH)
    survive = SurvivalProbability( diffuse );
    estimation - doing it properly, closely following
    if;
    radiance = SampleLight( &rand, I, &L, &align,
    e.x + radiance.y + radiance.z) > 0) && (depth <
    v = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiance
    random walk - done properly, closely following Small's
    vive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
    
```



↑  
↑  
Level 1  
Root node

BVH node array

Order of nodes in the node array:

*We will never find the parent of node  $X$  at a position greater than  $X$ .*

Therefore:

```

for( int i = N-1; i >= 0; i-- )
    nodeArray[i].AdjustBounds();
    
```





```

ics
& (depth < MAXDEPTH) {
    if (nt > nc) {
        nt = inside ? 1 + 1.25 * log(nc / nt) : 1;
        nt = nt / nc; ddn = ddn * nt;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    at a = nt - nc, b = nt * nc;
    at Tr = 1 - (R0 + (1 - R0) * b);
    (Tr) R = (D * nnt - N * (ddn * nnt - 1));
    E * diffuse;
    = true;
    -
    refl + refr)) && (depth < MAXDEPTH) {
        D, N );
        refl * E * diffuse;
        = true;
    }
    MAXDEPTH)
    survive = SurvivalProbability( diffuse );
    estimation - doing it properly, closely following
    if;
    radiance = SampleLight( &rand, I, &L, &lightPdf );
    e.x + radiance.y + radiance.z) > 0) && (dot( N, L ) > 0) {
        w = true;
        at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
        at3 factor = diffuse * INVPI;
        at weight = Mis2( directPdf, brdfPdf );
        at cosThetaOut = dot( N, L );
        E * ((weight * cosThetaOut) / directPdf) * (radiance
    }
    random walk - done properly, closely following Small's
    vive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    ion = true;

```

# Today's Agenda:

- Building Better BVHs
- Refitting
- Fast BVH Construction
- The Top-level BVH



# Binning

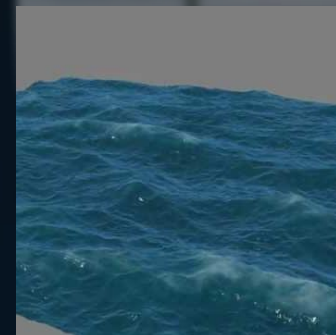
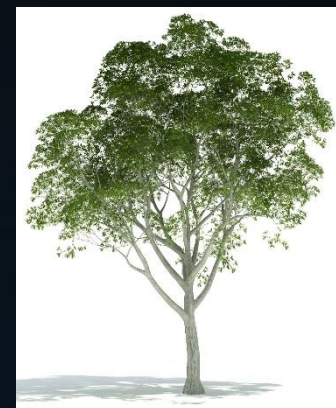
## Rapid BVH Construction

Refitting allows us to update hundreds of thousands of primitives in real-time. But what if topology changes significantly?

Rebuilding a BVH requires  $3N \log N$  split plane evaluations.

### Options:

1. Do not use SAH (significantly lower quality BVH)
2. Do not evaluate all 3 axes (minor degradation of BVH quality)
3. Make split plane selection independent of  $N$

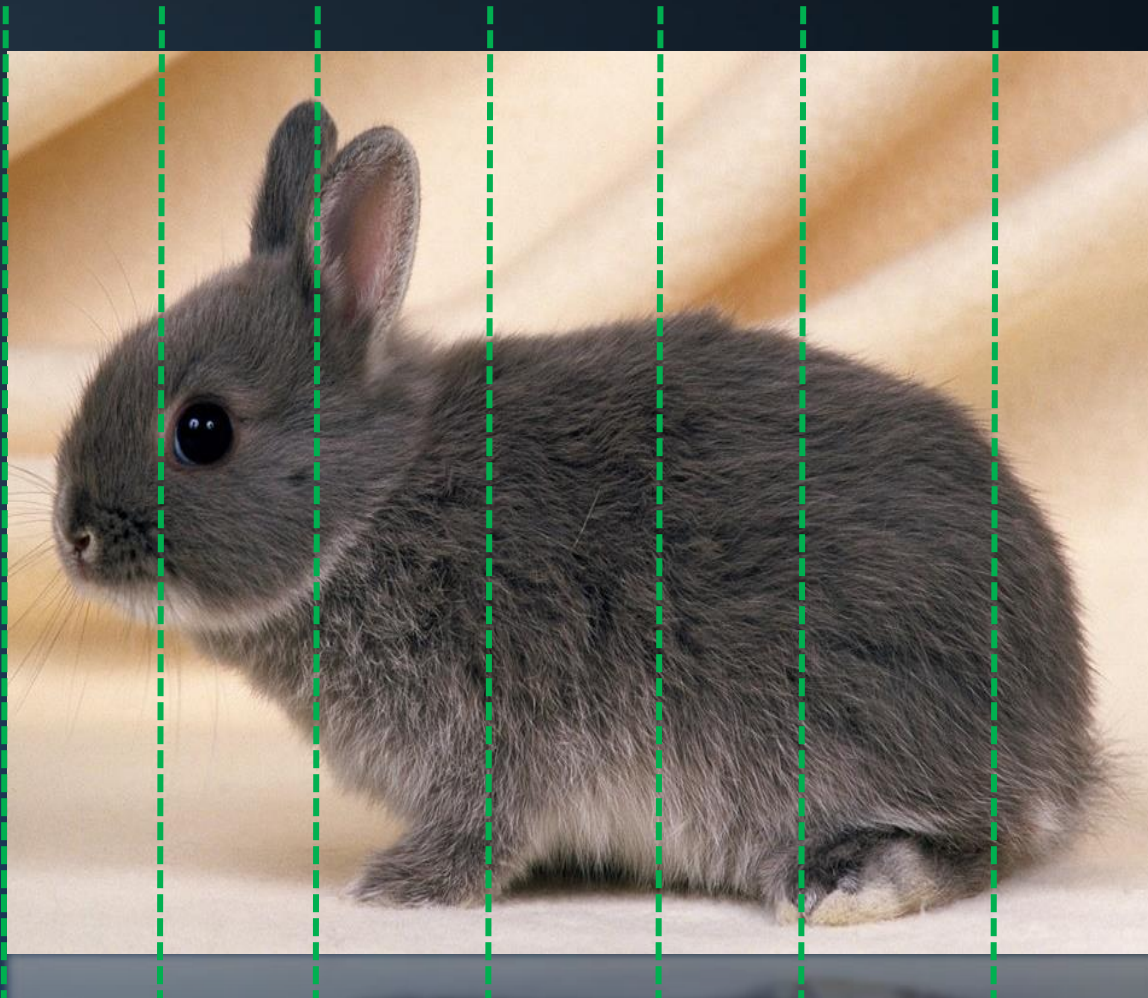


# Binning

```

ics
& (depth < MAXDEPTH)
{
    if (inside ? 1.0f : 0.0f)
    {
        nt = nt / nc; ddn = ddn * nc;
        cos2t = 1.0f - nnt * nnt;
        D, N );
    }
    else
    {
        at a = nt - nc, b = nt + nc;
        at Tr = 1 - (R0 + (1 - R0) * rand());
        Tr) R = (D * nnt - N * (ddn * cos2t));
    }
    E * diffuse;
    = true;
    =
    refl + refr)) && (depth < MAXDEPTH)
    {
        D, N );
        refl * E * diffuse;
        = true;
    }
    MAXDEPTH)
    survive = SurvivalProbability( diffuse );
    estimation - doing it properly, closely following Small's
    df;
    radiance = SampleLight( &rand, I, &L, &align, &pdf );
    e.x + radiance.y + radiance.z) > 0) && (depth < MAXDEPTH)
    {
        w = true;
        at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
        at3 factor = diffuse * INVPI;
        at weight = Mis2( directPdf, brdfPdf );
        at cosThetaOut = dot( N, L );
        E * ((weight * cosThetaOut) / directPdf) * (radiance
    }
    random walk - done properly, closely following Small's
    vive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;

```





# Binning

## Binned BVH Construction\*

Binned construction:

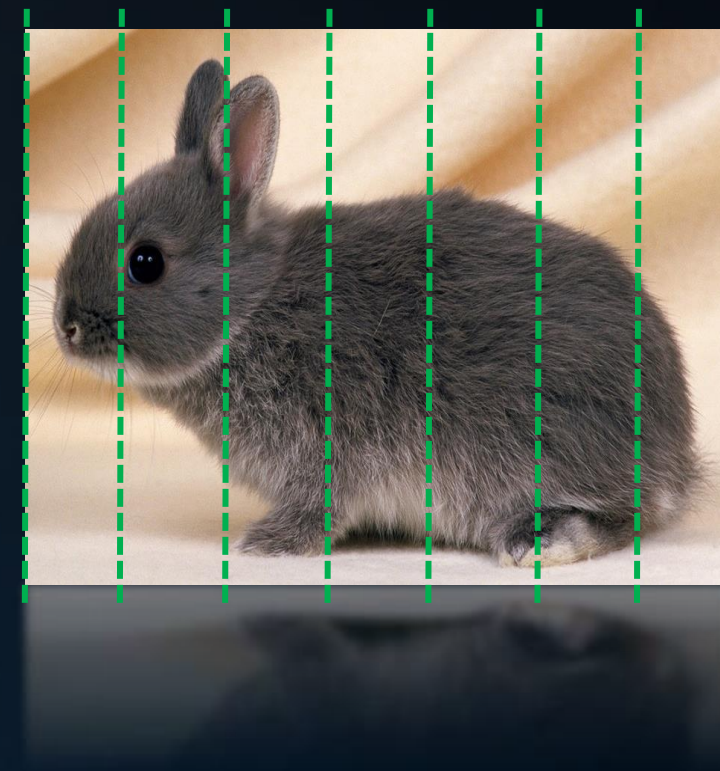
*Evaluate SAH at  $N$  discrete intervals.*

```

ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0)
    {
        nt = nt / nc; ddn = ddn * nc;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    {
        at a = nt - nc, b = nt + nc;
        at Tr = 1 - (R0 + (1 - R0) * a);
        Tr) R = (D * nnt - N * (ddn *
    }
    E * diffuse;
    = true;
    -
    refl + refr)) && (depth < MAXDEPTH)
    {
        D, N );
        refl * E * diffuse;
        = true;
    }
    MAXDEPTH)
    survive = SurvivalProbability( diffuse );
    estimation - doing it properly, closely following
    if;
    radiance = SampleLight( &rand, I, &L, &lightDir );
    e.x + radiance.y + radiance.z) > 0) && (depth <
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiance
    random walk - done properly, closely following Small's
    vive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, R
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    ion = true;

```

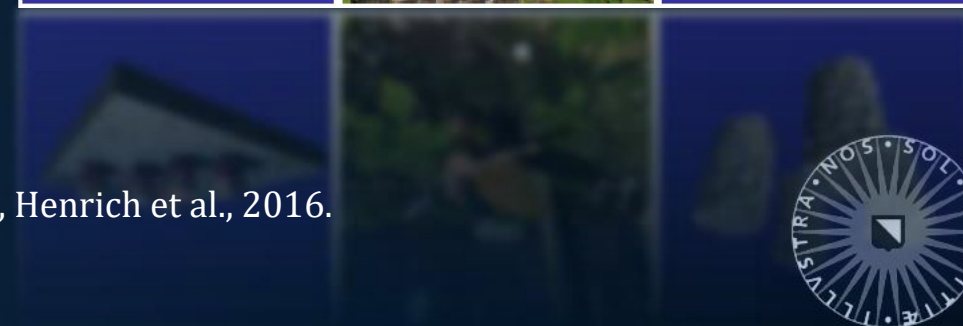
\*: On fast Construction of SAH-based Bounding Volume Hierarchies, Wald, 2007





## Binned BVH Construction

472ms 7.88M triangles (12 cores @ 2Ghz)\*.



\*: Parallel BVH Construction using Progressive Hierarchical Refinement, Henrich et al., 2016.



```

ics
& (depth < MAXDEPTH) {
    if (nt < nc) {
        nt = inside ? 1 + 1.2f * (1 - nt) : 1 - nt;
        nt = nt / nc; ddn = ddn * nt;
        cos2t = 1.0f - nnt * nnt;
        D, N );
    }
    // Russian roulette
    at a = nt - nc, b = nt * nc;
    at Tr = 1 - (R0 + (1 - R0) * sqrt(b));
    (Tr) R = (D * nnt - N * (ddn < 0));
    E * diffuse;
    = true;
    -
    refl + refr)) && (depth < MAXDEPTH) {
        D, N );
        refl * E * diffuse;
        = true;
    }
    MAXDEPTH)
    survive = SurvivalProbability( diffuse );
    estimation - doing it properly, closely following
    if;
    radiance = SampleLight( &rand, I, &L, &lightPdf );
    e.x + radiance.y + radiance.z) > 0) && (dot( N, L ) > 0) {
        w = true;
        at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
        at3 factor = diffuse * INVPI;
        at weight = Mis2( directPdf, brdfPdf );
        at cosThetaOut = dot( N, L );
        E * ((weight * cosThetaOut) / directPdf) * (radiance
    }
    random walk - done properly, closely following Small's
    (survive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
}

```

# Today's Agenda:

- Building Better BVHs
- Refitting
- Fast BVH Construction
- The Top-level BVH



# Top-level BVH

```
ics  
& (depth < MAXDEPTH)  
nt = inside ? 1 : 0;  
nt = nt / nc; ddn = dot(N, D);  
cos2t = 1.0f - nnt * ddn;  
D, N );  
0)  
at a = nt - nc, b = nt + nc;  
at Tr = 1 - (R0 + (1 - R0) * cosThetaOut);  
Tr) R = (D * nnt - N * (ddn * cosThetaOut));  
E * diffuse;  
= true;  
efl + refr)) && (depth < MAXDEPTH)  
D, N );  
refl * E * diffuse;  
= true;  
MAXDEPTH)  
survive = SurvivalProbability( diffuse, I, R );  
estimation - doing it properly, closely following  
df;  
radiance = SampleLight( &rand, I, &L, Align  
e.x + radiance.y + radiance.z ) > 0 ) && (depth < MAXDEPTH)  
w = true;  
at brdfPdf = EvaluateDiffuse( L, N ) * Psurf;  
at3 factor = diffuse * INVPI;  
at weight = Mis2( directPdf, brdfPdf );  
at cosThetaOut = dot( N, L );  
E * ((weight * cosThetaOut) / directPdf);  
random walk - done properly, closely following the path  
ive)  
at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );  
survive;  
pdf;  
n = E * brdf * (dot( N, R ) / pdf);  
sion = true;
```





# Top-level BVH

```
ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0)
    {
        nt = nt / nc; ddn = ddn * ddn;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    at a = nt - nc; b = nt * nc;
    at Tr = 1 - (R0 + (1 - R0) * cos2t);
    Tr) R = (D * nnt - N * (ddn * cos2t));
    E * diffuse;
    = true;
    refl + refr)) && (depth < MAXDEPTH)
    {
        D, N );
        refl * E * diffuse;
        = true;
    }
    MAXDEPTH)
    survive = SurvivalProbability( diffuse, I,
    estimation - doing it properly, closely
    df;
    radiance = SampleLight( &rand, I, &L, Align
    e.x + radiance.y + radiance.z) > 0) && (cos
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurf;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf);
    random walk - done properly, closely following
    (survive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf);
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
}
```





321 20:31 368

A B C D E

D  
86 m  
AMMO DEPOT

G  
19 m

30 45 60



GERMANY

DEFEND  
AMMO DEPOT

bkevend72 16426 HOLD

Aoi\_Fuuka

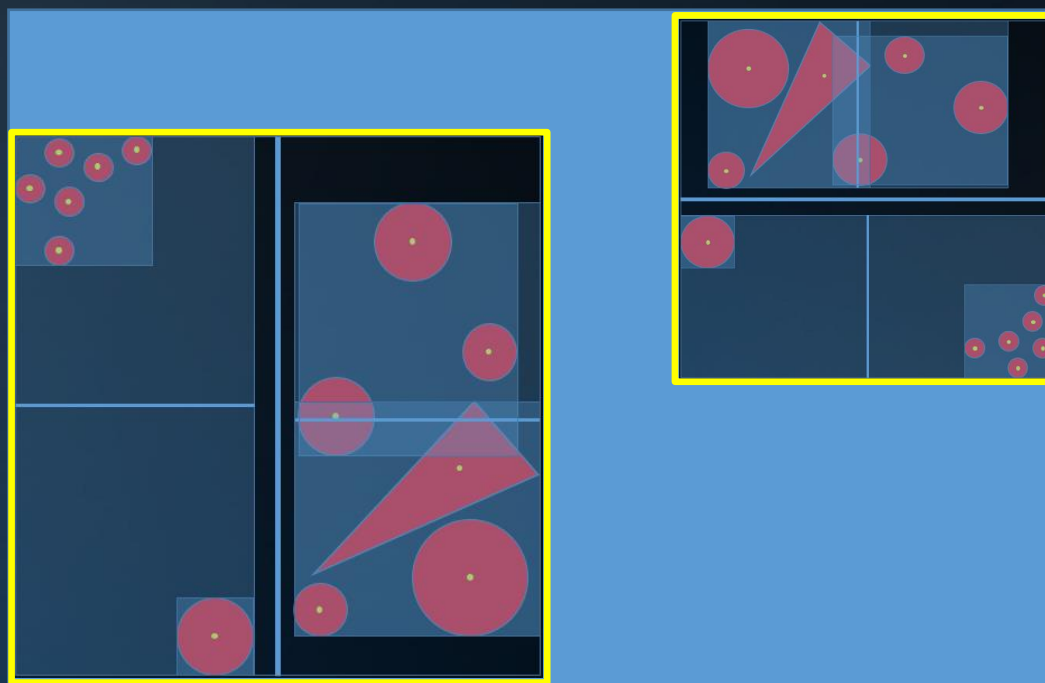
Din0Rock

rocktoni1

68 m 17 m

100 m 9 17 | 11

# Combining BVHs

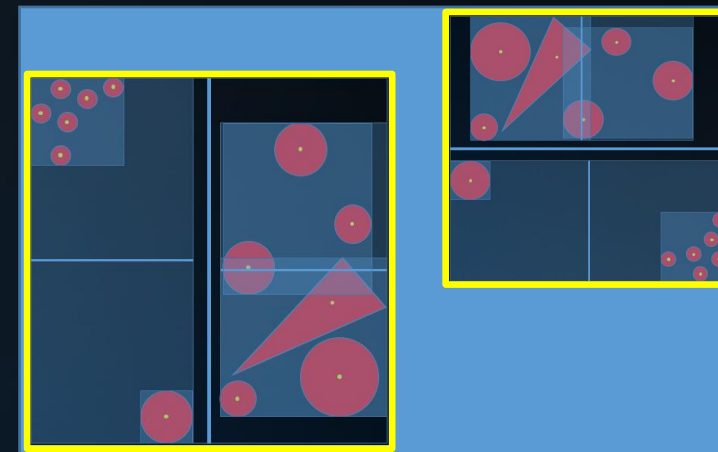


# Top-level BVH

## Combining BVHs

Two BVHs can be combined into a single BVH, by simply adding a new root node pointing to the two BVHs.

- This works regardless of the method used to build each BVH
- This can be applied repeatedly to combine many BVHs



```

ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0)
    {
        nt = nt / nc; ddn = ddn * nc;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
}

at a = nt - nc, b = nt + nc;
at Tr = 1 - (R0 + (1 - R0) *
Tr) R = (D * nnt - N * (ddn
0);

E * diffuse;
= true;

efl + refr)) && (depth < MAXDEPTH)
{
    D, N );
    refl * E * diffuse;
    = true;
}

MAXDEPTH)

survive = SurvivalProbability( diffuse );
estimation - doing it properly, closely following
if;
radiance = SampleLight( &rand, I, &L, &lightPdf );
e.x + radiance.y + radiance.z) > 0) && (dot( N, L ) > 0)
{
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiance
    random walk - done properly, closely following Small's
    vive)
};
at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
survive;
pdf;
n = E * brdf * (dot( N, R ) / pdf);
sion = true;

```



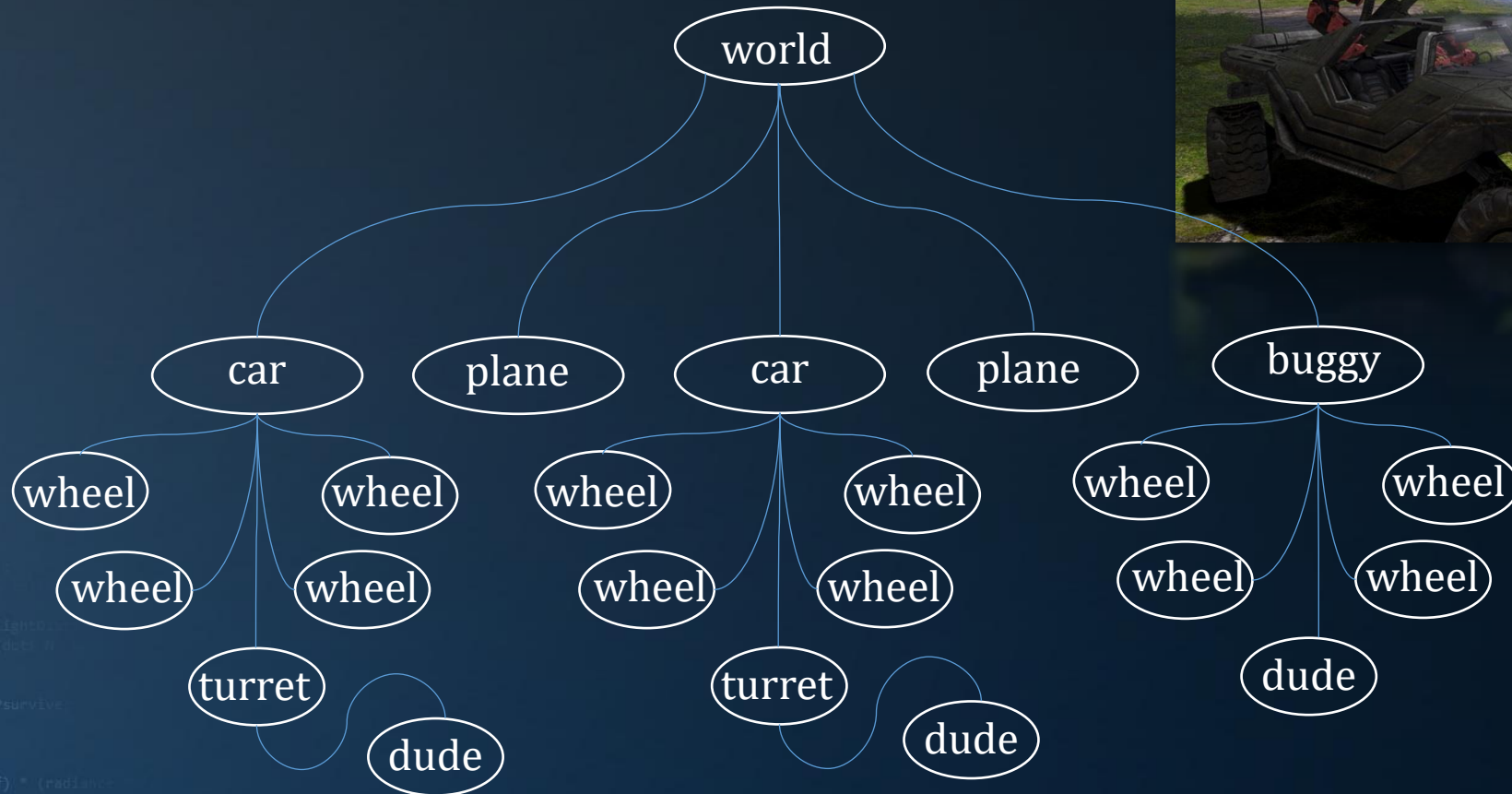






# Top-level BVH

## Scene Graph



# Top-level BVH

## Scene Graph

If our application uses a scene graph, we can construct a BVH for each scene graph node.

The BVH for each node is built using an appropriate construction algorithm:

- High-quality SBVH for static scenery (offline)
- Fast binned SAH BVHs for dynamic scenery

The extra nodes used to combine these BVHs into a single BVH are known as the *Top-level BVH*.



# Top-level BVH

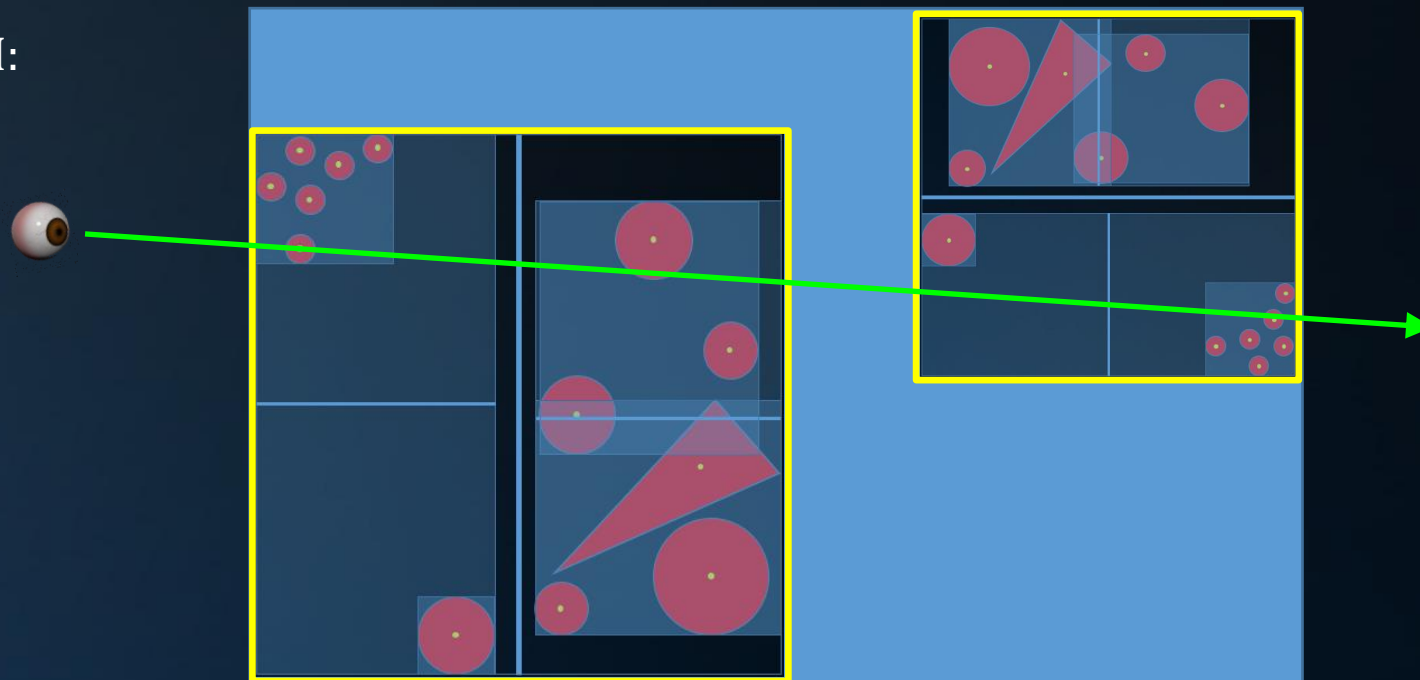
## Rigid Motion

Applying rigid motion to a BVH:

1. Refit the top-level BVH
2. Refit the affected BVH

```

ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0)
    {
        nt = nt / nc; ddn = ddn * nc;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    at a = nt - nc, b = nt * nc;
    at Tr = 1 - (R0 + (1 - R0) *
    Tr) R = (D * nnt - N * (ddn
    )
    E * diffuse;
    = true;
    =
    refl + refr)) && (depth < MAXDEPTH)
    {
        D, N );
        refl * E * diffuse;
        = true;
    }
    MAXDEPTH)
    survive = SurvivalProbability( diffuse );
    estimation - doing it properly, closely following
    if;
    radiance = SampleLight( &rand, I, &L, &light;
    e.x + radiance.y + radiance.z) > 0) && (depth <
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiance
    random walk - done properly, closely following Small
    vive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf;
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
    
```



# Top-level BVH

## Rigid Motion

Applying rigid motion to a BVH:

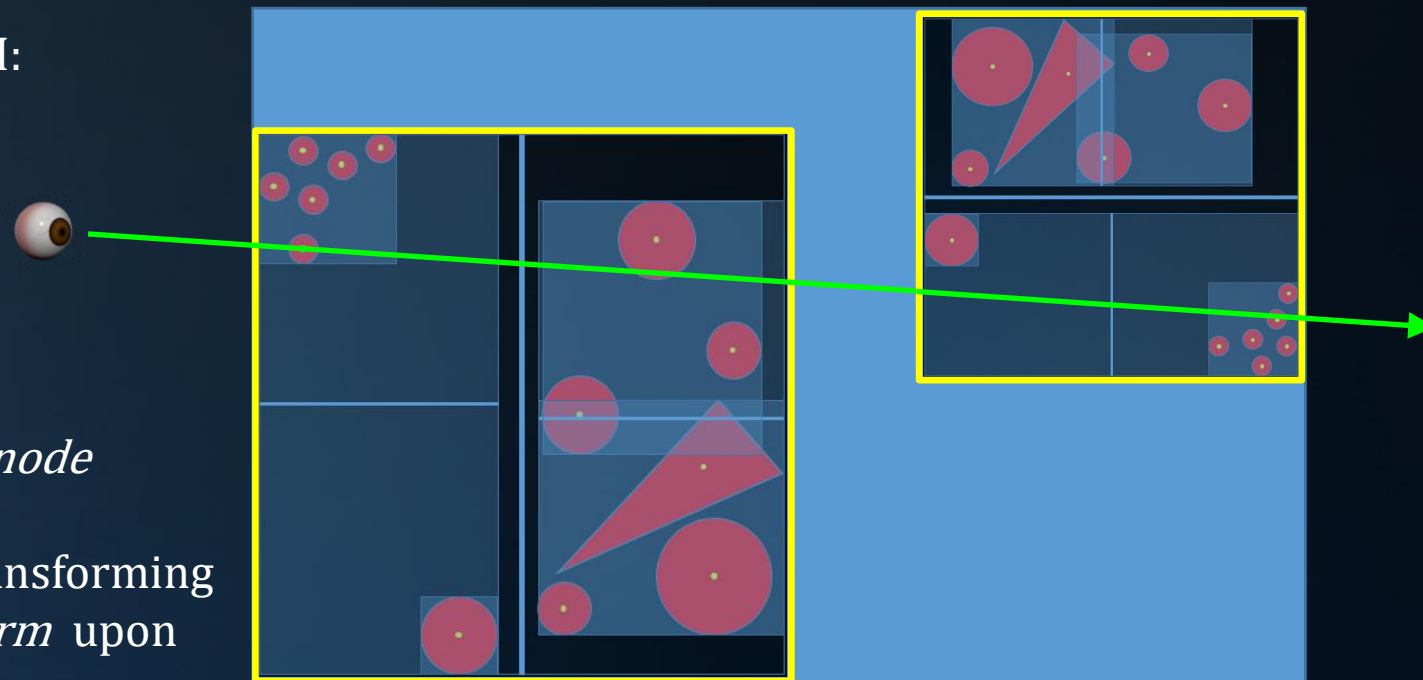
1. Refit the top-level BVH
2. Refit the affected BVH

or:

2. *Transform the ray, not the node*

Rigid motion is achieved by transforming the rays by the *inverse transform* upon entering the sub-BVH.

*(this obviously does not only apply to translation)*





# Top-level BVH

## The Top-level BVH - Construction

Input: *list of axis aligned bounding boxes for transformed scene graph nodes*

Algorithm:

1. Find the two elements in the list for which the AABB has the smallest surface area
2. Create a parent node for these elements
3. Replace the two elements in the list by the parent node
4. Repeat until one element remains in the list.

Note: algorithmic complexity is  $O(N^3)$ .

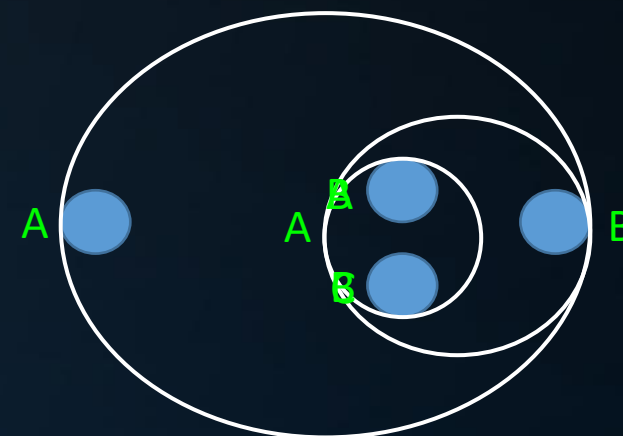


# Top-level BVH

## The Top-level BVH – Faster Construction\*

Algorithm:

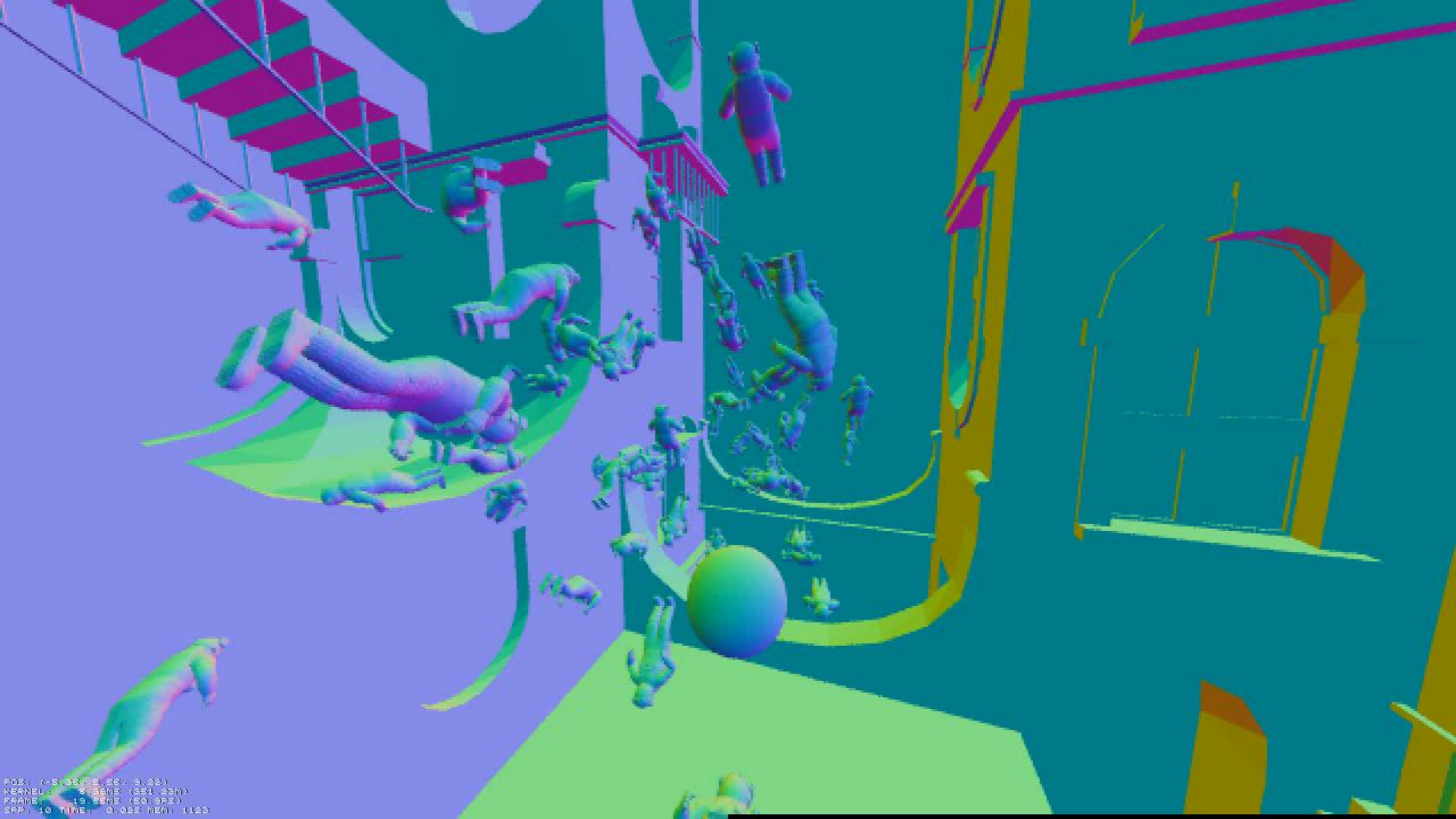
```
Node A = list.GetFirst();
Node B = list.FindBestMatch( A );
while (list.size() > 1)
{
    Node C = list.FindBestMatch( B );
    if (A == C)
    {
        list.Remove( A );
        list.Remove( B );
        A = new Node( A, B );
        list.Add( A );
        B = list.FindBestMatch( A );
    }
    else A = B, B = C;
}
```



\*: Fast Agglomerative Clustering for Rendering, Walter et al., 2008







POS: (-5.35, 15.55, 3.22)  
KERNEL: 4.58MS (351.23M)  
FRAME: 15.58MS (50.9PS)  
SPP: 10 TIME: 0.025 MEM: 1123



# Top-level BVH

## The Top-level BVH – Summary

The top-level BVH enables complex animated scenes:

- for static objects, it contains high-quality sub-BVHs;
- for objects undergoing rigid motion, it also contains high-quality sub-BVHs, with a transform matrix and its inverse;
- for deforming objects, it contains sub-BVHs that can be refitted;
- for arbitrary animations, it contains lower quality sub-BVHs.

Combined, this allows for efficient maintenance of a global BVH.

```

ics
& (depth < MAXDEPTH) {
    if ( ! inside ) {
        nt = nt / nc; ddn = ddn * nc;
        cos2t = 1.0f - nnt * nnt;
        D, N );
    }
    at a = nt - nc, b = nt + nc;
    at Tr = 1 - (R0 + (1 - R0) *
    Tr) R = (D * nnt - N * (ddn
    E * diffuse;
    = true;
    -
    refl + refr)) && (depth < MAXDEPTH) {
        D, N );
        refl * E * diffuse;
        = true;
    }
    MAXDEPTH)
    survive = SurvivalProbability( diffuse );
    estimation - doing it properly, closely following
    if;
    radiance = SampleLight( &rand, I, &L, &align,
    e.x + radiance.y + radiance.z) > 0) && (depth <
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiance
    random walk - done properly, closely following Small's
    vive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
    
```



# INFOMAGR – Advanced Graphics

Jacco Bikker - November 2021 - February 2022

## END of “The Perfect BVH”

next lecture: “Path Tracing”

