Universiteit Utrecht

[Faculty of **Science**
Information and Computing Sciences]

Constraint-based Type Error Diagnosis (Tutorial)

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About me

- Full professor and Head of Department, Heriot-Watt University, Edinburgh
- Before that, two decades at Utrecht University
- Topics of interest:
  - Static analysis of functional languages
    - Non-standard/type and effect systems
  - Program plagiarism detection, object-sensitive analysis, soft typing of dynamic languages, and switching classes
  - PhD students active in legacy system modernization, and testing
  - Type error diagnosis (for functional languages/EDSLs)
  - PhD positions in Edinburgh?

Credits

The following people have contributed to this talk:

- Alejandro Serrano Mena, current PhD student
- Bastiaan Heeren, PhD student between 2000-2004
- Patrick Bahr, visiting postdoc in 2014
- Atze Dijkstra, implementor of UHC
- Many master students
- Many people contributed to Helium

I. Introduction and Motivation

Static type systems

- Statically typed languages come equiped with an intrinsic type system, preventing some structurally correct programs from being compiled
- "well-typed programs can't go wrong"
- type incorrect programs ⇒ the need for diagnosis
- When type checking we typically assume various simple local properties to have been checked:
  - syntactic correctness
  - well-scopedness
  - definedness of variables
- Which properties it enforces, depends intimately on the language
  - Cf. does every function have the right number of arguments in C vs. Haskell

What is type error diagnosis?

- Type error diagnosis is the problem of communicating to the programmer that and/or why a program is not type correct
- This may involve information
  - that a program is type incorrect
  - which inconsistency was detected
  - which parts of the program contributed to the inconsistency
  - how the inconsistency may be fixed
- Traditionally, functional languages have more room for inconsistencies $\Rightarrow$ at least some attention was paid to type error diagnosis

Languages follow Lehmann's sixth law

- Java has seen the introduction of parametric polymorphism (and type errors suffered)
- Java has seen the introduction of anonymous functions (I have not dared look)
- Languages like Scala embrace multiple paradigms
- Odersky's "type wall": unless complicated type system features are balanced by better diagnosis, programmers will flock to dynamic languages
- In terms of maintainability of (sizable) programs, dynamic languages do not seem to scale well
- New trends: dynamic languages becoming more static
- Again, the need for diagnosis

Some simple Haskell

$reverse = foldr\ (flip\ (:))\ [\,]$
$palindrome\ xs = reverse\ xs == xs$

Is this program well typed?

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```
Occurs check: cannot construct the infinite type: t ~ [[t]]
  Expected type: [t]
  Actual type: [[[t]]]
In the second argument of '(==)', namely 'xs'
In the expression: reverse xs == xs
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What is wrong?


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- It does not point to the source of the error → not precise
- It's intimidating → not succint
- It shows an artifact of the implementation → mechanical
  - "Occurs check" is part of the unification algorithm
- Generally, message not very helpful
- Anyone know the likely fix?

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  - "Occurs check" is part of the unification algorithm
- Generally, message not very helpful
- Anyone know the likely fix? *foldr* should be *foldl*

Unresolved top-level overloading

$$
\begin{aligned}
&xxxx = xs : [4, 5, 6] \\
&\quad \textbf{where } len = length\ xs \\
&\qquad xs = [1, 2, 3]
\end{aligned}
$$

Unresolved top-level overloading

$$xxxx = xs : [4, 5, 6]$$
$$\textbf{where } len = length\ xs$$
$$xs = [1, 2, 3]$$

The Hugs message (GHC's message is just more verbose)

```
ERROR "Main.hs":1 - Unresolved top-level overloading
*** Binding             : xxxx
*** Outstanding context : (Num [b], Num b)
```

- Type classes make the type error message hard to understand
- The location of the mistake is rather vague
- No suggestions how to fix the program

Very old school parser combinators

$$
\begin{aligned}
&pExpr = pAndPrioExpr \\
&\quad <|> sem\_Expr\_Lam \\
&\quad\quad \langle\$ pKey\ \texttt{"\textbackslash\textbackslash"} \\
&\quad\quad \langle*\rangle pFoldr1\ (sem\_LamIds\_Cons, sem\_LamIds\_Nil)\ pVarid \\
&\quad\quad \langle*\rangle pKey\ \texttt{"->"} \\
&\quad\quad \langle*\rangle pExpr
\end{aligned}
$$

gives

```
ERROR "BigTypeError.hs":1 - Type error in application
*** Expression     : sem_Expr_Lam <$ pKey "\\" <*> pFoldr1 (sem_LamIds_Cons,sem_
LamIds_Nil) pVarid <*> pKey "->"
*** Term           : sem_Expr_Lam <$ pKey "\\" <*> pFoldr1 (sem_LamIds_Cons,sem_
LamIds_Nil) pVarid
*** Type           : [Token] -> [((Type -> Int -> [([Char],(Type,Int,Int))] -> I
nt -> Int -> [(Int,(Bool,Int))] -> (PP_Doc,Type,a,b,[c] -> [Level],[S] -> [S]))
-> Type -> d -> [([Char],(Type,Int,Int))] -> Int -> Int -> e -> (PP_Doc,Type,a,b
,f -> f,[S] -> [S]),[Token])]
*** Does not match : [Token] -> [([Char] -> Type -> d -> [([Char],(Type,Int,Int)
)] -> Int -> Int -> e -> (PP_Doc,Type,a,b,f -> f,[S] -> [S]),[Token])]
```

Order is arbitrary (in Hugs)

$$yyyy :: (Bool \mathrel{-\!\!>} a) \mathrel{-\!\!>} (a, a, a)$$
$$yyyy = \backslash\, f \mathrel{-\!\!>} (f\ True, f\ False, f\ [\,])$$

What's wrong with this program?

Order is arbitrary (in Hugs)

$$yyyy :: (Bool \mathrel{->} a) \mathrel{->} (a, a, a)$$
$$yyyy = \backslash f \mathrel{->} (f\ True, f\ False, f\ [\,])$$

What's wrong with this program?

```
ERROR "Main.hs":2 - Type error in application
*** Expression     : f False
*** Term           : False
*** Type           : Bool
*** Does not match : [a]
```

- There is a lot of evidence that `f False` is well typed
- The type signature is not taken into account
- The type inference process suffers from (**right-to-left**) bias

Order is arbitrary (in GHC)

$$zzzz = \backslash f \rightarrow (f\ [\,], f\ \mathit{True}, f\ \mathit{False})$$

```
Ov.hs:8:23:
    Couldn't match expected type '[t2]' with actual type 'Bool'
    Relevant bindings include
      f :: [t2] -> t (bound at Ov.hs:8:9)
      zzzz :: ([t2] -> t) -> (t, t, t) (bound at Ov.hs:8:1)
    In the first argument of 'f', namely 'True'
    In the expression: f True
```

- No signature to take into account
- Both *f True* and *f False* are found to be in error
- The type inference process suffers from (**left-to-right**) bias

Good Error Reporting Manifesto

From Improved Type Error Reporting by Yang, Trinder and Wells

1. Correct detection and correct reporting
2. Precise: the smallest possible location
3. Succint: maximize useful and minimize non-useful info
4. Does not depend on implementation, i.e., amechanical
5. Source-based: not based on internal syntax
6. Unbiased
7. Comprehensive: enough to reason about the error

II. Constraint-based Type Inference

Hindley-Milner (intuitive summary)

- Consider the expression $\backslash x \mathrel{-\!\!>} x + 2$.
- Hindley-Milner will
  - introduce a fresh α for x
  - look at the body $x + 2$: unify the arguments of $+$ with their formal types (here all *Int*)
  - α becomes *Int*, and the whole expression has type *Int* $\mathrel{-\!\!>}$ *Int*

Adding let-polymorphism to the mix

- Consider

  let $y = \backslash\, z \texttt{->} z$
  in $\backslash\, x \texttt{->} y\ x + 2$

- For z, α_1 is introduced, so that the body of y has type α_1
- Since α_1 does not show up in any other type (it is free) we may generalize over α_1 so that $y :: \forall\, \beta\ .\ \beta \texttt{->} \beta$
- Visit the body, introducing α for x, and instantiating β in y to, say, α_2 to give $\alpha_2 \texttt{->} \alpha_2$
- Unifying α with α_2 will identify the two, (arbitrarily) leading to $x :: \alpha$ and the instance of $y :: \alpha \texttt{->} \alpha$
- Then we perform the unifications of the previous slide

The polymorphic lamdba-calculus

$$\frac{\tau \prec \Gamma(x)}{\Gamma \vdash_{\mathrm{HM}} x : \tau}$$

$$\frac{\Gamma \vdash_{\mathrm{HM}} e_1 : \tau_1 \rightarrow \tau_2 \qquad \Gamma \vdash_{\mathrm{HM}} e_2 : \tau_1}{\Gamma \vdash_{\mathrm{HM}} e_1\ e_2 : \tau_2}$$

$$\frac{\Gamma \backslash x \cup \{x : \tau_1\} \vdash_{\mathrm{HM}} e : \tau_2}{\Gamma \vdash_{\mathrm{HM}} \lambda x \rightarrow e : (\tau_1 \rightarrow \tau_2)}$$

$$\frac{\Gamma \vdash_{\mathrm{HM}} e_1 : \tau_1 \qquad \Gamma \backslash x \cup \{x : \mathit{generalize}(\Gamma, \tau_1)\} \vdash_{\mathrm{HM}} e_2 : \tau_2}{\Gamma \vdash_{\mathrm{HM}} \mathrm{let}\ x = e_1\ \mathrm{in}\ e_2 : \tau_2}$$

- Algorithm $\mathcal{W}$ is a (deterministic) implementation of these typing rules.

Characteristics of Algorithm $\mathcal{W}$

- Can infer most general types for the let-polymorphic lambda-calculus
- Can deal with user-provided type information
- For extensions like higher-ranked types, type signatures must be provided
- Binding group analysis may need to be performed (always messy)
- Minor disadvantage: let-polymorphism does not integrate that well with some advanced type system features.
- Major disadvantage: algorithmic bias

What bias?

- Unifications are performed in a fixed order
- Order may be changed: many alternative implementations of HM exist
- Order of unification is unimportant for the resulting types,
- but it is important if you blame the first unification that is inconsistent with the foregoing.

How to cope

1. Investigate families of implementations (=solving orders) algorithm W, M, G, H,...
   - But which one to use when?

How to cope

1. Investigate families of implementations (=solving orders) algorithm W, M, G, H,...
   - But which one to use when?
2. Take a constraint-based approach, separating the unifications (=constraints) from the order in which they are solved.
   - generate and collect the constraints that describe the unifications that were to be performed, e.g., $\alpha == \mathit{Int}$
   - choose the order to solve them in some way that may be determined by the programmer, or by the program
   - Or even better: consider constraints a set at the time to identify situations that are known to often cause mistakes and suggest fixes

Constraint-based type inference

- Popular approach (see Pottier et al., Wells et al., OutsideIn(X), Pavlinovic et al.)
- A basic operation for type inference is unification. Property: let S be $\mathit{unify}(\tau_1, \tau_2)$, then $S\tau_1 = S\tau_2$

We can view unification of two types as a constraint.

Constraint-based type inference

- Popular approach (see Pottier et al., Wells et al., OutsideIn(X), Pavlinovic et al.)
- A basic operation for type inference is unification.
  Property: let S be $\mathit{unify}(\tau_1, \tau_2)$, then $S\tau_1 = S\tau_2$

We can view unification of two types as a constraint.

- An equality constraint imposes two types to be equivalent.
  Syntax: $\tau_1 \equiv \tau_2$
- We define satisfaction of an equality constraint as follows.
  $\mathcal{S}$ satisfies $(\tau_1 \equiv \tau_2) \quad =_{\mathsf{def}} \quad \mathcal{S}\tau_1 = \mathcal{S}\tau_2$
- Example:
  - $[\tau_1 := \mathit{Int}, \tau_2 := \mathit{Int}]$ satisfies $\tau_1 \rightarrow \tau_1 \equiv \tau_2 \rightarrow \mathit{Int}$

Bottom-up typing rules

$$\{x:\beta\},\ \emptyset\ \vdash_{\mathrm{BU}}\ x:\beta \qquad [\textsc{Var}]_{\mathrm{BU}}$$

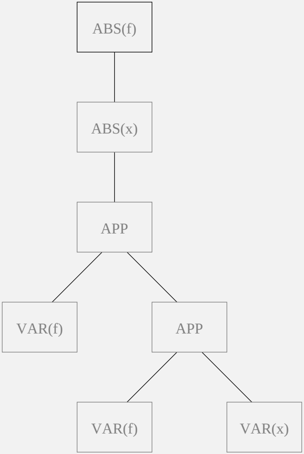
$$\frac{\mathcal{A}_1,\ \mathcal{C}_1\ \vdash_{\mathrm{BU}}\ e_1 : \tau_1 \qquad \mathcal{A}_2,\ \mathcal{C}_2\ \vdash_{\mathrm{BU}}\ e_2 : \tau_2}{\mathcal{A}_1 \cup \mathcal{A}_2,\ \mathcal{C}_1 \cup \mathcal{C}_2 \cup \{\tau_1 \equiv \tau_2 \to \beta\}\ \vdash_{\mathrm{BU}}\ e_1\ e_2 : \beta} \qquad [\textsc{App}]_{\mathrm{BU}}$$

$$\frac{\mathcal{A},\ \mathcal{C}\ \vdash_{\mathrm{BU}}\ e : \tau}{\mathcal{A}\backslash x,\ \mathcal{C} \cup \{\tau' \equiv \beta \mid x:\tau' \in \mathcal{A}\}\ \vdash_{\mathrm{BU}}\ \lambda x \to e : (\beta \to \tau)} \qquad [\textsc{Abs}]_{\mathrm{BU}}$$

- A judgement ($\mathcal{A},\ \mathcal{C}\ \vdash_{\mathrm{BU}}\ e : \tau$) consists of the following.
  - $\mathcal{A}$: assumption set (contains assigned types for the free variables)
  - $\mathcal{C}$: constraint set
  - e: expression
  - τ: asssigned type (variable)

Example

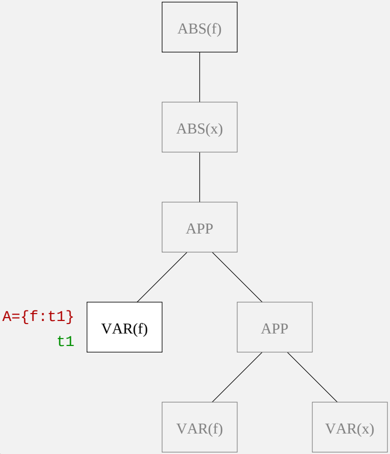
$$twice = \backslash f \; {-}{>} \; \backslash x \; {-}{>} \; f \; (f \; x)$$



Constraints

Example

$$twice = \backslash f \rightarrow \backslash x \rightarrow f\ (f\ x)$$



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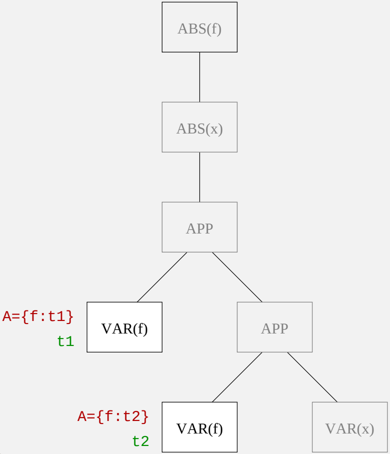

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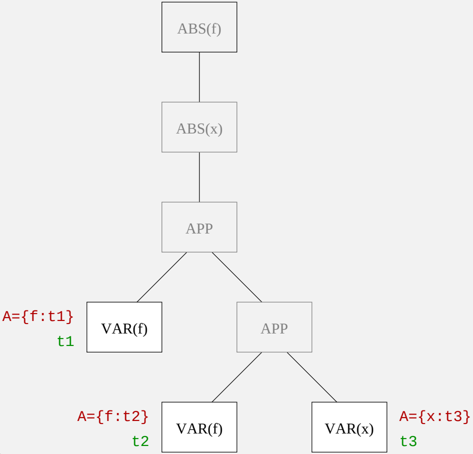

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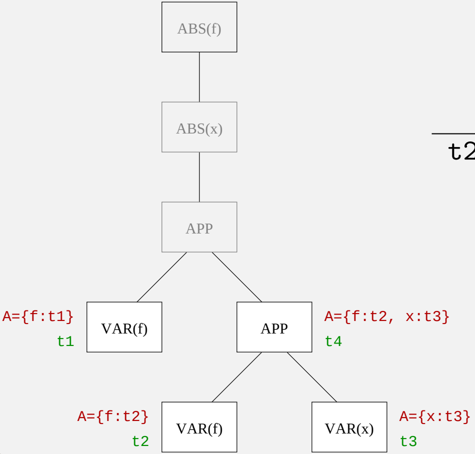

Constraints		
t2	≡	t3 -> t4

Example

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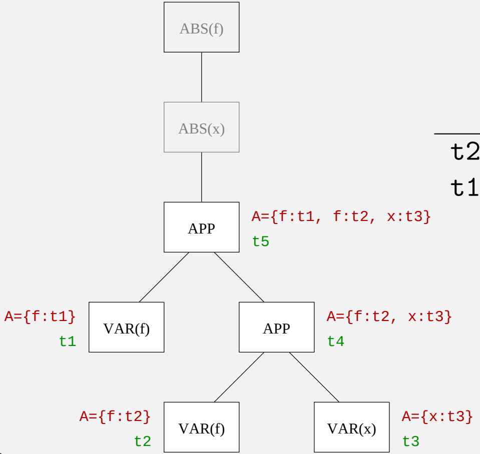

Constraints		
t2	≡	t3 -> t4
t1	≡	t4 -> t5

Example

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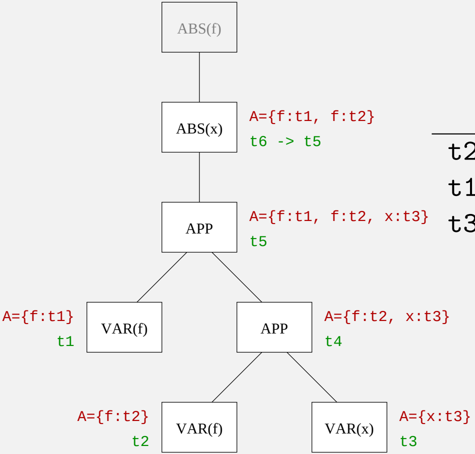

Constraints		
t2	≡	t3 -> t4
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t3	≡	t6

Example

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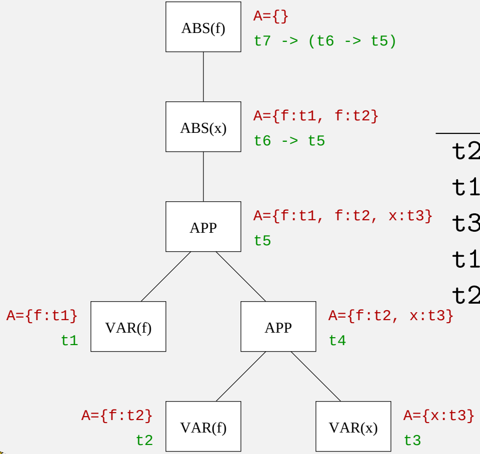

Constraints		
t2	≡	t3 -> t4
t1	≡	t4 -> t5
t3	≡	t6
t1	≡	t7
t2	≡	t7

Example

$$twice = \backslash f \rightarrow \backslash x \rightarrow f\ (f\ x)$$

- $\mathcal{C} = \begin{cases} \texttt{t2} \equiv \texttt{t3 -> t4} \\ \texttt{t1} \equiv \texttt{t4 -> t5} \\ \texttt{t3} \equiv \texttt{t6} \\ \texttt{t1} \equiv \texttt{t7} \\ \texttt{t2} \equiv \texttt{t7} \end{cases}$
- $\mathcal{S} = \begin{cases} \texttt{t1,t2,t7} := \texttt{t6 -> t6} \\ \texttt{t3,t4,t5} := \texttt{t6} \end{cases}$
- $\mathcal{S}$ satisfies $\mathcal{C}$ (moreover, $\mathcal{S}$ is a minimal substitution that satisfies $\mathcal{C}$). As a result, we have inferred the type

$$\mathcal{S}(\texttt{t7 -> t6 -> t5}) = \texttt{(t6 -> t6) -> t6 -> t6}$$

for `twice`.

Constraints and polymorphism

- Syntax of an instance constraint:

$$\tau_1 \leqslant_M \tau$$

- Semantics with respect to a substitution $\mathcal{S}$:

$$\mathcal{S} \text{ satisfies } (\tau_1 \leqslant_M \tau_2) \quad =_{\mathsf{def}} \quad \mathcal{S}\tau_1 \prec \mathit{generalize}(\mathcal{S}M, \mathcal{S}\tau_2)$$

- Example:
  - $[$`t1 := t2, t4 := t5 -> t5`$]$ satisfies `t4` $\leqslant_\emptyset$ `t1 -> t2`

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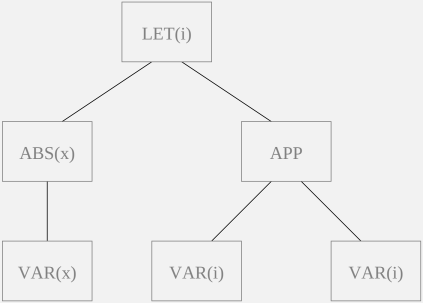
$$\frac{\mathcal{A}_1,\ \mathcal{C}_1 \ \vdash_{\mathrm{BU}}\ e_1 : \tau_1 \qquad\qquad \mathcal{A}_2,\ \mathcal{C}_2 \ \vdash_{\mathrm{BU}}\ e_2 : \tau_2}{\mathcal{A}_1 \cup \mathcal{A}_2 \backslash x,\ \mathcal{C}_1 \cup \mathcal{C}_2 \cup \{\tau' \leqslant_M \tau_1 \mid x : \tau' \in \mathcal{A}_2\} \ \vdash_{\mathrm{BU}}\ \textbf{let } x = e_1 \textbf{ in } e_2 : \tau_2} \quad [\textsc{Let}]_{\mathrm{BU}}$$

Example

$$identity = \textbf{let}\ i = \backslash x \rightarrow x\ \textbf{in}\ i\ i$$

Constraints



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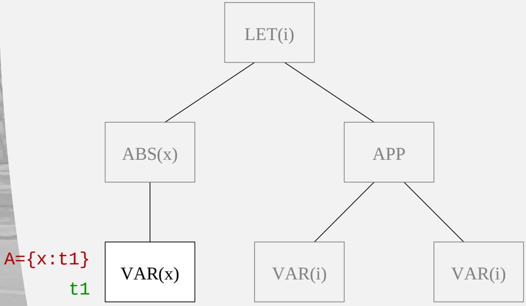

Example

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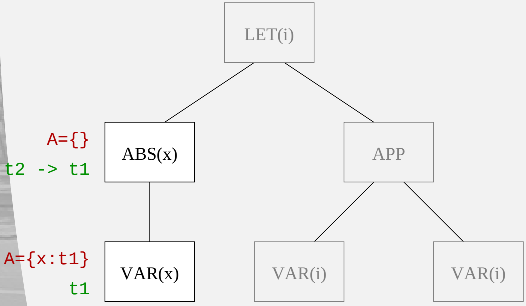
Constraints		
t1	≡	t2



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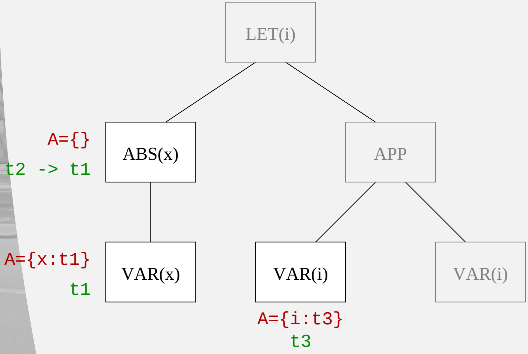

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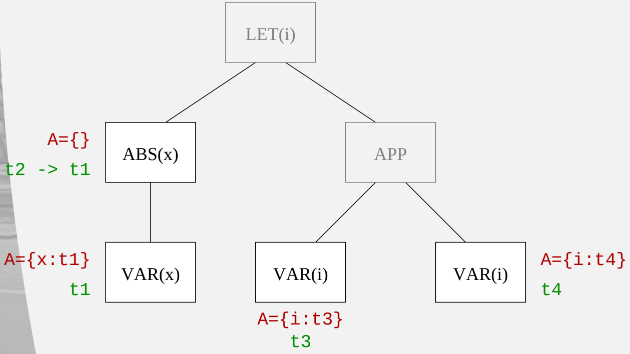
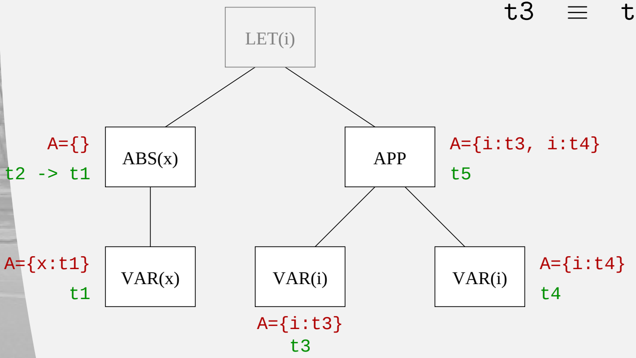


Example

identity = **let** *i* = \ *x* -> *x* **in** *i* *i*

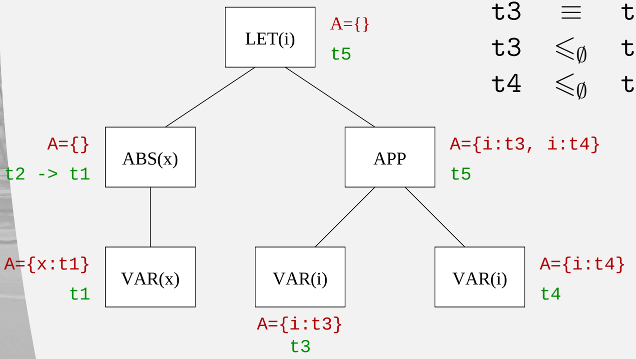
Constraints		
t1	≡	t2
t3	≡	t4 -> t5



Example

$$\textit{identity} = \textbf{let}\ i = \backslash\, x \rightarrow x\ \textbf{in}\ i\ i$$

Constraints		
t1	$\equiv$	t2
t3	$\equiv$	t4 -> t5
t3	$\leqslant_{\emptyset}$	t2 -> t1
t4	$\leqslant_{\emptyset}$	t2 -> t1



Example

$$identity = \textbf{let } i = \backslash x \texttt{ -> } x \textbf{ in } i\ i$$

- $\mathcal{C} = \begin{cases} \texttt{t1} & \equiv & \texttt{t2} \\ \texttt{t3} & \equiv & \texttt{t4 -> t5} \\ \texttt{t3} & \leqslant_{\emptyset} & \texttt{t2 -> t1} \\ \texttt{t4} & \leqslant_{\emptyset} & \texttt{t2 -> t1} \end{cases}$
- $\mathcal{S} = \begin{cases} \texttt{t1} & := & \texttt{t2} \\ \texttt{t3} & := & \texttt{(t6 -> t6) -> t6 -> t6} \\ \texttt{t4,t5} & := & \texttt{t6 -> t6} \end{cases}$
- $\mathcal{S}$ satisfies $\mathcal{C}$ (moreover, $\mathcal{S}$ is a minimal substitution that satisfies $\mathcal{C}$). As a result, we have inferred the type

$$\mathcal{S}(\texttt{t5}) = \texttt{t6 -> t6}$$

for `identity`.

III. Type Inferencing in Helium

The Helium compiler

- Constraint based approach to type inferencing
- Implements many heuristics, multiple solvers
- Existing algorithms/implementations can be emulated
- ```
  cabal install helium
  cabal install lvmrun
  ```
- Only: Haskell 98 minus type class and instance definitions
- And bias still exists from early binding groups to later ones
  - Others have addressed this issue
  ```

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  ```
- Only: Haskell 98 minus type class and instance definitions
- And bias still exists from early binding groups to later ones
  - Others have addressed this issue
- Supports domain specific type error diagnosis
- Details of the type rules: see Bastiaan Heeren's PhD
  ```

Some important compiler flags

- `--overloading` and `--no-overloading`
- `--enable-logging`, `--host` and `--port`
- `--algorithm-w` and `--algorithm-m`
- `--experimental` gives many more flags
  - `--kind-inferencing`
  - `--select-cnr` to select a particular constraint for blame
  - flags for choosing a particular solver
  - many other treewalks for ordering constraints

Constraints generated by Helium

For the program,

$allinc = \backslash\, xs \;{-}{>}\; map\ (+1)\ xs$

Helium generates ($-d$ option)

```
v5 := Inst(forall a b. (a -> b) -> [a] -> [b])
v9 := Inst(forall a. Num a => a -> a -> a)
Int == v10    : {literal}
v9 == v8 -> v10 -> v7    : {infix application}
v8 -> v7 == v6    : {left section}
v3 == v11    : {variable}
v5 == v6 -> v11 -> v4    : {application}
v3 -> v4 == v2    : {lambda abstraction}
v2 == v0    : {right-hand side}
v0 == v1    : {right hand side}
s22 := Gen([], v1)    : {Generalize allinc}
```

Greedy constraint solver

Given a set of type constraints, the greedy constraint solver returns a substitution that satisfies these constraints, and a list of constraint that could not be satisfied by the solver. The latter is used to produce type error messages.

- Advantages:
  - Efficient and fast
  - Straightforward implementation
- Disadvantage:
  - The order of the type constraints strongly influences the reported error messages. The type inference process is biased.

Ordering type constraints

- One is free to choose the order in which the constraints should be considered by the greedy constraint solver. (Although there is a restriction for an implicit instance constraint)
- Instead of returning a list of constraints, return a constraint tree that follows the shape of the AST.
- A tree-walk flattens the constraint tree and orders the constraints.
  - $\mathcal{W}$: almost a post-order tree walk
  - $\mathcal{M}$: almost a pre-order tree walk
  - Bottom-up: ...
  - Pushing down type signatures: ...

A realistic type rule

- Some constraints 'belong' to certain subexpressions:

$$\mathcal{T}_{\mathcal{C}} = [c_2, c_3] \mathbin{\underline{\Diamond}} \lbrace\!\!\lbrace c_1 \triangledown \mathcal{T}_{\mathcal{C}1}, \mathcal{T}_{\mathcal{C}2}, \mathcal{T}_{\mathcal{C}3} \rbrace\!\!\rbrace$$

$$c_1 = (\tau_1 \equiv Bool) \qquad c_2 = (\tau_2 \equiv \beta) \qquad c_3 = (\tau_3 \equiv \beta)$$

$$\frac{\mathcal{A}_1, \mathcal{T}_{\mathcal{C}1} \vdash e_1 : \tau_1 \qquad \mathcal{A}_2, \mathcal{T}_{\mathcal{C}2} \vdash e_2 : \tau_2 \qquad \mathcal{A}_3, \mathcal{T}_{\mathcal{C}3} \vdash e_3 : \tau_3}{\mathcal{A}_1 +\!\!+ \mathcal{A}_2 +\!\!+ \mathcal{A}_3, \mathcal{T}_{\mathcal{C}} \vdash \textbf{if } e_1 \textbf{ then } e_2 \textbf{ else } e_3 : \beta}$$

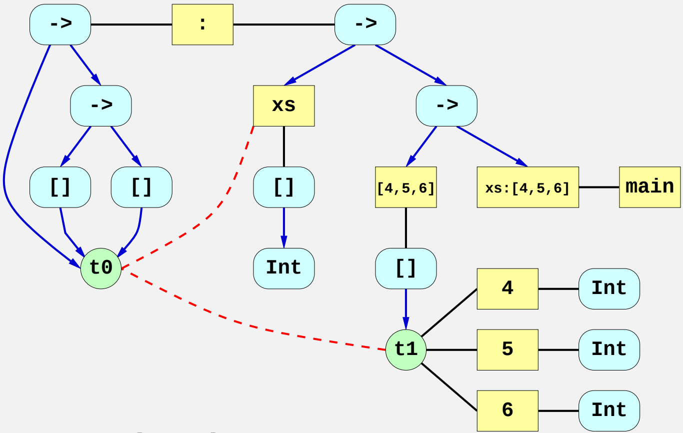
- c_1 is generated by the conditional, but associated with the boolean subexpression.
- Example strategy: left-to-right, bottom-up for then and else part, push down *Bool* (do c_1 before $\mathcal{T}_{\mathcal{C}1}$).

Global constraint solver

Uses type graphs allow us to solve the collected type constraints in a more global way. These can represent inconsistent sets of constraints.

- Advantages:
  - Global properties can be detected
  - A lot of information is available
  - The type inference process can be unbiased
  - It is easy to include new heuristics to spot common mistakes.
- Disadvantage:
  - Extra overhead makes this solver a bit slower
  - But: only for the first inconsistent binding group!

Type graphs (for $xs : [4, 5, 6]$)



$main = xs : [4, 5, 6]$
  where $len = length\ xs$
    $xs = [1, 2, 3]$

Type graph heuristics

If a type graph contains an inconsistency, then heuristics help to choose which location is reported as type incorrect.

- Examples:
  - minimal number of type errors
  - count occurrences of clashing type constants ($3 \times \mathit{Int}$ versus $1 \times \mathit{Bool}$)
  - reporting an expression as type incorrect is preferred over reporting a pattern
  - wrong literal constant (4 versus 4.0)
  - not enough arguments are supplied for a function application
  - permute the elements of a tuple
  - (:) is used instead of (++)

Heuristics in Helium


```
listOfHeuristics options siblings path =
  ...
  [avoidForbiddenConstraints   -- remove constraints that should NEVER be reported
  , highParticipation 0.95 path
  , phaseFilter                -- phasing from the type inference directives
  ] ++
  [Heuristic (Voting (
    [siblingFunctions siblings
    , siblingLiterals
    , applicationHeuristic
    , variableFunction   -- ApplicationHeuristic without application
    , tupleHeuristic     -- ApplicationHeuristic for tuples
    , fbHasTooManyArguments
    , constraintFromUser path   -- From .type files
    , unaryMinus (Overloading `elem` options)
    ] ++
    [similarNegation | Overloading `notElem` options] ++
    [unifierVertex | UnifierHeuristics `elem` options]))] ++
  [inPredicatePath | Overloading `elem` options] ++
  [avoidApplicationConstraints, avoidNegationConstraints
  , avoidTrustedConstraints, avoidFolkloreConstraints
  , firstComeFirstBlamed   -- Will delete all except the first
  ]
```

The Helium message

$$\mathit{main} = \mathit{xs} : [4,5,6]$$
$$\quad \textbf{where}\ \mathit{len} = \mathit{length}\ \mathit{xs}$$
$$\qquad \mathit{xs} = [1,2,3]$$

```
(2,9): Warning: Definition "len" is not used
(1,11): Type error in constructor
 expression         : :
   type             : a     -> [a  ] -> [a]
   expected type    : [Int] -> [Int] -> b
 probable fix       : use ++ instead
```

Example: permute function arguments

test :: *Parser Char String*
test = *option* "" (*token* "hello!")

In Helium:

```
(2,8): Type error in application
 expression          : option "" (token "hello!")
 term                : option
   type              : Parser a b -> b -> Parser a b
   does not match    : String -> Parser Char String -> c
 probable fix        : flip the arguments
```

Limitations of Helium

- The Helium language is relatively small
- A major limitation of the type inference process: consistent binding groups are never blamed.

  $$myfold\ f\ z\ [\,] = [z]$$
  $$myfold\ f\ z\ (x : xs) = myfold\ f\ (f\ z\ x)\ xs$$
  $$rev = myfold\ (flip\ (:))\ [\,]$$
  $$palin :: Eq\ a \Rightarrow [a] \rightarrow Bool$$
  $$palin\ xs = rev\ xs == xs$$

- Helium blames *palin*, some other systems can blame *myfold* instead. Signatures for *rev* and *myfold* improve Helium's message.
- Note: we use our intuition of what *rev* and *palin* do, a compiler (typically) cannot.

Who's to blame?

$$wrongxxx :: (Int \to Int) \to Int \to Int \to Int$$
$$wrongxxx\ f\ x\ y = \textbf{if}\ f\ (x + y)\ \textbf{then}\ x * y\ \textbf{else}\ x + y$$

Running `helium -d Constraintnr.hs` gets you (a.o.), after some early filters:

```
cnr   edge           ratio   info
----------------------------------------------------
#12*  (35-97)        100%    {conditional}
#1*   (26-80)        100%    {explicitly typed binding}
#2*   (28-31)        100%    {pattern of function binding}
#5*   (31-36)        100%    {variable}
#11*  (36-96)        100%    {application}
```

The error path

- $wrongxxx :: (Int \rightarrow Int) \rightarrow Int \rightarrow Int \rightarrow Int$
  $wrongxxx\ \overline{f}^{v28}\ x\ y = \textbf{if}\ \overline{f}^{v36}\ \overline{x + y}^{v37}$
  $\textbf{then}\ x * y\ \textbf{else}\ x + y$

- The error path goes from the explicit type for f as part of *wrongxxx*'s type signature, to the mismatch of the result type of f with the *Bool* the conditional expects:

  $\#\,1\ v26 := Inst\ ((Int \rightarrow Int) \rightarrow Int \rightarrow Int \rightarrow Int)$
  $\#\,2\ v28 == v31$
  $\#\,5\ v31 == v36$
  $\#\,11\ v36 == v37 \rightarrow v35$
  $\#\,12\ v35 == Bool$

- The constraint $v26 == v28 \rightarrow v29 \rightarrow v30 \rightarrow v27$ was exonerated earlier.

Blaming a constraint

$$wrongxxx :: (Int \rightarrow Int) \rightarrow Int \rightarrow Int \rightarrow Int$$

$$wrongxxx\ \overline{f}^{v28}\ x\ y = \textbf{if}\ \overline{f}^{v36}\ \overline{x + y}^{v37}$$
$$\textbf{then}\ x * y\ \textbf{else}\ x + y$$

Run `helium --select-cnr=12 ...` to blame $v35 == Bool$:

```
(9,21): Type error in conditional
 expression       : if f (x + y) then x * y else x + y
 term             : f (x + y)
   type           : Int
   does not match : Bool
```

$v35$ denotes the return type of f, the *Bool* is the one from the type rule for conditionals.

Blaming a constraint

$$wrongxxx :: (Int \rightarrow Int) \rightarrow Int \rightarrow Int \rightarrow Int$$

$$wrongxxx\ \overline{f}^{v28}\ x\ y = \textbf{if}\ \overline{f}^{v36}\ \overline{x + y}^{v37}\ \textbf{then}\ x * y\ \textbf{else}\ x + y$$

Constraint #11: $v36 == v37 \rightarrow v35$

```
(20,21): Type error in application
 expression       : f (x + y)
 term             : f
   type           : Int -> Int
   does not match : Int -> Bool
```

Blaming a constraint

$$\mathit{wrongxxx} :: (\mathit{Int} \rightarrow \mathit{Int}) \rightarrow \mathit{Int} \rightarrow \mathit{Int} \rightarrow \mathit{Int}$$

$$\mathit{wrongxxx}\ \overline{f}^{v28}\ x\ y = \textbf{if}\ \overline{f}^{v36}\ \overline{x + y}^{v37}\ \textbf{then}\ x * y\ \textbf{else}\ x + y$$

Constraint #5: $v31 == v36$

```
(9,21): Type error in variable
 expression        : f
   type            : Int -> Int
   expected type   : Int -> Bool
```

Blaming a constraint

$$wrongxxx :: (Int \rightarrow Int) \rightarrow Int \rightarrow Int \rightarrow Int$$

$$wrongxxx\ \overline{f}^{v28}\ x\ y = \textbf{if}\ \overline{f}^{v36}\ \overline{x + y}^{v37}\ \textbf{then}\ x * y\ \textbf{else}\ x + y$$

Constraint #2: $v28 == v31$

```
(9,10): Type error in pattern of function binding
 pattern             : f
   type              : Int -> Bool
   does not match    : Int -> Int
```

Blaming a constraint

$$wrongxxx :: (Int \rightarrow Int) \rightarrow Int \rightarrow Int \rightarrow Int$$

$$wrongxxx\ \overline{f}^{v28}\ x\ y = \textbf{if}\ \overline{f}^{v36}\ \overline{x + y}^{v37}$$

$$\textbf{then}\ x * y\ \textbf{else}\ x + y$$

Constraint #1:

$$v26 := Inst\ ((Int \rightarrow Int) \rightarrow Int \rightarrow Int \rightarrow Int)$$

```
(9,1): Type error in explicitly typed binding
 definition          : wrongxxx
   inferred type  : (a   -> Bool) -> a   -> a   -> a
   declared type  : (Int -> Int ) -> Int -> Int -> Int
```

v26 denotes the type inferred for *wrongxxx*'s implementation.
Not all knowledge about *a* has been used.

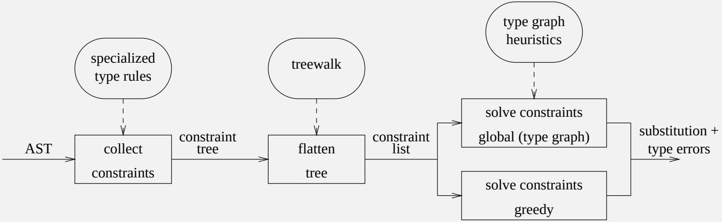
The next logical step...

- Put control over the order of constraint solving in the hands of the programmer
- Associate your own error message with a given constraint
- ⇒ domain-specific type error diagnosis

Summary

We have described a *parametric* type inferencer

- Constraint-based: specification and implementation are separated
- Standard algorithms can be simulated by choosing an order for the constraints
- Two implementations are available to solve the constraints
- Type graph heuristics help in reporting the most likely mistake



IV. Domain Specific Type Error Diagnosis

What is a DSL?

- Walid Taha:
  - the domain is well-defined and central
  - the notation is clear,
  - the informal meaning is clear,
  - the formal meaning is clear and implemented.

What is a DSL?

- Walid Taha:
  - the domain is well-defined and central
  - the notation is clear,
  - the informal meaning is clear,
  - the formal meaning is clear and implemented.
- Missing is:
  - and an implementation of the DSL can communicate with the programmer about the program in terms of the domain
- “domain-abstractions should not leak”

Embedded Domain Specific Languages

- Embedded (internal à la Fowler) Domain Specific Languages are achieved by encoding the DSL syntax inside that of a host language.
- Some (arguable) advantages:
  - familiarity host language syntax
  - escape hatch to the host language
  - existing libraries, compilers, IDE's, etc.
  - combining EDSLs
- At the very least, useful for prototyping DSLs
- According to Hudak "the ultimate abstraction"

What host language?

- Some languages provide extensibility as part of their design, e.g., Ruby, Python, Scheme
- Others are rich enough to encode a DSL with relative ease, e.g., Haskell, C++
- In most languages we just have to make do
- In Haskell, EDSLs are simply libraries that provide some form of "fluency"
    - Consisting of domain terms and types, and special operators with particular priority and fixity

Challenges for EDSLs

- How to achieve:
  - domain specific optimisations
  - domain specific error diagnosis
- Optimisation and error diagnosis are also costly in a non-embedded setting, but there we have more control.
- Can we achieve this control for error diagnosis?

Parser combinators

- Parser combinators (before *Applicative*): an EDSL for describing parsers
- An executable and extensible form of EBNF
  - Concatenation/juxtaposition: $p \langle * \rangle q$, and $p \langle * q$
  - Choice: $p <|> q$
  - Semantics: $f \langle \$ \rangle p$ and $f \langle \$ p$
  - Repetition: *many*, *many1*, ...
  - Optional: *option p* **default**
  - Literals: *token* `"text"`, *pKey* `"->"`
  - Others introduced as needed, and defined at will

```
pExpr = pAndPrioExpr
    <|> sem_Expr_Lam    -- a function of two arguments
      ⟨$ pKey "\\"
      ⟨*⟩pFoldr1 (sem_LamIds_Cons, sem_LamIds_Nil) pVarid
      ⟨*⟩pKey "->"
      ⟨*⟩pExpr
```

A small mistake

$$
\begin{aligned}
&pExpr = pAndPrioExpr \\
&\quad <|> sem\_Expr\_Lam \quad \text{-- Semantics for lambda expressions} \\
&\qquad \langle\$\ pKey\ \texttt{"\textbackslash\textbackslash"} \\
&\qquad \langle*\rangle pFoldr1\ (sem\_LamIds\_Cons, sem\_LamIds\_Nil)\ pVarid \\
&\qquad \langle*\rangle pKey\ \texttt{"->"} \\
&\qquad \langle*\rangle pExpr
\end{aligned}
$$

The error message that results:

```
ERROR "BigTypeError.hs":1 - Type error in application
*** Expression     : sem_Expr_Lam <$ pKey "\\" <*> pFoldr1 (sem_LamIds_Cons,sem_
LamIds_Nil) pVarid <*> pKey "->"
*** Term           : sem_Expr_Lam <$ pKey "\\" <*> pFoldr1 (sem_LamIds_Cons,sem_
LamIds_Nil) pVarid
*** Type           : [Token] -> [((Type -> Int -> [([Char],(Type,Int,Int))] -> I
nt -> Int -> [(Int,(Bool,Int))] -> (PP_Doc,Type,a,b,[c] -> [Level],[S] -> [S]))
-> Type -> d -> [([Char],(Type,Int,Int))] -> Int -> Int -> e -> (PP_Doc,Type,a,b
,f -> f,[S] -> [S]),[Token])]
*** Does not match : [Token] -> [([Char] -> Type -> d -> [([Char],(Type,Int,Int)
)] -> Int -> Int -> e -> (PP_Doc,Type,a,b,f -> f,[S] -> [S]),[Token])]
```

A closer look at the message


```
ERROR "BigTypeError.hs":1 - Type error in application
*** Expression     : sem_Expr_Lam <$ pKey "\\" <*> pFoldr1 (sem_LamIds_Cons,sem_
LamIds_Nil) pVarid <*> pKey "->"
*** Term           : sem_Expr_Lam <$ pKey "\\" <*> pFoldr1 (sem_LamIds_Cons,sem_
LamIds_Nil) pVarid
*** Type           : [Token] -> [((Type -> Int -> [([Char],(Type,Int,Int))] -> I
nt -> Int -> [(Int,(Bool,Int))] -> (PP_Doc,Type,a,b,[c] -> [Level],[S] -> [S]))
-> Type -> d -> [([Char],(Type,Int,Int))] -> Int -> Int -> e -> (PP_Doc,Type,a,b
,f -> f,[S] -> [S]),[Token])]
*** Does not match : [Token] -> [([Char] -> Type -> d -> [([Char],(Type,Int,Int)
)] -> Int -> Int -> e -> (PP_Doc,Type,a,b,f -> f,[S] -> [S]),[Token])]
```

- Message is large and looks complicated
- You have to discover why the types don't match yourself
- No mention of “parsers” in the error message
- It happens to be a common mistake, and easy to fix

The solution in a nutshell

1. Bring the type inference mechanism under control
   - by phrasing the type inference process as a constraint solving problem (see earlier)
2. Provide hooks in the compiler's type inference process to change the process for certain classes of expressions
   - specialize type error messages for a particular domain
   - control the order in which constraints are solved
   - drive heuristics that suggest fixes for often-made mistakes

The solution in a nutshell

1. Bring the type inference mechanism under control
   - by phrasing the type inference process as a constraint solving problem (see earlier)
2. Provide hooks in the compiler's type inference process to change the process for certain classes of expressions
   - specialize type error messages for a particular domain
   - control the order in which constraints are solved
   - drive heuristics that suggest fixes for often-made mistakes

- Changing the type system is forbidden!
  - Only the order of solving, and the provided messages can be changed

How is this organised in Helium?

- For a given source module Abc.hs, a DSL designer may supply a file Abc.type containing the directives
- The directives are automatically used when the module is imported
- The compiler will adapt the type error mechanism based on these type inference directives.
- The directives themselves are also a(n external) DSL!

The type inference process

- We piggy-back ride on Haskell's underlying type system
- Type rules for functional languages are often phrased as a set of logical deduction rules
- Inference is then implemented by means of an AST traversal
  - Ad-hoc or using attribute grammars

The rule for type checking applications

$$\frac{\Gamma \ \vdash_{\mathrm{HM}} \ f : \tau_a \rightarrow \tau_r \qquad\qquad \Gamma \ \vdash_{\mathrm{HM}} \ e : \tau_a}{\Gamma \ \vdash_{\mathrm{HM}} \ f\ e : \tau_r}$$

- Γ is an environment, containing the types of identifiers defined elsewhere
- Rules for variables, anonymous functions and local definitions omitted
- Algorithm $\mathcal{W}$ is a (deterministic) implementation of these typing rules.

Specializing a type rule (1/3) §IV

Applying the type rule for function application twice in succession results in the following:

$$\frac{\Gamma \vdash_{\mathrm{HM}} op : \tau_1 \to \tau_2 \to \tau_3 \qquad \Gamma \vdash_{\mathrm{HM}} x : \tau_1 \qquad \Gamma \vdash_{\mathrm{HM}} y : \tau_2}{\Gamma \vdash_{\mathrm{HM}} x \text{ `}op\text{` } y : \tau_3}$$

Specializing a type rule (1/3) §IV

Applying the type rule for function application twice in succession results in the following:

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Consider one of the parser combinators (pre-*Applicative*), for instance $<\$>$.

$$<\$> :: (a \rightarrow b) \rightarrow Parser\ s\ a \rightarrow Parser\ s\ b$$

We can now create a specialized type rule by filling in this type in the type rule

Applying the type rule for function application twice in succession results in the following:

$$\frac{\Gamma \;\vdash_{\mathrm{HM}}\; op : \tau_1 \rightarrow \tau_2 \rightarrow \tau_3 \qquad \Gamma \;\vdash_{\mathrm{HM}}\; x : \tau_1 \qquad \Gamma \;\vdash_{\mathrm{HM}}\; y : \tau_2}{\Gamma \;\vdash_{\mathrm{HM}}\; x \text{ `}op\text{` } y : \tau_3}$$

Consider one of the parser combinators (pre-*Applicative*), for instance <\$>.

$$<\$> :: (a \rightarrow b) \rightarrow Parser\ s\ a \rightarrow Parser\ s\ b$$

We can now create a specialized type rule by filling in this type in the type rule (x and y stand for arbitrary expressions of the given type)

$$\frac{\Gamma \;\vdash_{\mathrm{HM}}\; x : a \rightarrow b \qquad \Gamma \;\vdash_{\mathrm{HM}}\; y : Parser\ s\ a}{\Gamma \;\vdash_{\mathrm{HM}}\; x <\$> y : Parser\ s\ b}$$

Specializing a type rule (2/3) §IV

- Use equality constraints to make the restrictions that are imposed by the type rule explicit.
- Γ is unchanged, and therefore omitted from the rule
- Type rules are invalidated by shadowing, here, $\langle \$ \rangle$.

$$\frac{x : \tau_1 \qquad y : \tau_2}{x \mathbin{<\$>} y : \tau_3} \qquad \begin{cases} \tau_1 & \equiv \quad a \to b \\ \tau_2 & \equiv \quad \mathit{Parser}\ s\ a \\ \tau_3 & \equiv \quad \mathit{Parser}\ s\ b \end{cases}$$

- Use equality constraints to make the restrictions that are imposed by the type rule explicit.
- Γ is unchanged, and therefore omitted from the rule
- Type rules are invalidated by shadowing, here, $\langle\$\rangle$.

$$\frac{x : \tau_1 \qquad y : \tau_2}{x <\$> y : \tau_3} \qquad \begin{cases} \tau_1 & \equiv & a \to b \\ \tau_2 & \equiv & Parser\ s\ a \\ \tau_3 & \equiv & Parser\ s\ b \end{cases}$$

Split up the type constraints in "smaller" unification steps.

$$\frac{x : \tau_1 \qquad y : \tau_2}{x <\$> y : \tau_3} \qquad \begin{cases} \tau_1 & \equiv & a_1 \to b_1 & s_1 & \equiv & s_2 \\ \tau_2 & \equiv & Parser\ s_1\ a_2 & a_1 & \equiv & a_2 \\ \tau_3 & \equiv & Parser\ s_2\ b_2 & b_1 & \equiv & b_2 \end{cases}$$

Specializing a type rule (3/3) §IV

$$\frac{x : \tau_1 \qquad y : \tau_2}{x <\$> y : \tau_3} \qquad \begin{cases} \tau_1 \equiv a_1 \to b_1 & s_1 \equiv s_2 \\ \tau_2 \equiv \mathit{Parser}\ s_1\ a_2 & a_1 \equiv a_2 \\ \tau_3 \equiv \mathit{Parser}\ s_2\ b_2 & b_1 \equiv b_2 \end{cases}$$

```
  x :: t1;    y :: t2;
---------------------
    x <$> y :: t3;

t1 == a1 -> b1
t2 == Parser s1 a2
t3 == Parser s2 b2
s1 == s2
a1 == a2
b1 == b2
```

Special type error messages


```
  x :: t1;    y :: t2;
---------------------
    x <$> y :: t3;

t1 == a1 -> b1      : left operand is not a function
t2 == Parser s1 a2 : right operand is not a parser
t3 == Parser s2 b2 : result type is not a parser
s1 == s2 : parser has an incorrect symbol type
a1 == a2 : function cannot be applied to parser's result
b1 == b2 : parser has an incorrect result type
```

- Supply an error message for each type constraint. This message is reported if the corresponding constraint cannot be satisfied.

Example

test :: *Parser Char String*
test = *map toUpper*⟨$⟩"hello, world!"

This results in the following type error message:

```
Type error: right operand is not a parser
```

Example

$\mathit{test} :: \mathit{Parser}\ \mathit{Char}\ \mathit{String}$
$\mathit{test} = \mathit{map}\ \mathit{toUpper} \langle\$\rangle$ `"hello, world!"`

This results in the following type error message:

```
Type error: right operand is not a parser
```

Important context specific information is missing, for instance:

- Inferred types for (sub-)expressions, and intermediate type variables
- Pretty printed expressions from the program
- Position and range information

Error message attributes

The error message attached to a type constraint might now look like:

```
  x :: t1;   y :: t2;
---------------------
    x <$> y :: t3;
...
t2 == Parser s1 a2 :
 @expr.pos@: The right operand of <$> should be a
  expression       : @expr.pp@             parser
  right operand    : @y.pp@
    type           : @t2@
    does not match : Parser @s1@ @a2@
...
```

Example

test :: *Parser Char String*
test = *map toUpper*⟨$⟩`"hello, world!"`

This results in the following type error message (including the inserted error message attributes):

```
(2,21): The right operand of <$> should be a parser
 expression         : map toUpper <$> "hello, world!"
 right operand      : "hello, world!"
   type             : String
   does not match   : Parser Char String
```

Other facilities


```
   x :: t1;    y :: t2;
----------------------------
  x <$> y :: Parser s b;

constraints x
t1 == a1 -> b     : left operand is not a function
constraints y
t2 == Parser s a2 : right operand is not a parser
a1 == a2 : function cannot be applied to ...
```

- Interpolate constraints into the rule (cf. *Parser s b*): no effort for default behaviour
- Control over solving order wrt. subexpressions
- Automatic check for soundness and completeness
- Phase numbers for more control over solving order

Another directive: siblings

- Certain combinators are known to be easily confused:
  - cons (:) and append (++)
  - ⟨$⟩ and ⟨$
  - (.) and (++) (PHP programmers)
  - (+) and (++) (Java programmers)
- These combinations can be listed among the specialized type rules.

```
siblings   <$> , <$
siblings   ++ , +, .
```

- The siblings heuristic will try a sibling if an expression with such an operator fails to type check.

Example


```
data Expr = Lambda [String] Expr
pExpr
   = pAndPrioExpr
 <|> Lambda <$ pKey "\\"
            <*> many pVarid
            <* pKey "->"
            <* pExpr
```

Extremely concise:

```
(11,13): Type error in the operator <*
  probable fix: use <*> instead
```

V. Towards Haskell 2010 and onwards

Introducing DOMSTED

DOMain Specific Type Error Diagnosis

- Enable embedded DSL developers to control the error messages produced by the compiler
- Focus on those errors coming from ill-typed expressions
- Target a full-blown type system
  - Haskell 2010 + type classes, functional dependencies, type families, GADTs, kind polymorphism...
  - In the works: higher-rank and impredicative instantiation
- Constraint-based approach to typing

Why Haskell 98 is not complicated enough

Statistics computed some years back:

Extension	# Hackage	# Top 20
FlexibleInstances	332	10
MultiParamTypeClasses	321	9
FlexibleContexts	232	3
ScopedTypeVariables	192	3
ExistentialQuantification	149	6
FunctionalDependencies	139	4
TypeFamilies	114	1
OverlappingInstances	108	3
Rank2Types	100	3
GADTs	88	3
RankNTypes	81	1
UnboxedTuples	20	4
KindSignatures	20	0

What have we accomplished?

- Two-phase specialized type rules (ESOP 2016)
- Implementation on top of `OutsideIn(X)`
- We implemented some of our work into GHC

VI. Customizing type error diagnosis in GHC

Ondertussen, in GHC...


```
instance TypeError (Text "Cannot 'Show' functions." :$$:
                    Text "Perhaps a missing argument?")
      => Show (a -> b) where ...
```

- Leverages type-level programming techniques in GHC (Diatchki, 2015)
- Very restricted:
  - Only available for type class and family resolution
  - May not influence the ordering of constraints
  - Messages cannot depend on who generated the constraint

How far can we take this?

We provide

- control over the content of the type error message
  - the same constraint (to the solver) may result in different messages
- (some) control over the order in which constraints are checked
- Expression level error messages by type level programming
- GHC's abstraction facilities allow for reuse and uniformity
  - A type level embedded DSL for diagnosing embedded DSLs
- integrated as a patch in GHC version 8.1.20161202
- soundness and completeness for free

How much effort is involved?

- We get a lot for a few non-invasive changes to GHC, with *TypeError* and the *Constraint* kind as enablers
- Constraint resolution needs some changes to track messages, and deal with priorities
- A few additions to *TypeLits.hs* in the base library and a new module *TypeErrors.hs* (62 lines) that exposes the API
- One additional compiler pragma `CHECK_ARGS_BEFORE_FN`.
- We employ many language extensions:

  `DataKinds, TypeOperators, TypeFamilies, ConstraintKinds, FlexibleContexts, PolyKinds, UndecidableInstances, UndecidableSuperclasses`

  but the EDSL programmer only the first four, the EDSL user none.

A very stupid mistake

$$
\begin{aligned}
&\mathit{intid} :: \mathit{Int} \\
&\mathit{intid} = \mathit{id}'\ \mathit{True}
\end{aligned}
$$

A very stupid mistake

$$intid :: Int$$
$$intid = id'\ True$$

```
FormatEx.hs:17:9: error:
    * Hi! You must be Donald. Donald, please read
      this error message. It’s a great error message.
      The argument and result types of ’id’ do not
      coincide: Bool vs. Int
    * In the expression: id’ True
      In an equation for ’intid’: intid = id’ True
```

Our wrapped Donald-aware identity function

id' :: *CustomErrors*
   '['[a :≁: b
         :⇛: E.Text "Hi! You must be Donald. "
         :◇: E.Text "Donald, please read this error message."
         :◇: E.Text " It's a great error message."
         :$$:
        E.Text "The argument and result types of 'id'"
         :◇: E.Text " do not coincide: " :◇: VS a b]
     ] => a -> b
$id' = id$

- *E* qualifier to address type level *Text*
- id' is a type error aware wrapper for *id*
- $id' = id$ ensures id' is sound
- Completeness can be achieved too, dually
- With `{#- INLINE id' -#}` no run-time overhead

Type errors Class I: type inconsistencies

From the *diagrams* library (Yorgey, 2012/2016)

$$\begin{aligned} atop \ :: \ & (OrderedField\ n, Metric\ v, Semigroup\ m) \\ \Rightarrow\ & QDiagram\ b\ v\ n\ m \rightarrow \\ & QDiagram\ b\ v\ n\ m \rightarrow \\ & QDiagram\ b\ v\ n\ m \end{aligned}$$

writing *atop True* gives

```
Couldn't match type 'QDiagram b v n m' with type 'Bool'
```

or for *atop cube3d plane2d* might give

```
Couldn't match type 'V2' with type 'V3'
```

Type error Class II: left-undischarged errors

From the *persistent* library (Snoyman, 2012)

$$
\begin{aligned}
\mathit{insertUnique} \; :: \; & (\mathit{MonadIO}\ m, \mathit{PersistUniqueWrite}\ \mathit{backend}, \\
& \mathit{PersistEntity}\ \mathit{record}) \\
=> \; & \mathit{record} -> \\
& \mathit{ReaderT}\ \mathit{backend}\ m\ (\mathit{Maybe}\ (\mathit{Key}\ \mathit{record}))
\end{aligned}
$$

use of *insertUnique* gives rise to type class predicates that may be left undischarged, because the programmer forgot to write a *PersistEntity* instance.

We'd like to get something like:

```
Data type 'Person' is not declared as a Persistent
entity. Hint: entity definition can be automatically
derived. Read more at http://www.yesodweb.com/...
```

Type error Class III: ambiguous type errors

- Defaulting seems to be a more apt solution, or simply adding type annotations
- We wondered: are these ever "domain-specific"? We'd like to hear about it.
- Our work handles Class I and Class II errors

Before we go on: *Constraints*

GHC supports a special kind *Constraint* so that type level programming can be applied to constraints

type *JSONSerializable a* = (*FromJSON a*, *ToJSON a*)

and use type families as type-level functions:

```
type family All (c :: k -> Constraint) (xs :: [k]) where
  All c []       = ()
  All c (x : xs) = (c x, All c xs)
```

so we can write *All Show* [*Int*, *Bool*] instead of (*Show Int*, *Show Bool*)

This is what opens the door to manipulating constraints and type error messages in a reusable fashion.

The running example

$$\begin{aligned} atop \ :: \ & (OrderedField\ n, Metric\ v, Semigroup\ m) \\ \Rightarrow\ & QDiagram\ b\ v\ n\ m \rightarrow \\ & QDiagram\ b\ v\ n\ m \rightarrow \\ & QDiagram\ b\ v\ n\ m \end{aligned}$$

can also be written as

$$\begin{aligned} atop \ :: \ & (d_1 \sim QDiagram\ b_1\ v_1\ n_1\ m_1, \\ & \ d_2 \sim QDiagram\ b_2\ v_2\ n_2\ m_2, \\ & \ b_1 \sim b_2, v_1 \sim v_2, n_1 \sim n_2, m_1 \sim m_2, \\ & \ OrderedField\ n_1, Metric\ v_1, Semigroup\ m_1) \\ \Rightarrow\ & d_1 \rightarrow d_2 \rightarrow d_1 \end{aligned}$$

Failure to satisfy either $b_1 \sim b_2$ or $v_1 \sim v_2$ should lead to different messages.

Approach I: attaching hints to constraints

$$
\begin{aligned}
&atop \quad :: (\\
&\quad (d_1 \sim QDiagram\ b_1\ v_1\ n_1\ m_1)\\
&\quad `IH` (Text\ \texttt{"argument \#1 to 'atop' must be a diagram"}),\\
&\quad (d_2 \sim QDiagram\ b_2\ v_2\ n_2\ m_2)\\
&\quad `IH` (Text\ \texttt{"argument \#2 to 'atop' must be a diagram"}),\\
&\quad (b_1 \sim b_2)\\
&\quad `IH` (Text\ \texttt{"the diagrams must use the same back-end"}),\\
&\quad (v_1 \sim v_2)\\
&\quad `IH` (Text\ \texttt{"diagrams must live in the same vector space"}),\\
&\quad \ldots\ same\ for\ n_1, n_2, m_1\ and\ m_2\\
&\quad OrderedField\ n_1, Metric\ v_1, Semigroup\ m_1)\\
&\quad => d_1 -> d_2 -> d_1\\
&atop = Diagrams.Combinators.atop
\end{aligned}
$$

The constraint solving machinery propagates messages along with the associated type level error message. The *IH* annotations/predicates ensure the message is reported.

Some observations

- Message is attached as a hint if a constraint cannot be satisfied

  example = *atop* *True* 'c'

```
* Couldn't match type 'QDiagram b v n m' with 'Bool'
  ...
* In the expression: atop True 'c'
  ...
* Hint: argument #1 to 'atop' must be a diagram
```

- Very simple to implement
- May sometimes give unexpected results (more info in the paper)

Before we go on: a Class II example

We can also associate a hint with a type class predicate so that the hint is shown if that predicate is left undischarged:

```
insertUnique ::
    (  MonadIO m, PersistUniqueWrite backend,
       PersistEntity record `LeftUndischargedHint` (
       Text "Data type '"
         :<>: ShowType record
         :<>: Text "' is not declared as entity."
         :$$: Text "Hint: entity definition can be "
         :<>: "automatically derived."
         :$$: Text "Read more at http://www.yesodweb.com/..."
    )
  => record -> ReaderT backend m (Maybe (Key record))
```

Approach II: controlling the order

- The problem of Approach I arises from the order in which constraints may be solved by the constraint solver
- The solution is to give control over that order to the developer
- The basic combinator we introduce is *IfNot*

  $$\mathit{IfNot}\ (c :: \mathit{Constraint})\ (\mathit{fail} :: \mathit{Constraint})\ (\mathit{ok} :: \mathit{Constraint})$$

- **IMPORTANT:** the *ok* branch will also be chosen if the constraint c is not yet known to be consistent or not!
- E.g., if $c = \alpha \sim \beta$, we have to wait for more information.
- In other words: *IfNot* does not perform a unification.

Example


```
atop ::
  IfNot (d1 ~ QDiagram b1 v1 n1 m1)
    (TypeError "Arg. #1 to 'atop' must be a diagram")
    (IfNot (d2 ~ QDiagram b2 v2 n2 m2)
      (TypeError "Arg. #2 to 'atop' must be a diagram")
      (IfNot (b1 ~ b2)
        (TypeError "Back-ends do not coincide")
      ....))))
   => d1 -> d2 -> d1
```

- Better syntax later (defined on top of *IfNot*)

Controlling the solving order

- *IfNots* can be nested which induce a preferred solving order
- The constraint solver uses priorities to ensure solving obeys the dictated order (more details in the paper)
- The priorities cannot be generally controlled in relation to the rest of the program: too invasive
- We do offer one pragma: `CHECK_ARGS_BEFORE_FN`.
  - Ensures that we get the most out of arguments before looking at the application

From *IfNot* to *WhenApart*

- *WhenApart a b f o* represents *IfNot* $(a \sim b)$ *f o*
- *WhenApart* was introduced along with closed type families: the constraint is true if at this point *a* and *b* can never be reconciled.
- We cannot reduce *Int* :==: α until we know more about α, but if we have *Int* :==: $[\alpha]$ we can rewrite to *False* for the following type family:

  type *family a* :==: *b* :: *Bool* **where**
  a :==: *a* = *True*
  a :==: *b* = *False*

The EDSL-developer facing API (version 1)

Apartness is represented by the operator

infixl 5 $:\not\sim:$

We deal with two kinds of failure:

data *ConstraintFailure* $=$
$\forall\ t\ .\ t :\not\sim: t$ | *Undischarged Constraint*

A *CustomError* is then a failure and a message

infixl 4 $:\Rrightarrow:$
data *CustomError* $=$
ConstraintFailure $:\Rrightarrow:$ *ErrorMessage* | *Check Constraint*

The latter if we do not want a message.

Running back to our example

$$
\begin{aligned}
&\mathit{atop} :: \mathit{CustomErrors}\ [\\
&\quad d_1 :\not\sim: \mathit{QDiagram}\ b_1\ v_1\ n_1\ m_1\\
&\qquad :\Rrightarrow: \mathit{Text}\ \texttt{"Arg. \#1 to 'atop' must be a diagram"},\\
&\quad d_2 :\not\sim: \mathit{QDiagram}\ b_2\ v_2\ n_2\ m_2\\
&\qquad :\Rrightarrow: \mathit{Text}\ \texttt{"Arg. \#2 to 'atop' must be a diagram"},\\
&\quad b_1 :\not\sim: b_2\\
&\qquad :\Rrightarrow: \mathit{Text}\ \texttt{"Back-ends do not coincide"},\\
&\quad \ldots\\
&\quad \mathit{Check}\ (\mathit{OrderedField}\ n_1), \mathit{Check}\ (\mathit{Metric}\ v_1),\\
&\quad \mathit{Check}\ (\mathit{Semigroup}\ m_1)\\
&\quad] \Rightarrow d_1 \to d_2 \to d_1
\end{aligned}
$$

The *CustomErrors* type family traverses the list to build the constraint structure.

Abstraction and reuse

For consistency and conciseness we can define a type level implementation for the checks of back-ends, vector spaces, etc.

$$\textbf{type}\ \mathit{DoNotCoincide}\ \mathit{what}\ a\ b = \\ a :\not\sim: b :\Rrightarrow: \mathit{Text}\ \mathit{what} :\diamond: \mathit{Text}\ \texttt{" do not coincide: "} \\ :\diamond: \mathit{ShowType}\ a :\diamond: \mathit{Text}\ \texttt{" vs. "} :\diamond: \mathit{ShowType}\ b$$

Note that *ShowType* and type level *Texts* are provided by GHC.

The EDSL-developer facing API (version 2)

Some constraints can be checked independently: partition constraints into a list of lists.

$atop :: CustomErrors$ [
  [d_1 :$\not\sim$: $QDiagram\ b_1\ v_1\ n_1\ m_1$
      :⇛: $Text$ `"Arg. #1 to 'atop' must be a diagram"`,
   d_2 :$\not\sim$: $QDiagram\ b_2\ v_2\ n_2\ m_2$
      :⇛: $Text$ `"Arg. #2 to 'atop' must be a diagram"`],
  [$DoNotCoincide$ `"Back-ends"` $b_1\ b_2$,
   $DoNotCoincide$ `"Vector spaces"` $v_1\ v_2$,
   $DoNotCoincide$ `"Numerical fields"` $n_1\ n_2$,
   $DoNotCoincide$ `"Query annotations"` $m_1\ m_2$],
  [$Check\ (OrderedField\ n_1)$, $Check\ (Metric\ v_1)$,
   $Check\ (Semigroup\ m_1)$]
 ] => d_1 -> d_2 -> d_1

Return of the giant siblings

$$
\begin{aligned}
(\langle\$\rangle) :: \mathit{Sibling}\ & \texttt{"(<\$>)"}\ (\mathit{Applicative}\ f)\ ((a \to b) \to f\ a \to f\ b) \\
& \texttt{"(<\$)"}\ \qquad\qquad\qquad\quad (a \qquad\quad \to f\ b \to f\ a) \\
& \mathit{fn} \\
\Rightarrow\ & \mathit{fn}
\end{aligned}
$$

Given $f :: \mathit{Char} \to \mathit{Bool} \to \mathit{Int}$,
$f\ \langle\$\rangle\ [1 :: \mathit{Int}]\ \langle*\rangle\ \texttt{"a"}\ \langle*\rangle\ [\mathit{True}]$ leads to

```
* Type error in '(<$>)', do you mean '(<$)'
* In the first argument of '(<*>)', namely 'f <$> [1
  ...
```

What are these *fn*s doing there?

Implementation of siblings

We can define siblings on top of what we have (almost):

type *Sibling nameOk extra tyOk nameWrong tyWrong fn*
  = *IfNot* (*fn* ∼ *tyOk*)
    (*ScheduleAtTheEnd* (*IfNot* (*fn* ∼ *tyWrong*)
      (*fn* ∼ *tyOk*)
      (*TypeError* (*Text* `"Type error in '"` ...
              Text `"', do you mean '"`...))))
    extra

One caveat: we need *ScheduleAtTheEnd* to assign the lowest possible priority (otherwise *fn* ∼ *tyWrong* may succeed while other constraints in the set contradict it).

Alternatives and conversions

- *diagrams* distinguishes vectors from points
- You can compute the perpendicular of a vector (but not a point (pair)) with *perp*
- Can we provide a hint on how to convert a pair to a vector if the argument happens to be a pair?

```
* Expecting a 2D vector but got a tuple.
  Use 'r2' to turn the tuple into a vector.
```

As with siblings this may not be what the programmer intends, but the change will resolve the type error.

The new definition of *perp*

```
perp :: CustomErrors [
  [v :≁: V2 a :⇛?:
    ([v ~ (a, a) :⇛!:
       Text "Expecting a 2D vector but got a tuple."
       :$$: Text "Use r2 to turn a tuple into a vector."
       ],
       Text "Expected a 2D vector, but got "
       :◇: ShowType v)],
  [Check (Num a)]] => v -> v
```

With every apartness check we can associate a list of further checks on what in this case *v* might actually be.

A word of warning

- Why is the unification $v \sim (a, a)$ not so dangerous now?
- If we arrive there at all we know:
  - compilation will fail
  - we know the top level type constructor of v
- However: writing (a, a) does imply that a unification may take place.
- To be safe: only compare against $T\ a1 \mathinner{..} an$ with T a fixed type constructor, and all ai fresh.

What we did on our holidays

- We have worked out some rules for
  - *path* (Chris Done, 2015), appendix to the paper
  - *diagrams*
  - *persistent*
  - *map*, *Eq*, and making *foldr* and *foldl* siblings
  - *formatting* (Chris Done)
- They can be added to members of type classes too!

Wrapping up

Let's visit the Terminal/jEdit and take a look at *now* and (%).

Back to our slogan

Expression level type error messages
by
type level programming

- In retrospect, this makes a lot of sense
- Kind level programming for diagnosing type level programming?
- Possible relationships with dependently typed programming, staged programming, and higher-ranked analyses with effect operators
  - All provide a way to perform computations at the type level/compile time, with different restrictions.

Beyond the scope of this tutorial

- Type error diagnosis in Elm (with Falco Peijenburg and Alejandro Serrano)
- Type error diagnosis in LambdaPi (with Joey Eremondi and Wouter Swierstra)
- Refining type guards in Typescript (with Ivo Gabe de Wolff)
- Unification modulo type isomorphism (with Arjen Langebaerd and Bastiaan Heeren)
- Questions can be asked off-line

Recent and ongoing developments

- GADTs in Helium

Recent and ongoing developments

- GADTs in Helium
- Type families in Helium

Recent and ongoing developments

- GADTs in Helium
- Type families in Helium
- Formalizing system FC and row types

More plans

- Type class flavours in Helium

More plans

- Type class flavours in Helium
- Higher-ranked/impredicative types and error diagnosis in Helium

More plans

- Type class flavours in Helium
- Higher-ranked/impredicative types and error diagnosis in Helium
- Optimisation assistance in Helium

More plans

- Type class flavours in Helium
- Higher-ranked/impredicative types and error diagnosis in Helium
- Optimisation assistance in Helium
- TED for dependently typed languages

Thank you for your attention

VII. Other approaches

Type error slicing

- Sulzmann, Wazny, Stuckey: Chameleon system
  - Could deal with some language extensions
- Haack and Wells, and later also Rahli and a few others: type error slicing for (full) ML
- Thomas Schilling did something like this for Haskell

Heuristics

- IFL 2006, Helium's heuristics
- Nabil el Boustani translated it to Generic Java
- Danfeng Zhang and Andrew Myers (Bayesian predictor)
- Pavlinovic et al. (uses SMT solver to find optimal solutions)

Repair systems

- Suggesting fixes
- Helium's siblings
- McAdam's unification modulo type isomorphism
- Arjen Langebaerd MSc thesis (on Helium)

Black box systems

- Seminal (Lerner et al.): use type system on variations of the type incorrect program to determine how to diagnose the error
- Advantage: non-invasive, low effort and low risk

Counterfactual typing

- Erwig and Chen: counterfactual typing
- Allows the use of a type incorrect identifier to decide the correct type for the identifier
- The basis of the technology comes from feature selection

Beyond the intrinsic type system

- Weijers, Hage and Holdermans, Security type error diagnosis, SCP 2014
- Combines slicing (to get an over approximation of the locations), uses heuristics to further narrow down

Domain specific type error diagnosis

- Scripting The Type Inferencer by Heeren, Hage, Swierstra, 2003
- For Scala Hubert Plociniczak, Odersky and others did something similar
- Current work on Domsted

Post-processing

- David Raymond Christensen: better error diagnosis through post-processing in the dependently type Idris language