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Fusion in Accelerate

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Includes material from Accelerate source code and slides of Gabriele Keller

June 22, 2021

1. Accelerate

Accelerate


```
dotp :: Acc (Vector Float)
     -> Acc (Vector Float)
     -> Acc (Scalar Float)
dotp xs ys = fold (+) 0 (zipWith (*) xs ys)
```

Outline

- Introduction to Accelerate
- How fusion currently works
- Current research: extend fusion & in-place updates

Arrays

- Array programming
- `Acc`: array computation
- Multi-dimensional
- Shapes: `Z` or `sh :. Int`
- `type Scalar a = Array Z a`
- `type Vector a = Array (Z :. Int) a`
- `type Matrix a = Array (Z :. Int :. Int) a`

Combinators

Language only includes parallelizable constructs.

- Map
- Folds and scans
- Permute (scatter, random writes)
- Backpermute (gather, random reads)
- Stencil (map with neighbourhood)

And control flow to compose those parallel operations: conditionals, loops, let-bindings, tuples.

Domain specific language

- Combinators imply the parallel structure.
- No need for (fragile) analyses to recover parallel structure.

Embedded DSL

- Haskell gives polymorphism and support for higher order functions.
- In our internal AST, programs are monomorphic and first order.

☞ Analyses are simpler in this monomorphic combinator-based language.

Map


```
map :: (Shape sh, Elt a, Elt b)
    => (Exp a -> Exp b)
    -> Acc (Array sh a)
    -> Acc (Array sh b)
plusOne = map (+1)
```

- Each thread calculates one element of the array

Backpermute


```
backpermute :: (Shape sh, Shape sh', Elt a)
            => Exp sh'
            -> (Exp sh' -> Exp sh)
            -> Acc (Array sh  a)
            -> Acc (Array sh' a)

reverse xs = backpermute (shape xs)
    (\(I1 i) -> I1 (size xs - 1 - i)) xs
```

- Random reads

Transform


```
transform :: (Shape sh, Shape sh'
             , Elt a, Elt b)
          => Exp sh'
          -> (Exp sh' -> Exp sh)
          -> (Exp a -> Exp b)
          -> Acc (Array sh a)
          -> Acc (Array sh' b)

foo = reverse . plusOne
```

- Composition of `backpermute` and `map`.
- Not exposed to the user.
- Result of fusion.

Generate


```
generate :: (Shape sh, Elt a)
         => Exp sh
         -> (Exp sh -> Exp a)
         -> Acc (Array sh a)
generate (constant (I1 100))
         (\(I1 i) -> i * 2)
-- [0, 2, 4, ...]
```

- Each thread computes the value of one index.
- Could be used to implement `map`, `backpermute` and `transform`,
- but then we would lose some structure.

Folds


```
fold :: (Shape sh, Elt a)
     => (Exp a -> Exp a -> Exp a)
     -> Exp a
     -> Acc (Array (sh :. Int) a)
     -> Acc (Array sh a)
```

- Associative operator to enable parallel reduction.
- Other variants: fold without initial value, segmented folds, scans.

More combinators

- Permute: random writes
- Conditional
- While
- Zip, unzip
- Pairs
- Slice, replicate
- Stencil

AST

- Generalized algebraic datatypes (GADT).
- Typed environment and result.
- Pattern functor: type argument acc for recursive positions.

```
data PreOpenAcc acc aenv a where
  Avar  :: ArrayVar           aenv (Array sh e)
        -> PreOpenAcc acc aenv (Array sh e)
  Map   :: TupleType e'
        -> Fun                aenv (e -> e')
        -> acc                aenv (Array sh e)
        -> PreOpenAcc acc aenv (Array sh e')
  ...
newtype OpenAcc aenv t =
  OpenAcc (PreOpenAcc OpenAcc aenv t)
```

Current pipeline

- Sharing recovery
- Simplify tuples
- Fusion
- Code generation with LLVM

Runtime compiler

- Accelerate operates at Haskell runtime
- We compile and run the program at Haskell runtime
- Allows meta programming
- Compilation at Haskell compile time is possible with Template Haskell

Sharing recovery

- The combinators construct an AST representing the computation
- Some nodes may be used multiple times in the tree
- Make this explicit by adding let bindings

```
let
  xs = generate ..
in
  T2 xs (fold (+) 0 xs)
```

2. Fusion in the current pipeline

Fusion

- Programming model advocates splitting the program into many kernels.
- A naive implementation would result in the creation of many intermediate arrays.
- Fusion: combine multiple kernels into one.
- Mandelbrot had a speed up of 1000%, typically 50%.

Minimize:

- Number of kernels
- Number of (intermediate) arrays
- Number of memory operations

Example

$y = \textbf{map}\ f\ \$\ \textbf{map}\ g\ x$

- Why create the intermediate array for the result of the right `map`?
- Fusion will prevent the creation of that array.
- Result: `y = map (f . g) x`

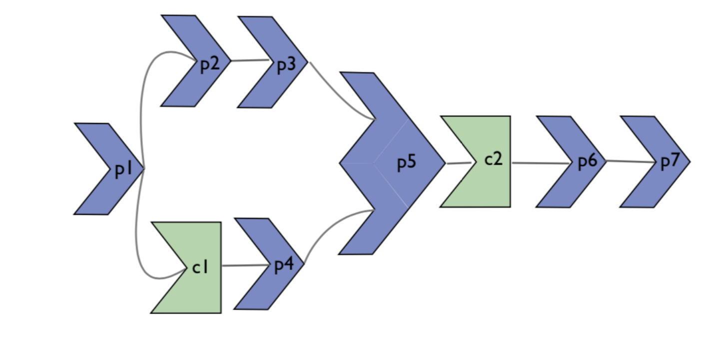
Example


```
dotp xs ys = fold (+) 0 (zipWith (*) xs ys)
```

- Why create the intermediate array for the result of `zipWith`?
- Fusion will prevent the creation of that array.
- We cannot represent the result in the same AST data type.

Elementwise and collective operations

- *Elementwise*: Each element of the result depends on at most one element of input array (e.g, map, backpermute, generate).
- *Collective*: Each element of result depends on multiple elements of input array (e.g., folds, scans, stencil operations).
- Fusion treats them separately:
  - Elementwise/Elementwise fused via program transformation.
  - Elementwise/Collective during code generation.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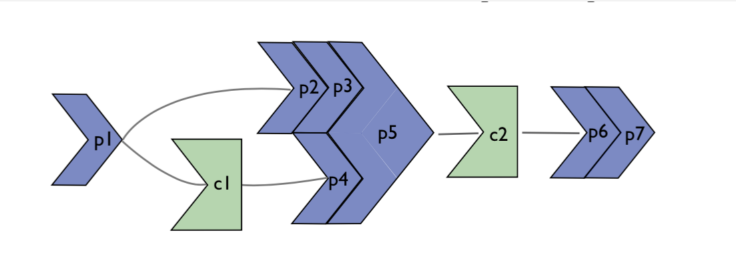
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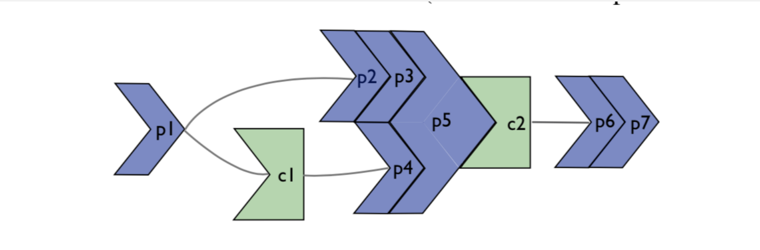
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Elementwise operations as functions

- Represent elementwise operations as a function from index to value.
- In the dot product example:
  `\ix -> (xs ! ix) * (ys ! ix)`.
- We also need an expression of the size (shape) of the array.
- We use this for elementwise/collective fusion.
- However, after elementwise/elementwise fusion we still want to know which elementwise operation we have.

Cunctation

We convert an array computation into a cunctation, which is either:

Done : Terms which we cannot fuse.

Yield : An expression of its size and a function from index to value. Similar to `generate`.

Step : An array variable, an index transform and a value transform. Similar to `transform`.

☞ Step $<$ Yield $<$ Done

Cunctation of `map`

- Map of a Step becomes a Step
- Map of a Yield becomes a Yield
- Map of a Done: manifest the argument array, then return a Step

Elementwise/Elementwise fusion

- Convert cunctations back to elementwise operations.
- `Yield` becomes a `generate`.
- `Step` becomes:
  - a `backpermute` if the value transform was an identity function.
  - a `map` if the index transform was an identity function (and the size is preserved).
  - a `transform` otherwise.

☞ A `map` may allow more other optimisations than a `transform`, like in-place updates.

Pattern functors

AST is defined as a pattern functor, closed by OpenAcc:

```
newtype OpenAcc aenv t =
  OpenAcc (PreOpenAcc OpenAcc aenv t)
```

Use a different data type to close it:

```
data DelayedOpenAcc aenv a where
  Manifest :: PreOpenAcc DelayedOpenAcc aenv a
           -> DelayedOpenAcc aenv a
  Delayed  :: ArrayR (Array sh e)
           -> Exp aenv sh
           -> Fun aenv (sh  -> e)
           -> Fun aenv (Int -> e)
           -> DelayedOpenAcc aenv (Array sh e)
```

Pattern functors

- Allows to store information anywhere in the tree.
- Type and effects: you could store variables of the proof tree this way.
- Sharing recovery does that in multiple steps: `UnscopedAcc`, `ScopedAcc`, before going to the resulting `Acc`.
- Disadvantage: We could store a `Delayed` at locations where we really expect a manifest array.

Code generation

- Delayed terms stay present in the AST until code generation.
- Embed the code of elementwise function into the code of the collective operation.
- Example of dot product: instead of performing indexing in the code of the fold, add code of
  `\ix -> (xs ! ix) * (ys ! ix)`.

After the break

What is the best way to compile this to imperative code?

```
as = map f1 x
bs = map f2 x

cs = map (\y -> bs !! y) as
ds = map (\y -> as !! y) bs
```

Note that (!!) is indexing in arrays, so constant time.

3. Current research

Current research

- Horizontal and diagonal fusion like (map f xs, map g xs).
- Collective/Collective fusion
- In-place updates in more cases

This results in many options how a program can be transformed. Greedy approach doesn't work anymore.

An example


```
as = map f1 xs
bs = map f2 xs

cs = map (\y -> bs !! y) as
ds = map (\y -> as !! y) bs
```

Fusion becomes a clustering problem. Options include:

- $\{as, bs\}$, $\{cs, ds\}$ (horizontal fusion)
- $\{as\}$, $\{bs, ds\}$, $\{cs\}$ (diagonal fusion)
- $\{bs\}$, $\{as, cs\}$, $\{ds\}$ (diagonal fusion)

Horizontal fusion

$\{as, bs\}, \{cs, ds\}$

```
as, bs, cs, ds = new arrays;
parallel for i in 0..n
  { x = xs[i]; as[i] = f1(x); bs[i] = f2(x); }
parallel for i in 0..n
  { cs[i] = bs[as[i]]; ds[i] = as[bs[i]]; }
```

Diagonal fusion

$\{as\}$, $\{bs, ds\}$, $\{cs\}$

```
as, bs, cs, ds = new arrays;
parallel for i in 0..n
  { as[i] = f1(xs[i]); }
parallel for i in 0..n
  { b = f2(xs[i]; bs[i] = b; ds[i] = as[b]; }

parallel for i in 0..n
  { cs[i] = bs[as[i]]; }
```

In-place updates

- Reuse an input array instead of allocating a new array.
- The input should be a unique reference.
- A map can reuse the input array if it has the same element type (or more general, the same element size).
- A permute in general has to copy the defaults array. If we can perform in-place updates, we don't need to copy that.
- For permute, in-place updates can result in a lower time complexity.

Uniqueness?

We can safely perform in-place updates on unique references.

- Uniqueness: there is only one reference to a variable.
- Temporal uniqueness: there is only one reference to a variable *at the time we perform the in-place update*.

Thus, we can perform in-place updates if we can reorder the program such that other uses of the array happen earlier.

In-place updates in the example


```
as = map f1 xs
bs = map f2 xs

cs = map (\y -> bs !! y) as
ds = map (\y -> as !! y) bs
```

The map of c may perform an in-place update on a if the map of d is executed earlier.

Diagonal fusion

$\{as\}$, $\{bs, ds\}$, $\{cs\}$

```
as, bs, cs, ds = new arrays;
parallel for i in 0..n
  { as[i] = f1(xs[i]); }
parallel for i in 0..n
  { b = f2(xs[i]; bs[i] = b; ds[i] = as[b]; }

parallel for i in 0..n
  { cs[i] = bs[as[i]]; }
```

Diagonal fusion, with in-place updates

$\{as\}, \{bs, ds\}, \{cs\}$

```
as, bs, ds = new arrays;
parallel for i in 0..n
  { as[i] = f1(xs[i]); }
parallel for i in 0..n
  { b = f2(xs[i]; bs[i] = b; ds[i] = as[b]; }
cs = as;
parallel for i in 0..n
  { cs[i] = bs[as[i]]; }
```

What is the best option?

- First option: 2 loops, 4 arrays
- Second option: 3 loops, 3 arrays
- And 9 other options, of which 3 are maximal

Interaction with in-place updates

- The best option for fusion prevents in-place updates
- The best option for in-place updates prevents fusion
- Can we decide on them in one analysis?

Analysis

- Fusion becomes a clustering problem
- Modelled as an ILP (Integer Linear Program, not instruction level parallellism!)
- Extend the ILP with in-place updates

Objective function

Minimize:

- Number of kernels
- Number of (intermediate) arrays
- Number of memory operations

Decision variables

- For each combinator, at which point in time it should be executed
- For an input of map or permute, whether we perform in-place updates

Constraints

- Fusion: enforce dependencies in order
- In-place: if we perform in-place updates, ensure that other uses of the array happen earlier

Implementation work

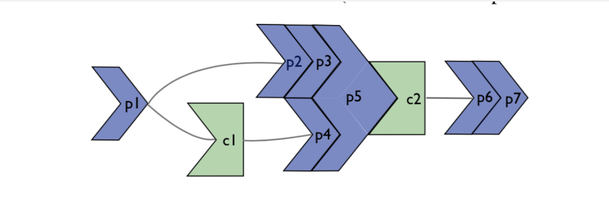
- After solving the ILP, we must transform the program accordingly
- Add synchronisation points to ensure temporal uniqueness
- Task parallelism: execute multiple kernels at the same time
- Typed environment makes large program transformations difficult

Other goals of the new pipeline

- Lower runtime overhead
- More backend specific optimisations

Let bindings

- Array computations can be bound to variables.
- Fusing those may duplicate work.
- Currently: only fuse those if the variable is used once.
- Desired: fuse those in many cases, as the cost of memory is usually higher than the computational cost.



Conclusion

- Program analyses are easier on the DSL
- Some analyses are not needed at all: the combinators imply the parallel structure
- Being an EDSL we also need to perform other analysis than a classic compiler (sharing recovery)
- Many ways to transform a program with fusion and in-place updates
- Use an ILP to make the decision