

```
ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0.25)
    {
        nt = nt / nc; ddn = ddn * ddn;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    {
        at a = nt - nc, b = nt + nc;
        at Tr = 1 - (R0 + (1 - R0) * ddn);
        Tr) R = (D * nnt - N * (ddn * cos2t));
    }
    E * diffuse;
    = true;
    {
        refl + refr)) && (depth < MAXDEPTH)
    {
        D, N );
        refl * E * diffuse;
        = true;
    }
    MAXDEPTH)
    survive = SurvivalProbability( diffuse );
    estimation - doing it properly, closely following
    if;
    radiance = SampleLight( &rand, I, N, align );
    e.x + radiance.y + radiance.z );
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiance
    random walk - done properly, closely following Small's
    vive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
}
```

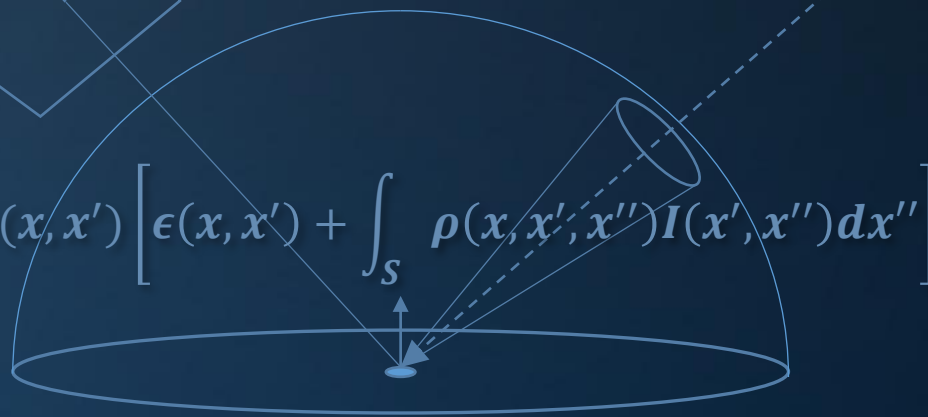


# INFOMAGR – Advanced Graphics

Jacco Bikker - November 2021 - February 2022

## *Lecture 12 - “Probability Theory”*

Welcome!



$$I(x, x') = g(x, x') \left[ \epsilon(x, x') + \int_S \rho(x, x', x'') I(x', x'') dx'' \right]$$



```

k (depth < MAXDEPTH) {
    if (nt > nc) {
        nt = inside ? 1 + 1.0f / nc : 1;
        nnt = nt * nt;
        ddh = ddot(N,N);
        cos2t = 1.0f - nnt * ddh;
        D, N );
        R = (D * nnt - N * (ddh * nnt)) / cos2t;
        E * diffuse;
        = true;
    } else {
        refl + refr)) && (depth < MAXDEPTH) {
            D, N );
            refl * E * diffuse;
            = true;
        } else {
            MAXDEPTH) {
                survive = SurvivalProbability( diffuse );
                // Monte Carlo estimation - doing it properly, closely following Small's paper
                df;
                radiance = SampleLight( &rnd, I, &L, &lightPdf, &pdf );
                e.x + radiance.y + radiance.z) > 0) && (depth < MAXDEPTH) {
                    w = true;
                    brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
                    at3 factor = diffuse * INVPI;
                    at weight = Mis2( directPdf, brdfPdf );
                    at cosThetaOut = dot( N, L );
                    E * ((weight * cosThetaOut) / directPdf) * (radiance
                        random walk - done properly, closely following Small's paper
                        survive)
                    ;
                    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
                    survive;
                    pdf;
                    n = E * brdf * (dot( N, R ) / pdf);
                    sion = true;

```

# Previously in Advanced Graphics...





# Today's Agenda: *Monte Carlo*



## Sampling an Area Light with One Ray



# Sampling Multiple Area Lights with One Ray



## Difficult Cases: Spherical Lights, Occluded Lights



# The Random Walk



## Random Walk with Next Event Estimation



# ~~Russian Roulette~~



## P3 Topics



# Lights

## Case 1: Point Light

### Situation:

- surface point: location  $p$ , normal  $\vec{N}$ ;
- point light: location  $e$ , intensity  $I$  (in Watt, or joule per second);
- distance between  $p$  and  $e$ :  $d$ .
- unit vector from  $p$  to  $e$ :  $\vec{L}$ .

Flux leaving  $e$ :  $I$  joules per second.

Flux arriving at a sphere, radius  $r$ , surface  $4\pi r^2$  around  $e$ :  $I$

(Ir)radiance arriving on that sphere:  $\frac{I}{4\pi r^2}$  ( $W/m^2$ )

Flux arriving per steradian:  $\frac{I}{4\pi}$

Steradians for a unit area surface patch at location  $p$ :  $\sim \frac{\vec{N} \cdot \vec{L}}{d^2}$

This is the solid angle of the unit area surface patch as seen from  $e$ , or:

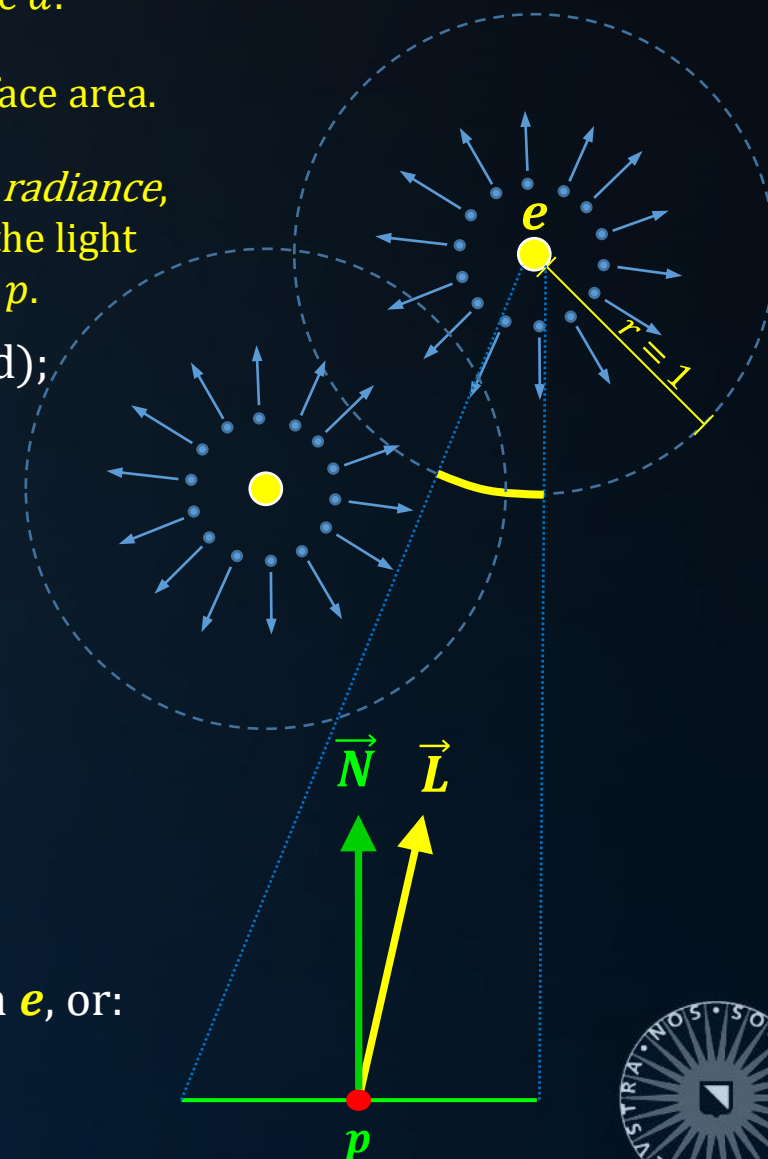
The area of the patch projected on the unit sphere around  $e$ .

Light arriving at  $p$  from a point light at distance  $d$ :

$$I \frac{\vec{N} \cdot \vec{L}}{4\pi d^2} \text{ per unit surface area.}$$

This is the *projected radiance*, i.e. *irradiance* from the light at  $e$  arriving at point  $p$ .

The contribution of multiple lights is summed.





# Lights

## Case 2: Area Light

Situation:

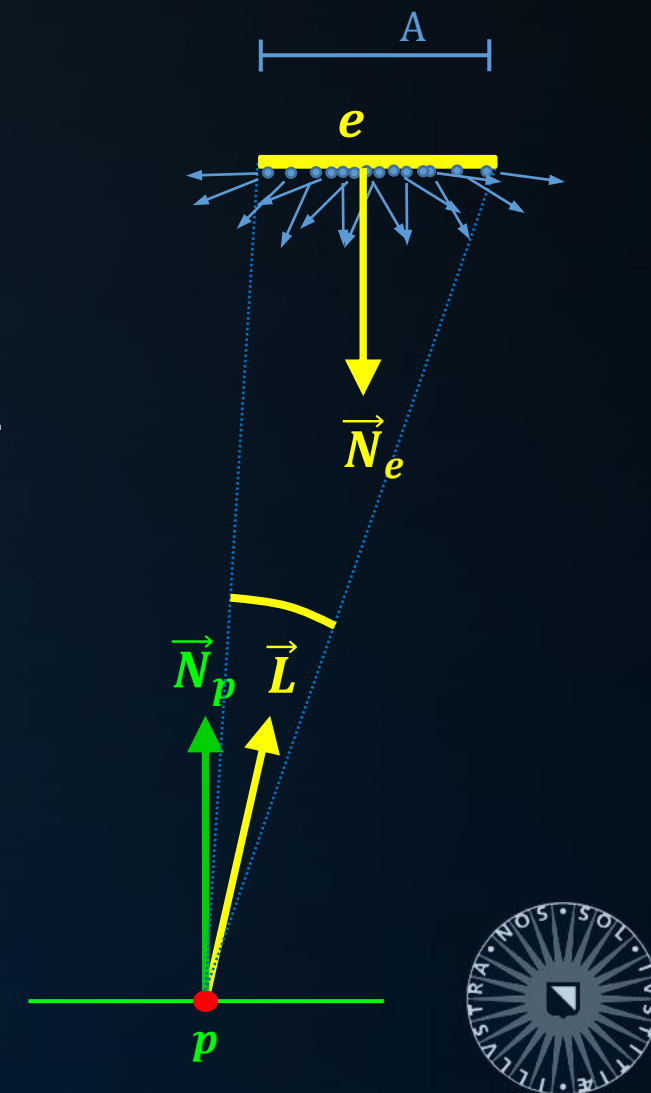
- surface point, location  $p$ , normal  $\vec{N}_p$ ;
- single-sided area light  $e$ , intensity  $I$ , area  $A$ , normal  $\vec{N}_e$ .

Steradians for the area light, as seen from  $p$ :  $\frac{A(\vec{N}_e \cdot -\vec{L})}{d^2}$  (approximately).

The *radiance* arriving from  $e$  at  $p$  is:  $I SA$ ,  $SA \approx \frac{A(\vec{N}_e \cdot -\vec{L})}{d^2}$

The *irradiance* (joules per second per unit area) is:

$$I SA (\vec{N}_p \cdot \vec{L}) \approx I \frac{A(\vec{N}_e \cdot -\vec{L})(\vec{N}_p \cdot \vec{L})}{d^2}.$$



# Lights

## Sampling an Area Light

The irradiance (joules per second per unit area) is:  $I = \frac{A_{visible}(\vec{N}_e \cdot -\vec{L})(\vec{N}_p \cdot \vec{L})}{d^2}$ .

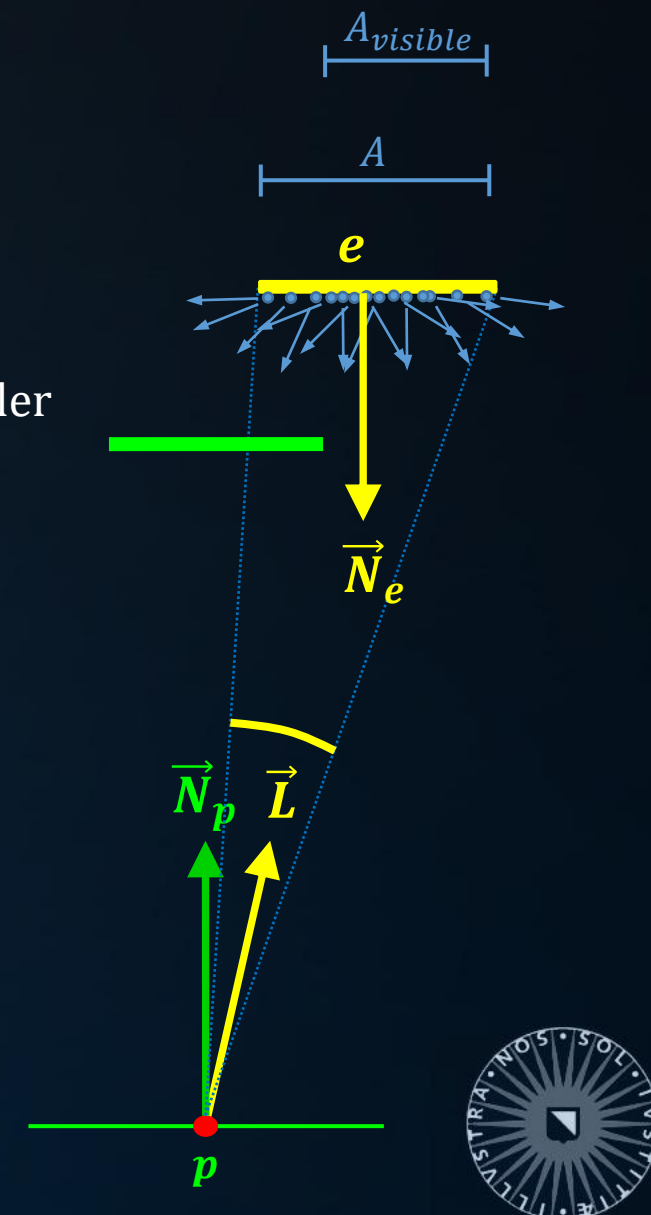
Here,  $A_{visible}$  is the *visible* area of the light source.  $A_{visible}$  may be smaller than  $A$  in the presence of occluders.

We send 1 million rays to the light source.  $N$  rays reach the light source.

The visible area is estimated as  $A_{visible} = A \frac{N}{1,000,000}$ .

Now, we send a *single* ray to the light source. The probability of a ray reaching the light source is  $\rho$ . Now,  $A_{visible} = A \rho$ .

For this single ray, the answer is usually wrong. However, *on average* the answer is correct.



# Today's Agenda: *Monte Carlo*



Sampling an Area Light with One Ray



Sampling Multiple Area Lights with One Ray



Difficult Cases: Spherical Lights, Occluded Lights



The Random Walk



Random Walk with Next Event Estimation



Russian Roulette



P3 Topics



```
ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0)
    {
        nt = nt / nc; ddn = ddn * ddn;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    at a = nt - nc, b = nt * nc;
    at Tr = 1 - (R0 + (1 - R0) *
    Tr) R = (D * nnt - N * (ddn
    E * diffuse;
    = true;
    -
    refl + refr)) && (depth
    D, N );
    refl
    = true;
    MAXDEP
    survi
    estim
    df;
    radian
    e.x + radian
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiant
    random walk - done properly, closely following Small
    vive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
```



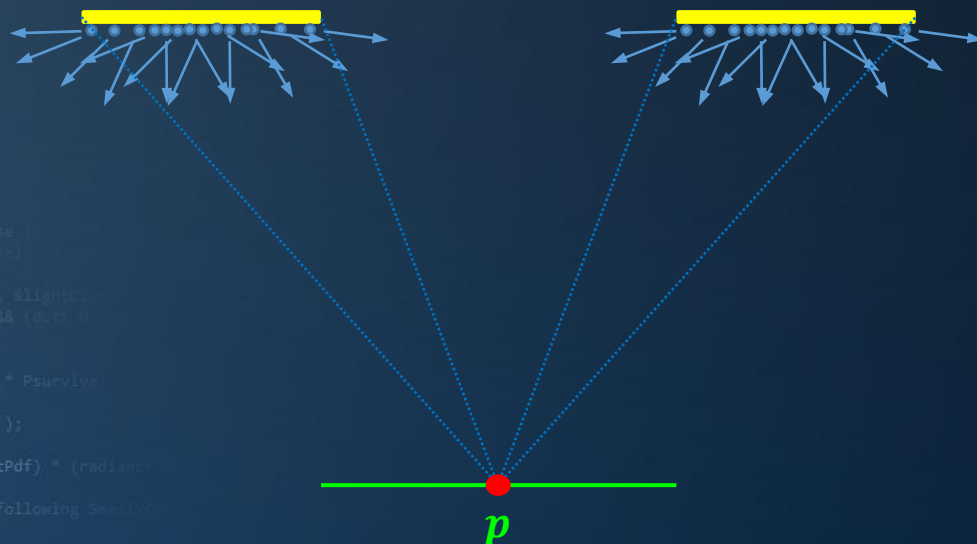


# Lights

## Sampling Multiple Lights

*“To sample  $N$  lights with a single ray, chose a random light, and multiply whatever the ray returns by  $N$ ”*

Situation: *two lights.*



Generalized:

If we have  $N$  lights, and we sample each with a probability  $\rho_i$ , we scale the contribution by  $\frac{1}{\rho_i}$  to get an unbiased sample of the set of  $N$  lights.

Any  $\rho_i$  is valid, as long as  $\sum \rho_i = 1$  and  $\rho_i > 0$  unless we know the sample will yield 0.

Mental steps:

- If the lights would have been point lights, we would have sampled both and summed the results.
- We know that we can sample an area light with a single ray. So, we sample both using a single ray, and sum the results.
- Using one ray, we could sample alternating lights. Since each light is now sampled in half the cases, we should increase the result we get each time by 2.
- Or, we can sample a randomly selected light. On average, each light is again sampled in half of the cases, so we scale by 2.
- In other words, we scale by  $1/50\% = 2$ , where 50% is the probability of selecting a light.



# Today's Agenda: *Monte Carlo*



Sampling an Area Light with One Ray



Sampling Multiple Area Lights with One Ray



Difficult Cases: Spherical Lights, Occluded Lights



The Random Walk



Random Walk with Next Event Estimation



Russian Roulette



P3 Topics



```
ics
& (depth < MAXDEPTH)
{
    if (nt < 1e-5)
    {
        nt = nt / nc; ddn = ddn * nc;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    at a = nt - nc, b = nt * nc;
    at Tr = 1 - (R0 + (1 - R0) * b);
    (Tr) R = (D * nnt - N * (ddn *
    E * diffuse;
    = true;
    -
    refl + refr)) && (depth
    D, N );
    refl
    = true;
    MAXDEP
    survi
    estim
    df;
    radian
    e.x + radi
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiant
    random walk - done properly, closely following Small
    vive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
```



# Lights

## Sampling a Spherical Light

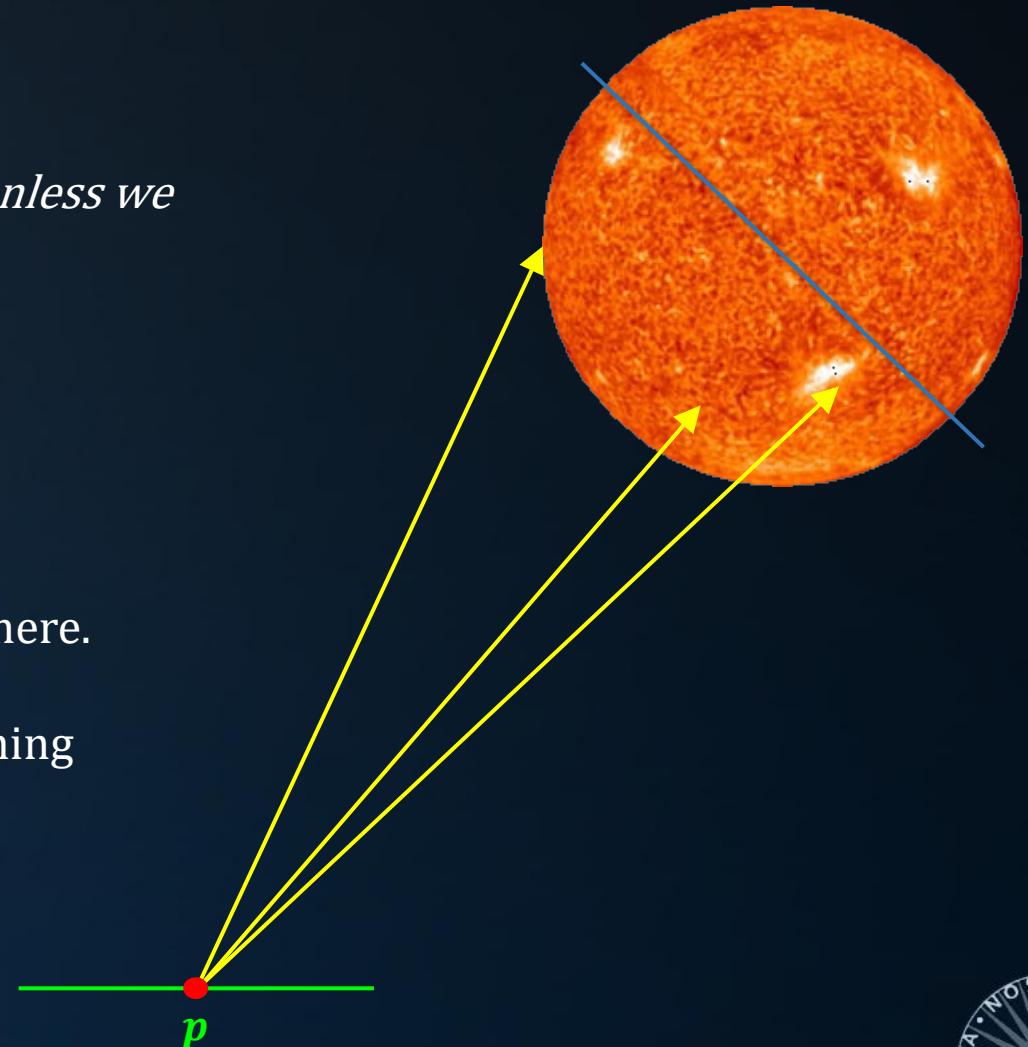
*“Any  $\rho_i$  is valid, as long as  $\sum \rho_i = 1$  and  $\rho_i > 0$  unless we know that the sample will yield 0.”*

Situation: *spherical light source.*

Starting point: *selecting  $N$  points on the sphere, uniformly, so  $\rho_i = 1/N$ .*

- Now, suppose we skip points on one hemisphere.
- Skipping points means  $\rho_i = 0$ .
- To ensure that  $\sum \rho_i = 1$ : double  $\rho_i$  for remaining points, to 0.2.
- Before:  $E = \frac{1}{N} \sum_{i=1}^N \frac{V_{i=0.5}}{\rho_{i=0.1}}$ , after:  $\frac{1}{N} \sum_{i=1}^N \frac{V_{i=1}}{\rho_{i=0.2}}$

Similar situation: when evaluating the Lambertian BRDF, we skip the hemisphere below the surface. We account for this by reducing the domain to  $2\pi$ .



# Lights

## Sampling Occluded Lights

### Situation 1:

*We have no information about occlusion.*

*NEE probes each light with 50% probability.*

- samples are scaled up by  $1/50\% = 2$ ;
- rays to light 2 always yields 0;

→ point  $p$  receives energy from light 1 in 50% of the cases, but the light is multiplied by 2.

### Situation 2:

*We know light 2 is occluded.*

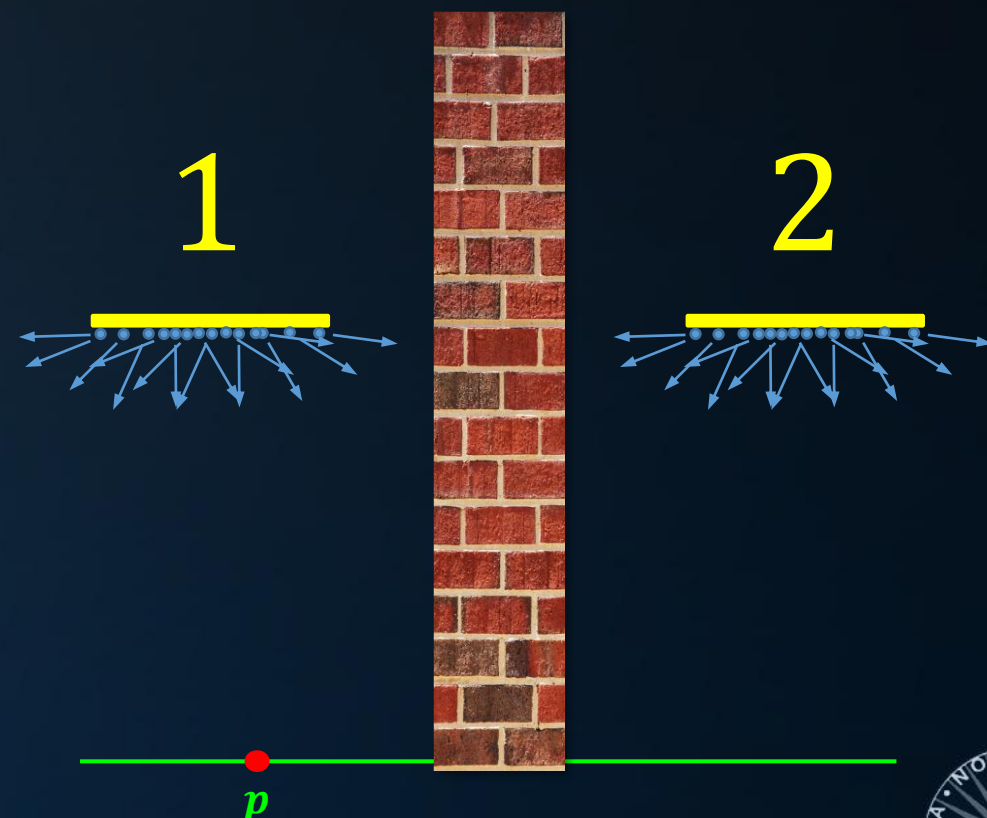
*NEE probes light 1 with 100% probability.*

- samples are scaled by 1;

→ point  $p$  receives energy from light 1 in 100% of the cases, multiplier is 1.

The only difference between situation 1 and 2 is variance: in situation 1, we get twice the energy each time we sample light 1, but it gets sampled in only 50% of the cases.

In situation 2, we get a much more even amount of energy for each sample.





# Today's Agenda: *Monte Carlo*



Sampling an Area Light with One Ray



Sampling Multiple Area Lights with One Ray



Difficult Cases: Spherical Lights, Occluded Lights



The Random Walk



Random Walk with Next Event Estimation



Russian Roulette



P3 Topics



```
ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0)
    {
        nt = nt / nc; ddn = ddn * ddn;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    at a = nt - nc, b = nt * nc;
    at Tr = 1 - (R0 + (1 - R0) *
    Tr) R = (D * nnt - N * (ddn
    E * diffuse;
    = true;
    -
    refl + refr)) && (depth
    D, N );
    refl
    = true;
    MAXDEP
    survi
    estim
    df;
    radian
    e.x + radian
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiant
    random walk - done properly, closely following Small
    vive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
```



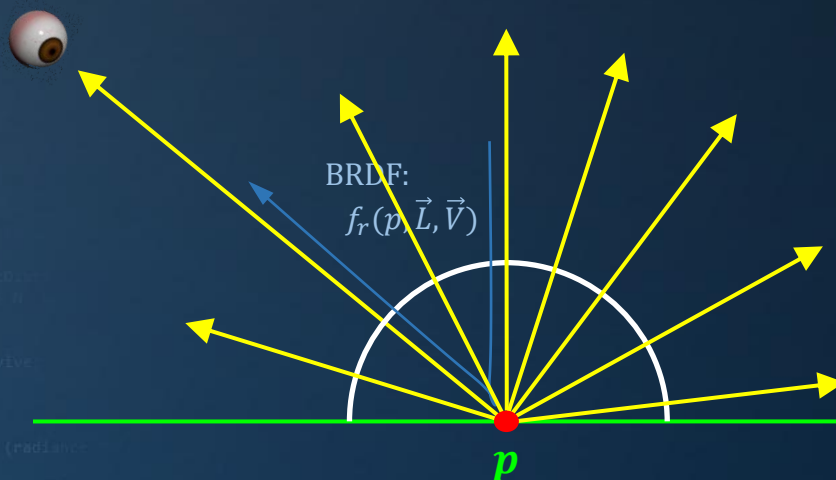


# Walk

## The Random Walk

How much light gets transported to the eye?

1. The light that **p** emits towards the eye (typically: nothing); plus:
2. The light that **p** reflects towards the eye.



Reflected energy:

Light (radiance) coming from all directions, reflected in a single direction; i.e:

$$L_o = \int_{\Omega} f_r(p, \theta_o, \theta_i) \cos \theta_i L_i$$

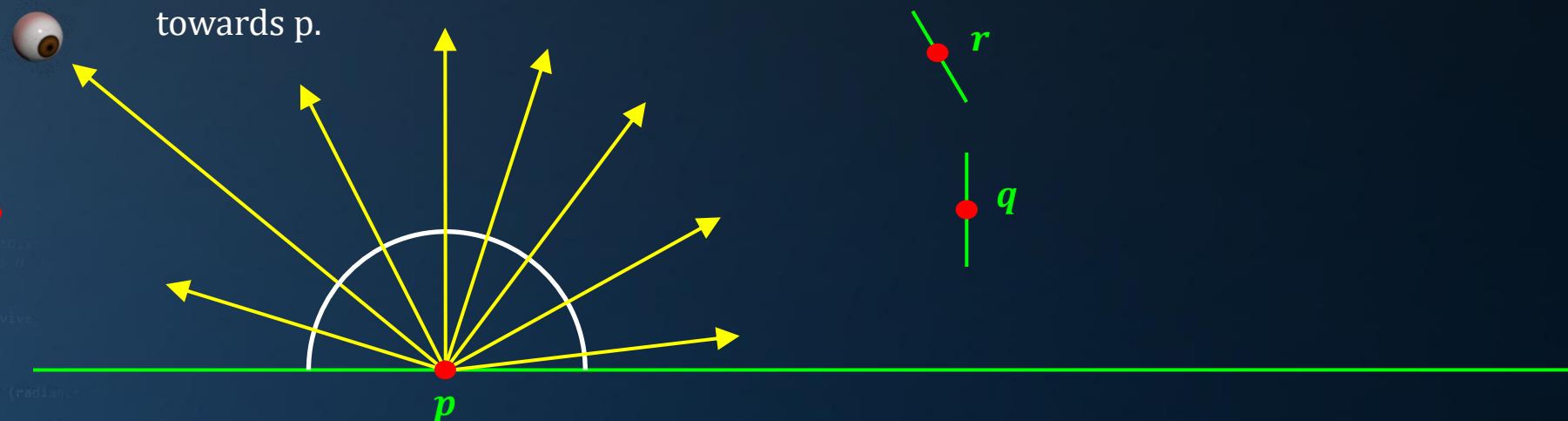


# Walk

## The Random Walk

How much light gets transported to the eye?

1. The light that **p** emits towards the eye (typically: nothing);
2. The light that **p** reflects towards the eye:
  - a) That is: the light that q, r, s emit towards p, plus
  - b) The light that q, r, s (and all other scene surface points) reflect towards p.



Regarding 2b:

- The further away a point, the lower the probability that a random ray from  $p$  strikes it.
- The probability is also proportional to  $\vec{N}_{s,p,q} \cdot -\vec{L}$ .
- At  $p$ , we scale by  $\vec{N}_p \cdot \vec{L}$  to compensate for the fact that we sample radiance, while in fact we need irradiance for the BRDF.



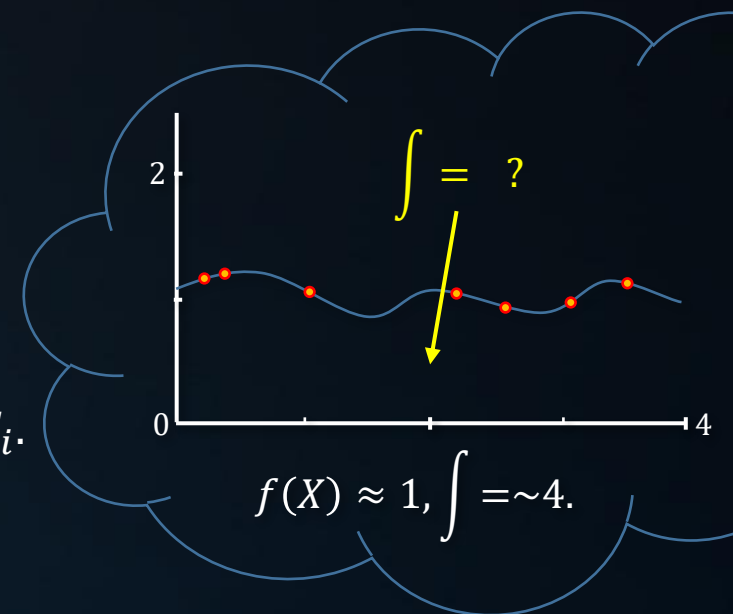
# Walk

## Sampling the Hemisphere using a Single Ray

The light being reflected towards the eye is the light arriving from all directions over the hemisphere, scaled by the BRDF:  $\int_{\Omega} f_r(p, \theta_o, \theta_i) E_i$ .

Sampling the integral using a single random ray:

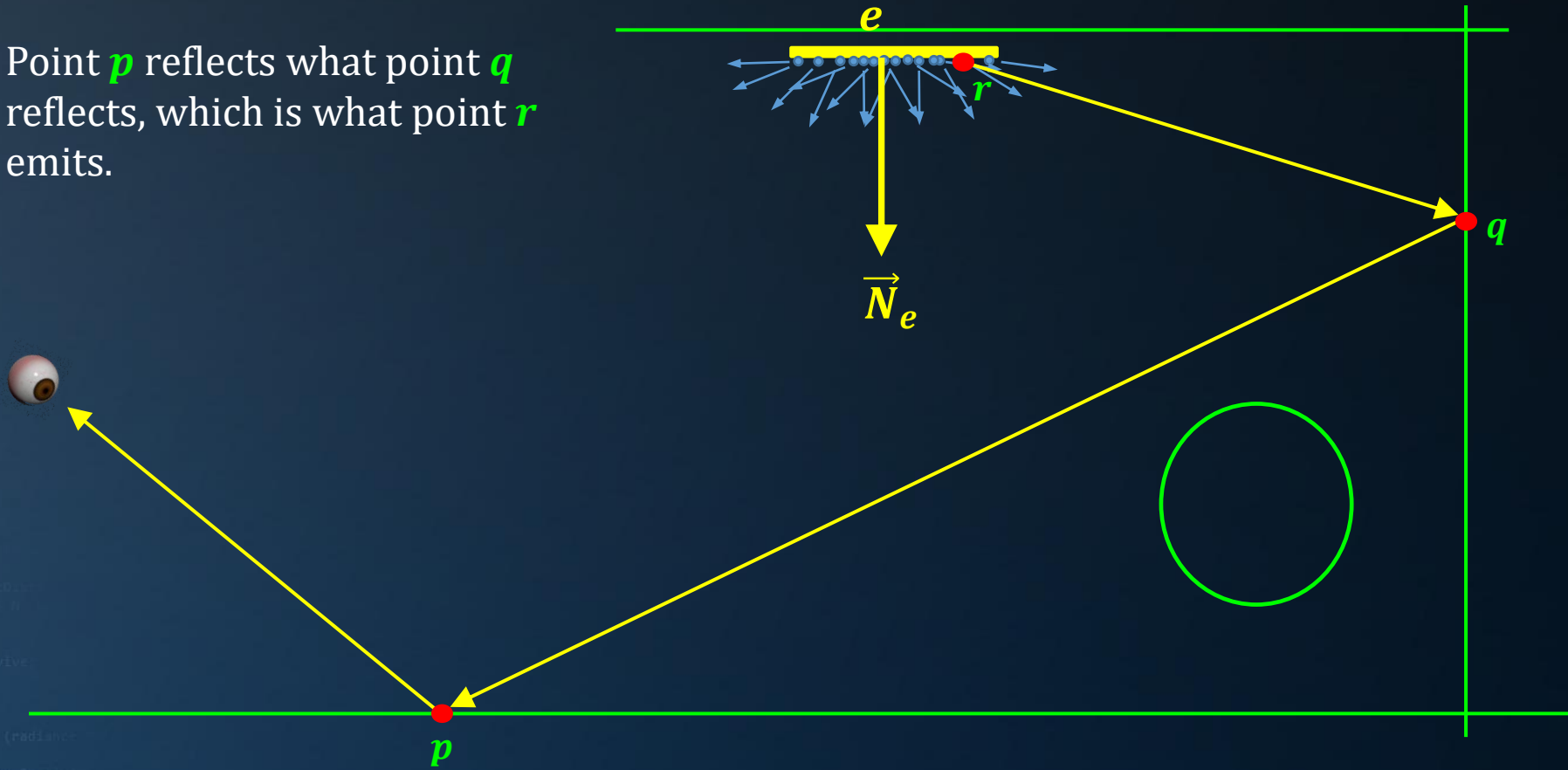
*Scale up by  $2\pi$ .*



# Walk

## Random Walk

Point  $p$  reflects what point  $q$  reflects, which is what point  $r$  emits.



# Random Walk

[illegible]



# Today's Agenda: *Monte Carlo*



Sampling an Area Light with One Ray



Sampling Multiple Area Lights with One Ray



Difficult Cases: Spherical Lights, Occluded Lights



The Random Walk



Random Walk with Next Event Estimation



Russian Roulette



P3 Topics



```
ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0)
    {
        nt = nt / nc; ddn = ddn * nc;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    at a = nt - nc, b = nt * nc;
    at Tr = 1 - (R0 + (1 - R0) * a);
    (Tr) R = (D * nnt - N * (ddn *
    E * diffuse;
    = true;
    -
    refl + refr)) && (depth
    D, N );
    refl
    = true;
    MAXDEP
    survi
    estim
    df;
    radian
    e.x + radian
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiant
    random walk - done properly, closely following Small's
    vive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
```



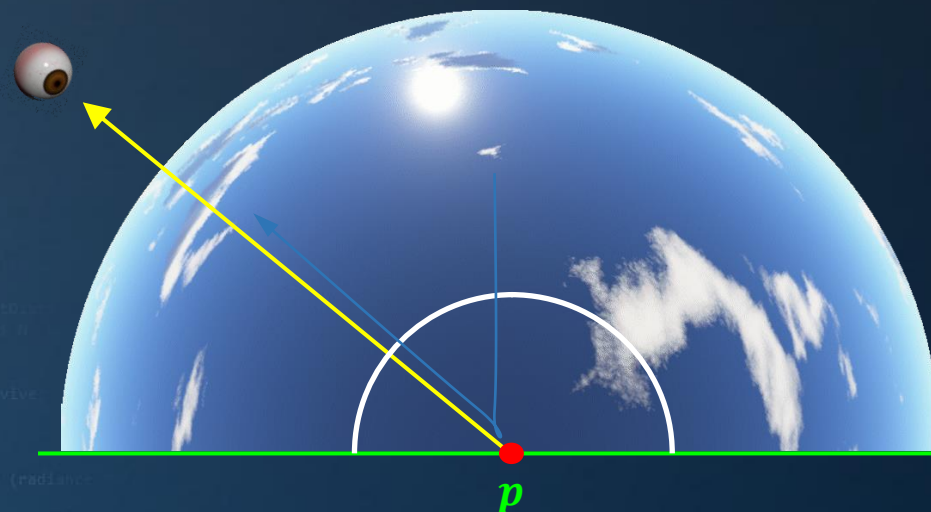
# NEE

$$E(f(X)) \approx \frac{1}{N} \sum_{i=1}^N \frac{f(X)}{p(X)}$$

## Importance-Sampling the Hemisphere

How much light gets transported to the eye?

1. The light that **p** emits towards the eye (typically: nothing); plus:
2. The light that **p** reflects towards the eye.
3. In practice: we have no idea. But we can guess.



Reflected energy:

Light (radiance) coming from all directions, reflected in a single direction; i.e:

$$L_o = \int_{\Omega} f_r(p, \theta_o, \theta_i) \cos \theta_i L_i$$



# NEE

```

ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0)
    {
        nt = nt / nc; do
        {
            pos2t = 1.0f - nm
            D, N );
        }
    }
    if (a = nt - nc,
    {
        at Tr = 1 - (R0
        Tr) R = (D * nnt
    }
    E * diffuse;
    = true;
    -
    (refl + refr)) &&
    D, N );
    refl * E * diffu
    = true;
}
MAXDEPTH)

```

```

survive = SurvivalProbability( diffuse );
estimation - doing it properly, closely following
df;
radiance = SampleLight( &rand, I, &L, &align,
e.x + radiance.y + radiance.z > 0) && (octa
w = true;
at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
at3 factor = diffuse * INVPI;
at weight = Mis2( directPdf, brdfPdf );
at cosThetaOut = dot( N, L );
E * ((weight * cosThetaOut) / directPdf) * (radiance

```

```

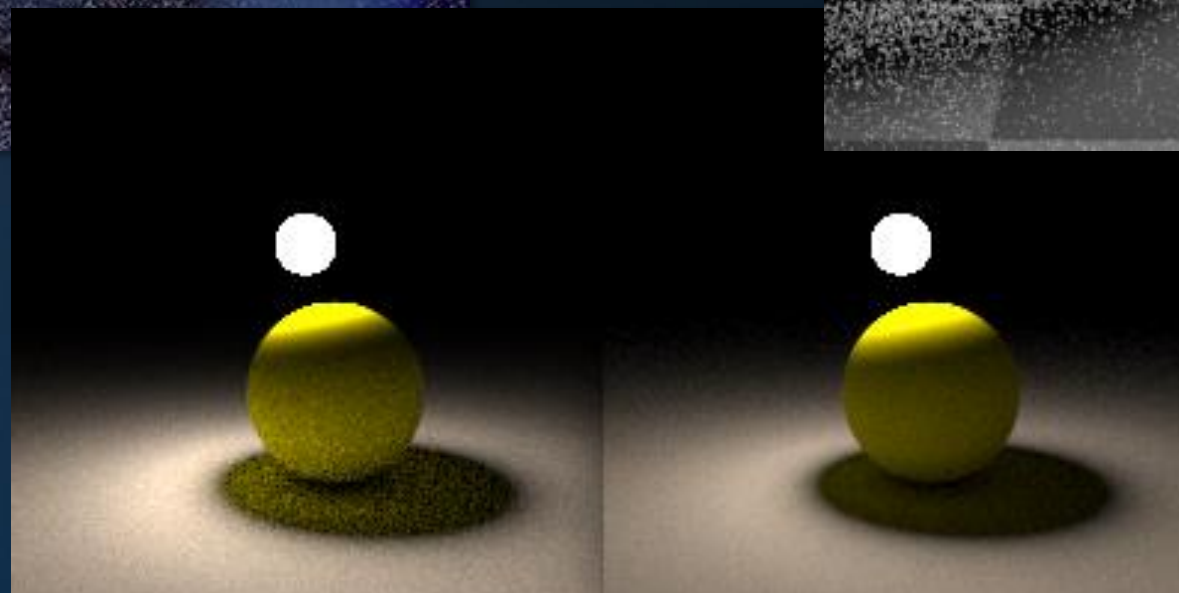
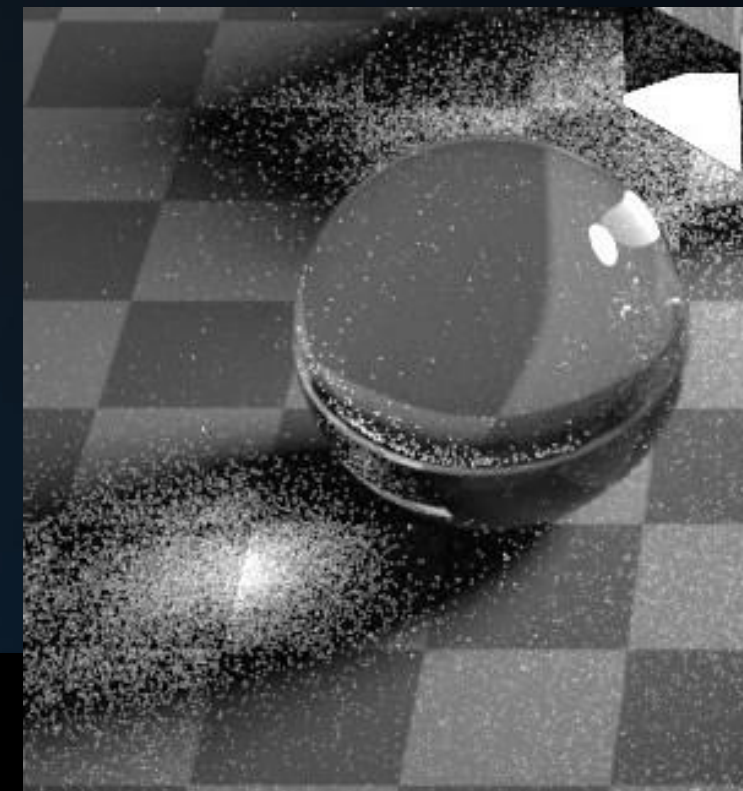
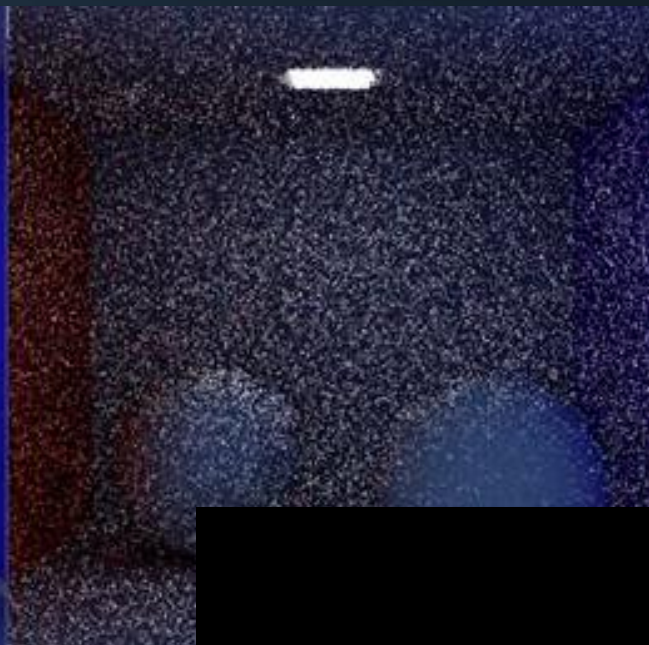
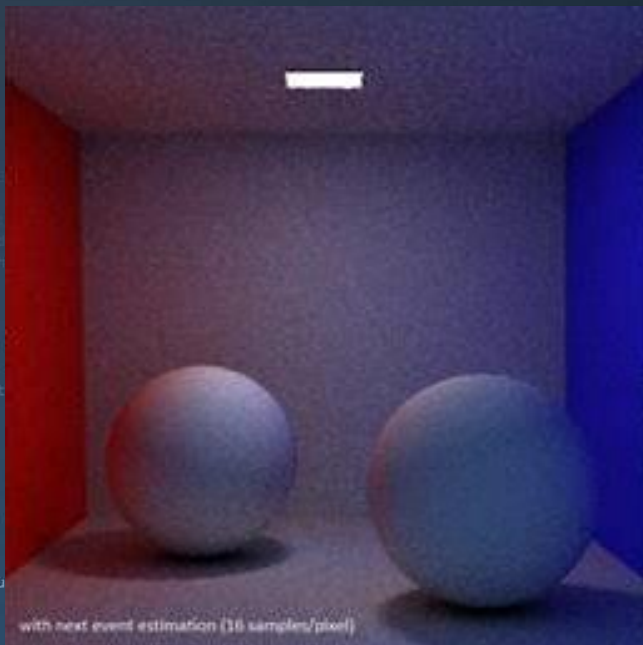
random walk - done properly, closely following
vive)
;
at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
survive;
pdf;
n = E * brdf * (dot( N, R ) / pdf);
sion = true;

```

```

with next event estimation (16 samples/pixel)

```

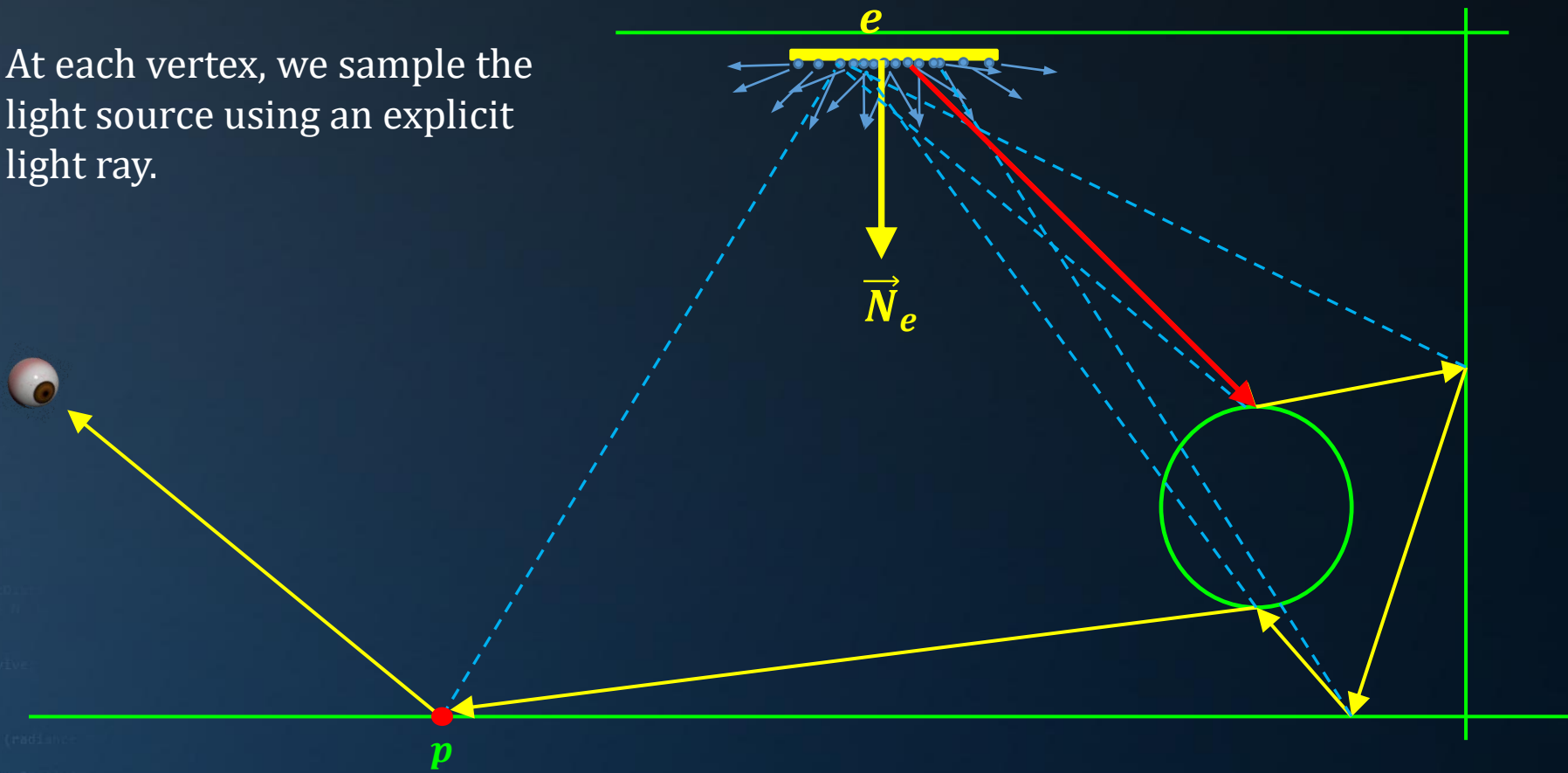




## NEE

## Next Event Estimation

At each vertex, we sample the light source using an explicit light ray.



```

ics
& (depth < MAXDEPTH)
{
    nt = inside ? 1.0 : 0.0;
    nt = nt / nc; ddn = dot(N, D);
    cos2t = 1.0f - nnt * nnt;
    D, N );
    0);
    at a = nt - nc, b = nt * nc;
    at Tr = 1 - (R0 + (1 - R0) *
    Tr) R = (D * nnt - N * (ddn
    E * diffuse;
    = true;
    efl + refr)) && (depth < MAXDEPTH)
    D, N );
    refl * E * diffuse;
    = true;
    MAXDEPTH)
    survive = SurvivalProbability( diffuse );
    estimation - doing it properly, closely
    if;
    radiance = SampleLight( &rand, I, &L, &lightP;
    e.x + radiance.y + radiance.z) > 0) && (dot(N,
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiant
    random walk - done properly, closely following Small
    vive)
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf;
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;

```



## NEE

## Next Event Estimation

## Why does this work?

“The light arriving via point  $p$  is the light reflected by point  $p$ , plus the light emitted by point  $p$ .”

And thus:

The light reflected by point  $p$  is the light arriving at  $p$  originating from light sources (1), plus the light reflected towards  $p$  (2).

1: Direct light at point  $p$ .

2: Indirect light at point  $p$ .





## NEE

## Next Event Estimation

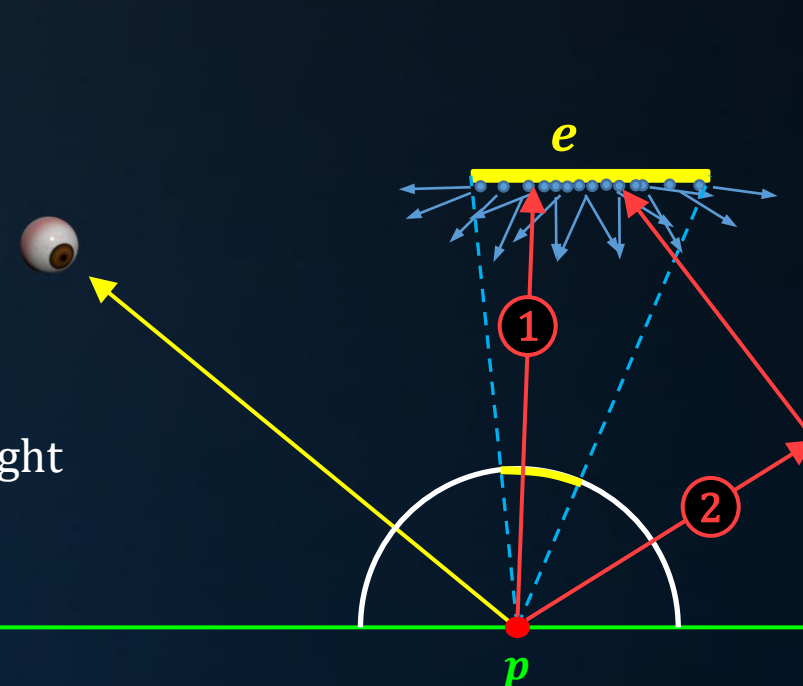
If we send out a ray in a random direction over the hemisphere of  $p$ , this ray may return two types of illumination:

- 1: Direct: the ray hit the light source;
- 2: Indirect: the ray missed the light source.

If we ignore all random rays that hit a light source (as in: terminate them, return 0), we remove the direct light arriving at  $p$ .

If we sample just the lights, we remove the indirect light arriving at  $p$ .

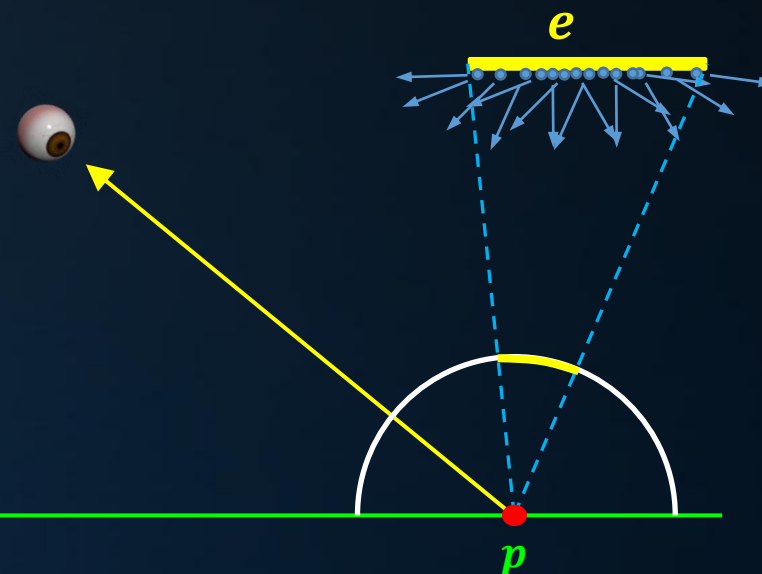
Since the contributions show no overlap, we can sample them individually, using two rays, and sum the result.



## Next Event Estimation

This works for any point, not just the primary hit:

E.g., the light that point **q** reflects towards point **p** is the direct lighting reflected by **q** towards **p**, plus the indirect lighting reflected by **q**.



## NEE

## Next Event Estimation

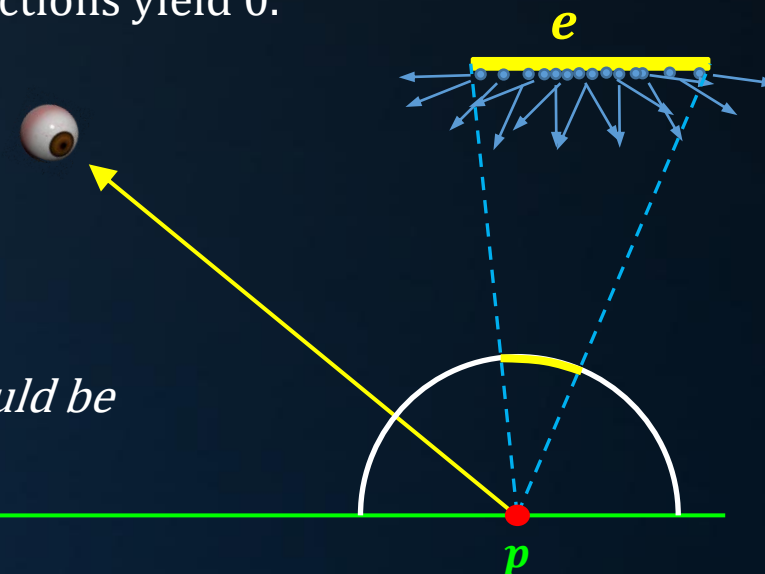
## 1. Why don't we use next event estimation for a specular surface?

Explicit light sampling still requires evaluation of the BRDF. For a specular surface, the BRDF for  $\theta_o$  is  $\infty$  for a single  $\theta_i$ , which is why we continue the (not so) random walk in that direction. All other directions yield 0.

## Consequence:

Since we do not send out an explicit light ray in this case, the random walk may now return direct illumination: there is no overlap.

*In fact, if we didn't accept direct illumination, we would be missing energy.*



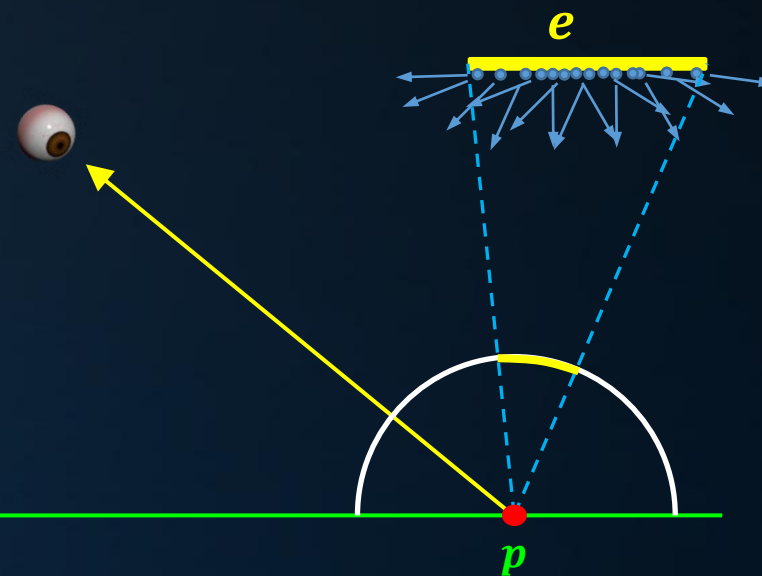
## NEE

## Next Event Estimation

## 2. Why should we return direct illumination for the primary ray?

The eye vertex did not send out an explicit light ray. Since direct illumination is not sampled separately, the random walk may return this illumination.

*The eye is thus considered a specular vertex.*



# NEE

## Next Event Estimation

Also think about it like this:

The eye looks directly at a light source.

*If we terminate those paths, the light will look black.*

The eye looks at a light in a mirror.

*If we terminate those paths, we see a black light in the mirror.*

In all other cases, we send out an explicit light ray. To compensate for that extra ray:

- The extra ray may only sample direct illumination. It doesn't bounce.
- Any other way of sampling direct illumination is blocked.

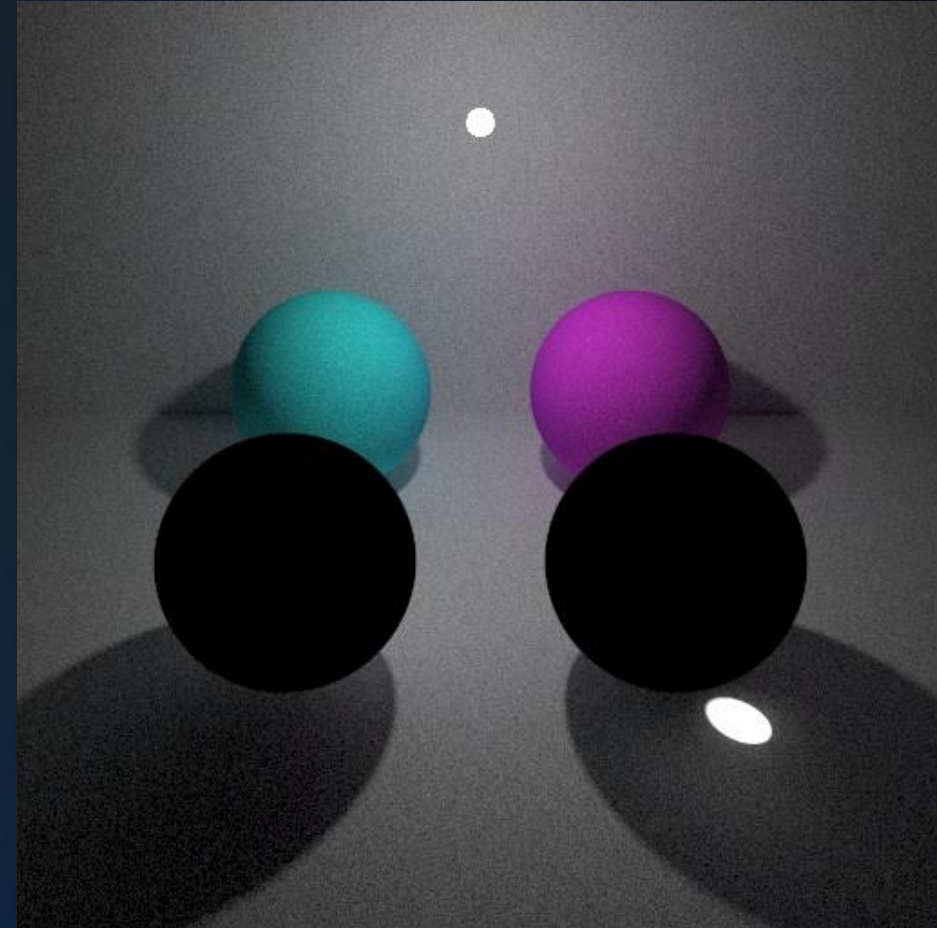
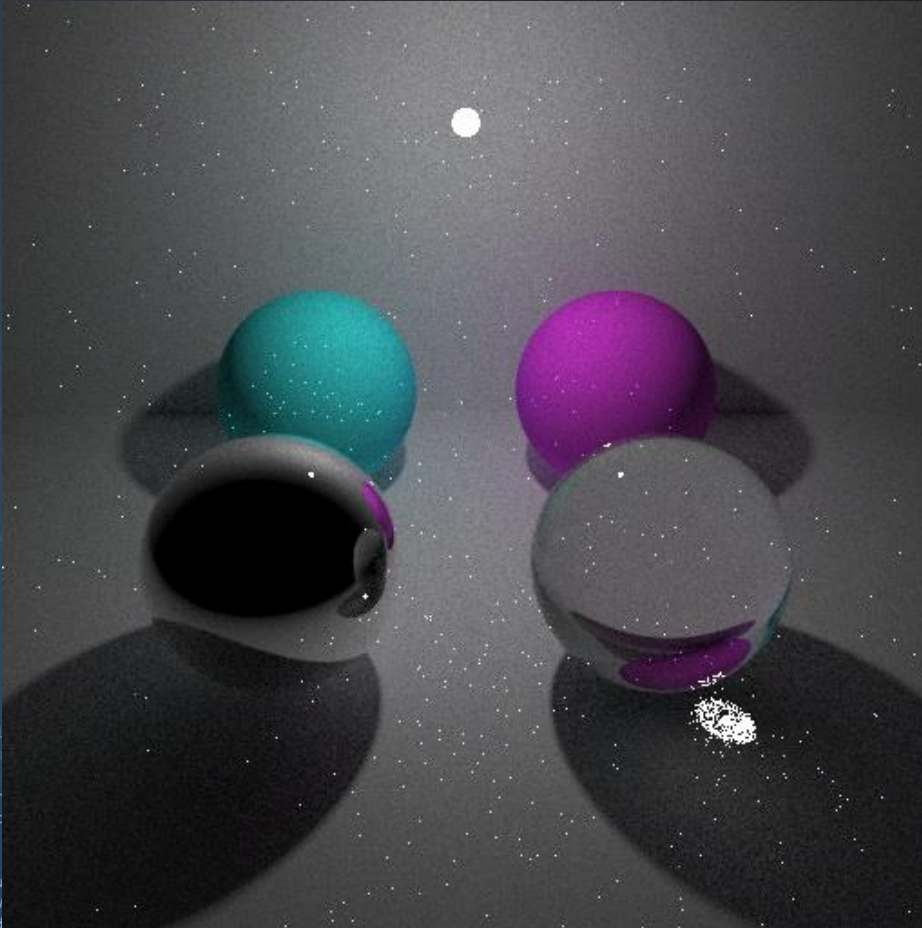




# NEE

```

ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0)
    {
        nt = nt / nc; ddn = sqrt(1 - nt * nt);
        pos2t = 1.0f - nnt * pos2t;
        D, N );
    }
    if (a = nt - nc, b = nt + nc)
    {
        at Tr = 1 - (R0 + (1 - R0) * ddn);
        (Tr) R = (D * nnt - N * (ddn *
    }
    E * diffuse;
    = true;
    = refl + refr)) && (depth < MAXDEPTH)
    {
        D, N );
        refl * E * diffuse;
        = true;
    }
    MAXDEPTH)
    survive = SurvivalProbability(
    estimation - doing it properly
    df;
    radiance = SampleLight( &r,
    e.x + radiance.y + radiance.z);
    w = true;
    at brdfPdf = EvaluateDiffuse(
    at3 factor = diffuse * INV_PI;
    at weight = Mis2( directPdf,
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiance
    random walk - done properly, closely following Small's
    vive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
    
```



<http://sjbrown.co.uk/2011/01/03/two-way-path-tracing>



## Next Event Estimation

## Important:

Earlier slides discussed the mathematical foundation of NEE. Make sure you understand it from the theoretical point of view as well.

```

    if( !nc ) {
        nc = inside ? 1 : -1; // outside == 0
        nt = nt / nc, dn = dn / nc;
        cos2t = 1.0f - nnt * nnt;
        float r = sqrt(cos2t);
        N = (D * N + N * (dn * r));
        E * diffuse;
        bool refl = true;
        if( !refl || !inside ) {
            refl = false;
            R = (D * nnt - N * (dn * r));
            E * diffuse;
            bool refl = true;
        }
        refl = refl && (depth < MAXDEPTH);
        D, N );
        refl * E * diffuse;
        bool refl = true;
    }
    MAXDEPTH)
    survive = SurvivalProbability( diffuse
estimation - doing it properly); Close
df;
    radiance = SampleLight( &rand, I, &t,
2.x + radiance.y + radiance.z) > 0.) &
w = true;
    brdfPdf = EvaluateDiffuse( L, N );
at3 factor = diffuse * INVPI;
brdfPdf = Mis2( directPdf, brdfPdf
cosThetaOut = dot( N, L );
E * ((weight * cosThetaOut) / direct
andom walk - done properly, closely f
ive)
;
at3 brdf = SampleDiffuse( diffuse, N,
survive;
pdf;
n = E * brdf * (dot( N, R ) / pdf);
ision = true;

```

# Today's Agenda: *Monte Carlo*



Sampling an Area Light with One Ray



Sampling Multiple Area Lights with One Ray



Difficult Cases: Spherical Lights, Occluded Lights



The Random Walk



Random Walk with Next Event Estimation



~~Russian Roulette~~ *Digest*



P3 Topics



```
ics
& (depth < MAXDEPTH)
{
    if (inside ? 1 : 0)
    {
        nt = nt / nc; ddn = ddn * ddn;
        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    at a = nt - nc, b = nt * nc;
    at Tr = 1 - (R0 + (1 - R0) *
    Tr) R = (D * nnt - N * (ddn
    E * diffuse;
    = true;
    -
    refl + refr)) && (depth
    D, N );
    refl
    = true;
    MAXDEP
    survi
    estim
    df;
    radiance
    e.x + radiance
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiance
    random walk - done properly, closely following Small
    vive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
```



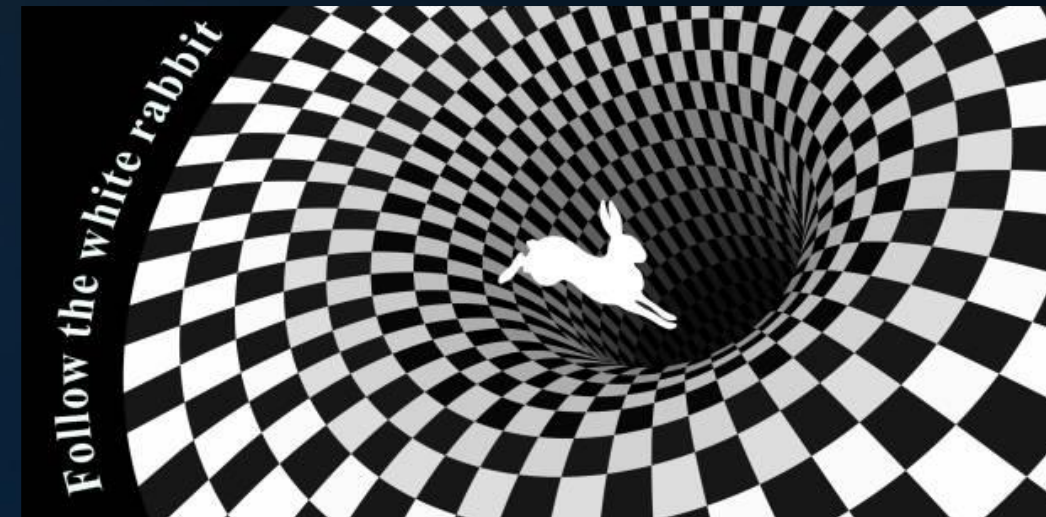


# Efficiency

## Efficiency in Monte-Carlo:

We must use importance sampling.

- We are not tracing photons backwards, we are establishing importance.
- A path that cannot be importance sampled is a noisy path.
- We can still trace from the light to the camera ('forward path tracing' aka light tracing).
- Consider using Multiple Importance Sampling.
- *(it works for NEE, but also when combining forward and backward path tracing)*
- Any pdf is valid, as long as ... and ....
- We can play with this stuff.
- We can *learn* importance.



# Today's Agenda: *Monte Carlo*



Sampling an Area Light with One Ray



Sampling Multiple Area Lights with One Ray



Difficult Cases: Spherical Lights, Occluded Lights



The Random Walk



Random Walk with Next Event Estimation



~~Russian Roulette~~ *Digest*



P3 Topics



```
ics
& (depth < MAXDEPTH)
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        cos2t = 1.0f - nnt * ddn;
        D, N );
    }
    at a = nt - nc, b = nt * nc;
    at Tr = 1 - (R0 + (1 - R0) * a);
    (Tr) R = (D * nnt - N * (ddn *
    E * diffuse;
    = true;
    -
    efl + refr)) && (depth
    D, N );
    refl = diffuse;
    = true;
    MAXDEP
    survi = Prob
    estim
    df;
    radiance
    e.x + radiance
    w = true;
    at brdfPdf = EvaluateDiffuse( L, N ) * Psurvive;
    at3 factor = diffuse * INVPI;
    at weight = Mis2( directPdf, brdfPdf );
    at cosThetaOut = dot( N, L );
    E * ((weight * cosThetaOut) / directPdf) * (radiance
    random walk - done properly, closely following Small
    vive)
    ;
    at3 brdf = SampleDiffuse( diffuse, N, r1, r2, &R, &pdf );
    survive;
    pdf;
    n = E * brdf * (dot( N, R ) / pdf);
    sion = true;
```





# Future Work

## Materials

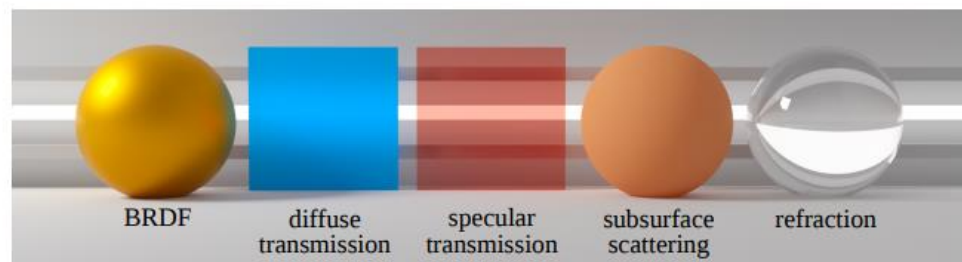


Figure 1: Rendered examples using our new BSDF.

### Extending the Disney BRDF to a BSDF with Integrated Subsurface Scattering

by Brent Burley, Walt Disney Animation Studios

#### 1 Introduction

We introduced a new physically based shading model on Wreck-It Ralph [Bur12], and we used this single, general-purpose BRDF on all materials (except hair). This model, which has come to be known as the Disney BRDF, is able to reproduce a wide range of materials with only a few parameters. For our next film, Frozen, we continued to use this BRDF unmodified, but effects like refraction and subsurface scattering were computed separately from the BRDF, and indirect illumination was approximated using point clouds. All of these effects were combined through ad hoc shading in an additive way.

Starting with Big Hero 6 in 2014, we switched from ad hoc lighting and shading to path-traced global illumination where refraction, subsurface scattering, and indirect illumination were all integrated

Other resources:

Implementing the Disney BSDF  
[schuttejoe.github.io/post/disneybsdf](https://schuttejoe.github.io/post/disneybsdf)

Efficient Rendering of Layered Materials using an Atomic Decomposition with Statistical Operators. Belcour, 2018.

Eric Heitz's Research Page:  
[eheimresearch.wordpress.com/research](https://eheimresearch.wordpress.com/research)  
 (all his work is good)



# Future Work

## Spectral Rendering

### Stratified Wavelength Clusters for Efficient Spectral Monte Carlo Rendering

Glenn F. Evans      Michael D. McCool

Computer Graphics Laboratory  
Department of Computer Science  
University of Waterloo

#### Abstract

Wavelength dependent Monte Carlo rendering can correctly and generally capture effects such as spectral caustics (rainbows) and chromatic aberration. It also improves the colour accuracy of reflectance models and of illumination effects such as colour bleeding and metamerism.

The stratified wavelength clustering (SWC) strategy carries several wavelength stratified radiance samples along each light transport path. The cluster is split into several paths or degraded into a single path only if a specular refraction at the surface of a dispersive material is encountered along the path. The overall efficiency of this strategy is high since the fraction of clusters that need to be split or degraded in a typical scene is low, and also because specular dispersion tends to decrease the source colour variance, offsetting the increased amortized cost of generating each path.

**Keywords:** Monte Carlo methods, wavelength dependent (spectral) rendering, caustics, rainbows, refraction, reflectance, global illumination.

#### 1 Introduction

The faults of using three component colour models for computing reflectance, absorption and dispersion are well known

per we demonstrate not only that this is feasible, but also that with a strategy we call *stratified wavelength clustering* (SWC) the marginal cost is negligible for Monte Carlo ray tracing and bidirectional path tracing.

#### 2 Outline

Prior work on wavelength dependent and spectral rendering is surveyed in Section 3. Section 4 presents the stratified wavelength cluster strategy. Three splitting strategies are also presented and compared. In Section 5 a bidirectional path tracer that uses stratified wavelength clusters is described. Finally, in Section 6 we present our results, comparing and contrasting crude Monte Carlo integration over wavelength, quasi Monte Carlo integration over wavelength using Halton sequences, and the stratified wavelength cluster strategy.

#### 3 Background

Before presenting our approach for extending Monte Carlo global illumination algorithms to wavelength-dependent (spectral) rendering, we first review the relationship of spectral power densities to perceived colour and review wavelength-dependent phenomena that have a significant effect on the generated image. Previous algorithms and representations for spectral ren-

Other resources:

Hero Wavelength Spectral Sampling. Wilie et al., 2014.

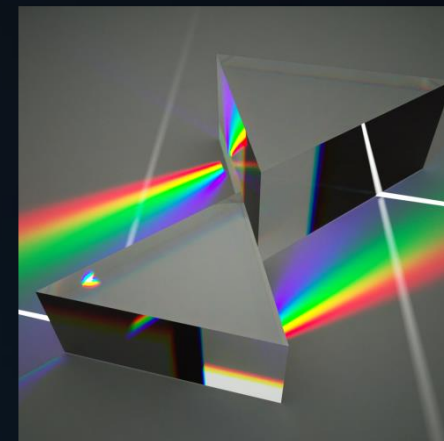
Physically Meaningful Rendering using Tristimulus Colours. Meng et al., 2015.

PBRT. Pharr et al., 2004-2018.

<https://www.pbrt.org>

Mitsuba Renderer. W. Jakob, 2014.

<https://www.mitsuba-renderer.org>





# Future Work

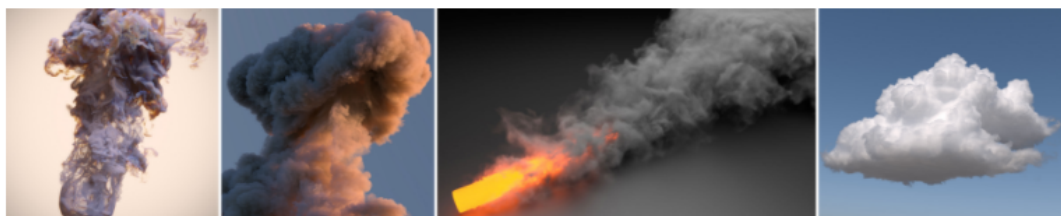
## Participating Media

### Monte Carlo methods for physically based volume rendering

Jan Novák<sup>1</sup> Iliyan Georgiev<sup>2</sup> Johannes Hanika<sup>3</sup> Jaroslav Krřivánek<sup>4</sup> Wojciech Jarosz<sup>5</sup>

<sup>1</sup>Disney Research <sup>2</sup>Solid Angle <sup>3</sup>Karlsruhe Institute of Technology <sup>4</sup>Charles University, Prague <sup>5</sup>Dartmouth College

In ACM SIGGRAPH Courses, 2018



Various media rendered with Monte Carlo methods for physically based simulation of light transport in volumes. The three images on the left are courtesy of Lee Griggs.

Abstract

Downloads

Cite

Related

### Abstract

We survey methods that utilize Monte Carlo (MC) integration to simulate light transport in scenes with participating media. The goal of this course is to complement a recent Eurographics 2018 state-of-the-art report providing a broad overview of most techniques developed to date, including a few methods from neutron transport, with a focus on concepts that are most relevant to CG practitioners.

The wide adoption of path-tracing algorithms in high-end realistic rendering has stimulated many diverse research initiatives aimed at efficiently rendering scenes with participating media. More computational power has enabled holistic approaches that tie volumetric effects and surface scattering together and simplify authoring workflows. Methods that were previously assumed to be incompatible have been unified to allow renderers to benefit from each method's respective strengths. Generally, investigations have shifted away from specialized solutions, e.g. for single- or multiple-scattering approximations or analytical methods, towards the more versatile Monte Carlo algorithms that are currently enjoying a widespread success in many production settings.

The goal of this course is to provide the audience with a deep, up-to-date understanding of key techniques for free-path sampling, transmittance estimation, and light-path construction in participating media, including those that are presently utilized in production rendering systems. We present a coherent overview of the fundamental building blocks and we contrast the various advanced methods that build on them, providing attendees with guidance for implementing existing solutions and developing new ones.

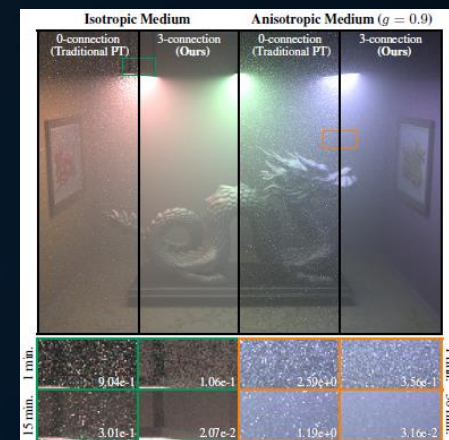
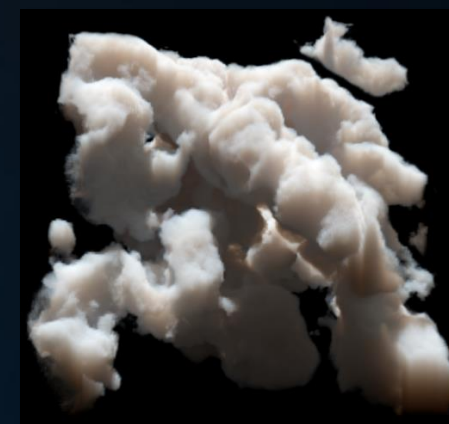
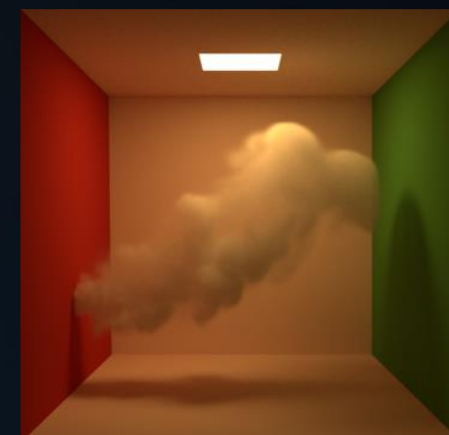
### Downloads

Other resources:

Importance Sampling Techniques for Path Tracing in Participating Media. Kulla and Fajardo, 2014.

Area Light Equi-Angular Sampling on ShaderToy:

<https://www.shadertoy.com/view/ldXGzS>



# Future Work

## Light Transport

### ROBUST MONTE CARLO METHODS FOR LIGHT TRANSPORT SIMULATION

A DISSERTATION  
SUBMITTED TO THE DEPARTMENT OF COMPUTER SCIENCE  
AND THE COMMITTEE ON GRADUATE STUDIES  
OF STANFORD UNIVERSITY  
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS  
FOR THE DEGREE OF  
DOCTOR OF PHILOSOPHY

by  
Eric Veach  
December 1997

Other resources:

The Rendering Equation. James T. Kajiya, 1986.

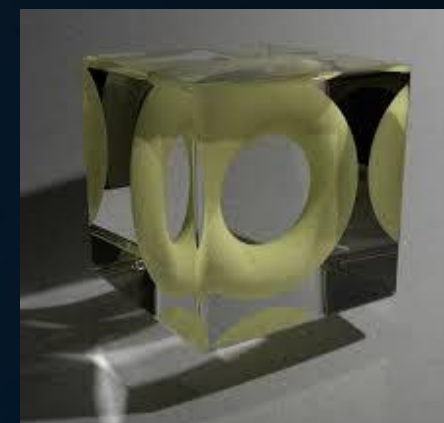
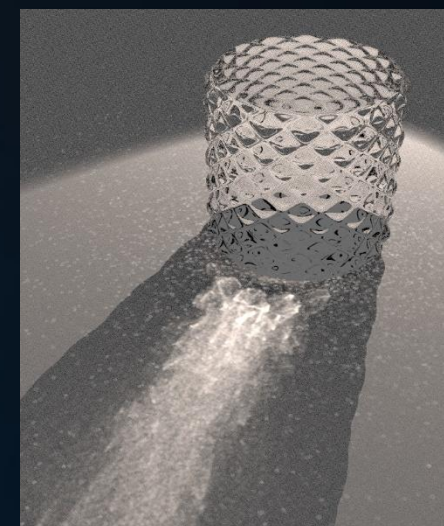
Bidirectional Path Tracing. Michal Vlas, 2018 (student paper).

Global Illumination using Photon Maps. Jensen, 1996.

Progressive Photon Mapping. Hachisuka et al., 2008.

(article on) Vertex Connection and Merging, Georgev et al., 2012.

<https://schuttejoe.github.io/post/vertexconnectionandmerging>





# Future Work

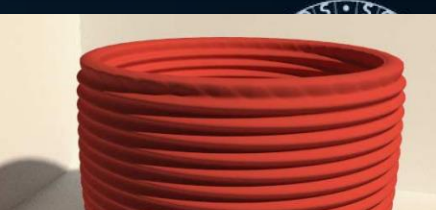
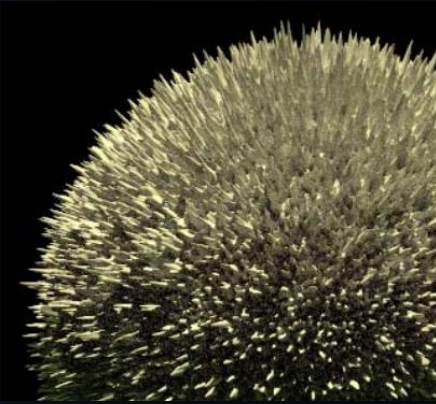
## Primitives

Other resources:

Direct Ray Tracing of Smoothed and Displacement Mapped Triangles. Smits et al., 2000.

Direct Ray Tracing of Phong Tessellation. Ogaki and Tokuyoshi, 2011.

Two-Level Ray Tracing with Reordering for Highly Complex Scenes. Hanika et al., 2010.



### Phantom Ray-Hair Intersector

Alexander Reshetov, David Luebke  
NVIDIA

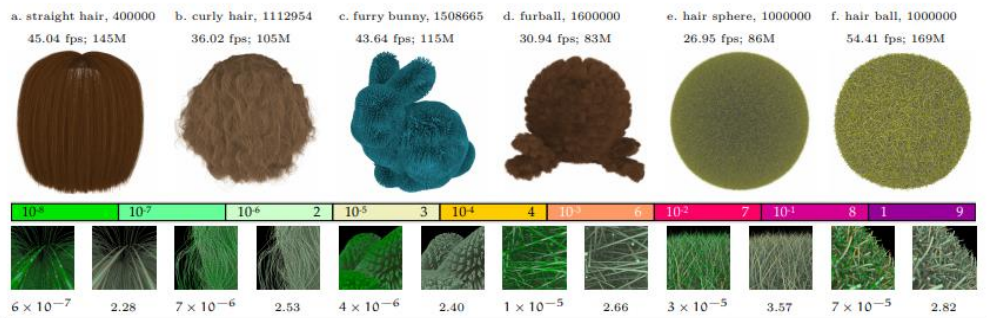


Figure 1. Different models rendered with our technique and number of curves in the model. Second line: the achieved frame rate on a Titan Xp and the total number of rays traced per second, including one primary ray for each pixel at 1000×1000 screen resolution and four ambient occlusion rays for each hit point. For each model, the bottom row gives the average error in curve parameter t (left) and the number of iterations (right). The corresponding inserts show these values for each primary ray according to the heatmap strip in the middle.

#### ABSTRACT

We present a new approach to ray tracing swept volumes along trajectories defined by cubic Bézier curves. It performs at two-thirds of the speed of ray-triangle intersection, allowing essentially even treatment of such primitives in ray tracing applications that require hair, fur, or yarn rendering.

At each iteration, we approximate a radially symmetric swept volume with a tangential cone. A distance from the ray-cone intersection to the cone's base is then used to compute the next curve parameter t. When this distance is zero, the ray intersects the swept volume and the cone at the same point and we stop the iterations. To enforce continuity of the iterative root finding, we introduce "phantom" intersection

#### CCS CONCEPTS

- Computing methodologies → Ray tracing; Parametric curve and surface models;

#### KEYWORDS

Ray tracing, Bézier curves, swept volumes, generalized cylinders, hair and fur rendering

#### ACM Reference Format:

Alexander Reshetov, David Luebke. 2018. Phantom Ray-Hair Intersector. In *Proceedings of HPG'18*. ACM, New York, NY, USA, 12 pages. <https://doi.org/10.1145/3233307>

#### 1 INTRODUCTION AND PRIOR ART



# Future Work

## Production

### Arnold: A Brute-Force Production Path Tracer

ILIYAN GEORGIEV, THIAGO IZE, MIKE FARNSWORTH, RAMÓN MONTTOYA-VOZMEDIANO, ALAN KING, BRECHT VAN LOMMEL, ANGEL JIMENEZ, OSCAR ANSON, SHINJI OGAKI, ERIC JOHNSTON, ADRIEN HERUBEL, DECLAN RUSSELL, FRÉDÉRIC SERVANT, and MARCOS FAJARDO, Solid Angle



Fig. 1. Arnold is a path-tracing renderer used for the production of photo-realistic and art-directed visual effects in feature films (left), commercials (middle), animated films (right), television series, music videos, game cinematics, motion graphics, and others. *Gravity* ©2013 Warner Bros. Pictures, courtesy of Framestore; *Racing Faces* ©2016 Opel Motorsport, courtesy of The Mill; *Captain Underpants* ©2017 DreamWorks Animation.

Arnold is a physically based renderer for feature-length animation and visual effects. Conceived in an era of complex multi-pass rasterization-based workflows struggling to keep up with growing demands for complexity and realism, Arnold was created to take on the challenge of making the simple and elegant approach of brute-force Monte Carlo path tracing practical for production rendering. Achieving this required building a robust piece of ray-tracing software that can ingest large amounts of geometry with detailed shading and lighting and produce images with high fidelity, while scaling well with the available memory and processing power.

Arnold's guiding principles are to expose as few controls as possible, provide rapid feedback to artists, and adapt to various production workflows. In this article, we describe its architecture with a focus on the design

#### ACM Reference format:

Iliyan Georgiev, Thiago Ize, Mike Farnsworth, Ramón Montoya-Vozmediano, Alan King, Brecht Van Lommel, Angel Jimenez, Oscar Anson, Shinji Ogaki, Eric Johnston, Adrien Herubel, Declan Russell, Frédéric Servant, and Marcos Fajardo. 2018. Arnold: A Brute-Force Production Path Tracer. *ACM Trans. Graph.* 37, 3, Article 32 (July 2018), 12 pages. <https://doi.org/10.1145/3182160>

#### 1 INTRODUCTION

At a purely technical level, visual realism in computer-generated imagery boils down to two areas: (1) amount of scene detail, e.g., number of geometric primitives, amount of textures, variety of ma-

### Other resources:

The Iray Light Transport Simulation and Rendering System. Keller et al., 2017.

The Design and Evolution of Disney's Hyperion Renderer. Burley et al., 2018.

Manuka: A batch-shading architecture for spectral path tracing in movie production. Fascione et al., 2018.

Mitsuba 2: A Retargetable Forward and Inverse Renderer. Nimier-David, 2019.





# Future Work

## Real-time

### Real-time Ray Tracing through the Eyes of a Game Developer

Jacco Bikker\*  
IGAD / NHTV University of Applied Sciences  
Breda, The Netherlands



Figure 1: Real-time ray traced images from our experimental ARAUNA engine, used in a student project.

#### ABSTRACT

There has been and is a tremendous amount of research on the topic of ray tracing, spurred by the relatively recent advent of real-time ray tracing and the inevitable appearance of consumer hardware capable of handling this rendering algorithm. Besides researchers, the prospect of a brave new world attracts hobbyists (such as demo coders) and game developers: Ray tracing promises an elegant and fascinating alternative to z-buffer approaches, as well as more intuitive graphics and games development. This article provides a view from the inside on ray tracing in games and demos, where the emphasis is on performance and short-term practical usability. It covers the way science is applied, the unique contribution of these developers to the field and their link with the research community.

The Arauna ray tracer, developed at the NHTV university of applied science, is used as an example of a ray tracer that has been specifically built with games and performance in mind. Its purpose and architecture, as well as some implementation details are presented.

**Index Terms:** L3.7 [Computer Graphics]: Three-dimensional Graphics and Realism—Ray Tracing

#### 1 INTRODUCTION

like Heaven7 [5] and the RealStorm benchmark [6] displayed the capabilities of this somewhat invisible demoscene movement [24].

The importance of ray tracing for games has also been recognised by researchers [17]. Ray tracing has been used extensively for offline rendering of game graphics (e.g. Myst and Riven [36]) and for lighting preprocessing as in e.g. Quake 2 [12]. Games are also frequently used as a test-case for real-time ray tracing: At the Breakpoint 2005 demo party, a fully playable real-time ray traced version of Quake 2 was shown by Wächter and Keller [14]. Ray traced versions of animated Quake 3 [19] and Quake 4 [20] walkthroughs were shown running on a PC cluster, and even the PS3 gameconsole is already being used for ray tracing [21].

Today, it has become clear that advancing ray tracing performance is not just a matter of better high-level algorithms: Low level optimization plays a crucial role. It is also clear that ray tracing performance is not dependent of fast ray traversal alone: Shading cost plays a significant role in real-time ray tracing [21, 31].

This article presents a description of the construction of the Arauna ray tracer, which was built as an exercise in applied science and with performance in mind. The Arauna ray tracer project started as an attempt to implement the ideas presented in Wald's PhD thesis on real-time ray tracing [30]. While previous attempts at interactive ray tracing used approximations (e.g., the Render Cache [34], the holodesk ray cache [15]), the OoepRT ray tracer [23]

Other resources:

OptiX: A General Purpose Ray Tracing Engine. Parker et al., 2010.

REAL-TIME RAYTRACING WITH NVIDIA RTX. Stich, 2018.

(not a lot of research so far...)



Ray Tracing Gems 1 & 2,

<https://research.nvidia.com/publication/2020-07-Spatiotemporal-reservoir-resampling>,  
<https://research.nvidia.com/publication/2019-07-Temporally-Dense-Ray>,  
<https://research.nvidia.com/publication/2019-03-Improving-Temporal-Antialiasing>, ...



Temporally Dense Ray Tracing

Semantic Image Synthesis with Spatially-Adaptive Normalization

A Style-Based Generator Architecture for Generative Adversarial Networks

Dynamic Diffuse Global Illumination with Ray-Traced Irradiance Fields

Near-Eye Display and Tracking Technologies for Virtual and Augmented Reality

NVGaze: An Anatomically-Informed Dataset for Low-Latency, Near-Eye Gaze Tracking

Manufacturing Application-Driven Foveated Near-Eye Displays

Ray Tracing Gems

Improving Temporal Antialiasing with Adaptive Ray Tracing

Cool Patches: A Geometric Approach to Ray/Bilinear Patch Intersections

Precision Improvements for Ray/Sphere Intersection

A Fast and Robust Method for Avoiding Self-Intersection

Texture Level of Detail Strategies for Real-Time Ray Tracing

Simple Environment Map Filtering Using Ray Cones and Ray Differentials

Efficient Generation of Points that Satisfy Two-Dimensional Elementary

Massively Parallel Path Space Filtering

Massively Parallel Construction of Radix Tree Forests for the Efficient Sampling

Fast, High Precision Ray/Fiber Intersection using Tight, Disjoint Bounding

Massively Parallel Stackless Ray Tracing of Catmull-Clark Subdivision Surfaces

A Ray-Box Intersection Algorithm and Efficient Dynamic Voxel Rendering

FocusAR: Auto-focus Augmented Reality Eyeglasses for both Real World and

Image Inpainting for Irregular Holes Using Partial Convolutions

Machine Learning and Rendering

Towards Virtual Reality Infinite Walking: Dynamic Saccadic Redirection

Correlation-Aware Semi-Analytic Visibility for Antialiased Rendering

Adaptive Temporal Antialiasing

Phantom Ray-Hair Intersector

Other resources:

OptiX: A General Purpose Ray Tracing Engine. Parker et al., 2010.

REAL-TIME RAYTRACING WITH NVIDIA RTX. Stich, 2018.

(not a lot of research so far...)

Ray Tracing Gems,

<https://research.nvidia.com/publication/2020-07-Spatiotemporal-reservoir-resampling>,

<https://research.nvidia.com/publication/2020-07-Temporally-Dense-Ray>,

<https://research.nvidia.com/publication/2020-03-Improving-Temporal-Antialiasing>, ...



# Future Work

## Massive Scenes

<https://pharr.org/matt/blog/2018/07/16/moana-island-pbrt-all.html>



Moana Island scene. Heather Pritchett & Rasmus Tamstorf, 2016. *~15 billion primitives.*





# Today's Agenda: *Monte Carlo*



## Sampling an Area Light with One Ray



# Sampling Multiple Area Lights with One Ray



## Difficult Cases: Spherical Lights, Occluded Lights



# The Random Walk



## Random Walk with Next Event Estimation



# ~~Russian Roulette~~



## P3 Topics





# INFOMAGR – Advanced Graphics

Jacco Bikker - November 2021 - February 2022

## END of “Probability”

next up: “Bidirectional”



*That's all Folks!*

