

Introduction to Auctions

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Motivation

- ▶ Auctions are mechanisms for **allocating resources among self-interested agents**
- ▶ Very widely used
 - ▶ government sale of resources
 - ▶ privatization
 - ▶ stock market
 - ▶ request for quote
 - ▶ real estate sales
 - ▶ eBay
- ▶ **Resource allocation** is a fundamental problem in CS
- ▶ Increasing importance of studying distributed systems with heterogeneous agents

A Taxonomy

- ▶ **Single Unit Auctions** (where one good is involved);
- ▶ **Multiunit Auctions** (where more tokens of the same goods are involved);
- ▶ **Combinatorial Auctions** (where more tokens of different goods are involved);
- ▶ We will assume that participants can either be **buyers** or **sellers**, i.e. we do not talk about **exchanges**;
- ▶ For all the categories, a classification will be provided, together with formal definitions and some theoretical results.

Single Unit Auctions

- ▶ There is **one good for sale**, one seller, and multiple potential buyers;
- ▶ Each buyer has his own valuation for the good, and each wishes to purchase it at the lowest possible price.
- ▶ **Desirable Properties**
 - ▶ There are auction maximizing the expected revenue of the auctioneer;
 - ▶ There are auction that guarantees that the potential buyer with the highest valuation ends up with the good (no winner's curse).
- ▶ Types of Single Unit Auctions
 - ▶ English
 - ▶ Japanese
 - ▶ Dutch
 - ▶ First- en Second-price Sealed-bid

English Auction

- ▶ The auctioneer sets a starting price for the good;
- ▶ Agents then have the option to announce successive bids;
- ▶ Each bid must be higher than the previous one;
- ▶ The final bidder must purchase the good for the amount of his final bid.

Japanese Auction

- ▶ The auctioneer sets a starting price for the good;
- ▶ Each agent must chose whether he is **in** or **out** for that price; dropping out is irrevocable.
- ▶ The auctioneer calls increasing prices in a regular fashion;
- ▶ The auction ends when exactly one agent is **in**, who must purchase the product.

Dutch Auction

- ▶ The auctioneer sets a starting price for the good;
- ▶ Each agent has the option to buy the good for that price;
- ▶ The auctioneer calls decreasing prices in a regular fashion;
- ▶ The auction ends when exactly an agent purchases the product.

Sealed-Bid Auctions

- ▶ Each agent submits to the auctioneer a secret bid for the good that is not accessible to any of the other agents;
- ▶ The agent with the highest bid must purchase the good;
 - ▶ In **first-price** auctions, the price is the value of highest bid;
 - ▶ In **second-price** auctions (**Vickrey Auction**), the price is the value of the second-highest bid.

Auctions as Structured Negotiations

A negotiation mechanism that is:

- ▶ **market-based** (determines an exchange in terms of currency)
- ▶ **mediated** (auctioneer)
- ▶ **well-specified** (follows rules)

Defined by three kinds of rules:

- ▶ rules for bidding
- ▶ rules for what information is revealed
- ▶ rules for clearing

Auctions as Structured Negotiations

Defined by three kinds of rules:

- ▶ rules for **bidding**
 - ▶ who can bid, when
 - ▶ what is the form of a bid
 - ▶ restrictions on offers, as a function of:
 - ▶ bidder's own previous bid
 - ▶ auction state (others' bids)
 - ▶ eligibility (e.g., budget constraints)
 - ▶ expiration, withdrawal, replacement
- ▶ rules for what information is revealed
- ▶ rules for clearing

Auctions as Structured Negotiations

Defined by three kinds of rules:

- ▶ rules for bidding
- ▶ rules for **what information is revealed**
 - ▶ when to reveal what information to whom
- ▶ rules for clearing

Auctions as Structured Negotiations

Defined by three kinds of rules:

- ▶ rules for bidding
- ▶ rules for what information is revealed
- ▶ rules for **clearing**
 - ▶ when to clear
 - ▶ at intervals
 - ▶ on each bid
 - ▶ after a period of inactivity
 - ▶ allocation (who gets what)
 - ▶ payment (who pays what)

Intuitive comparison of 5 auctions

	English	Dutch	Japanese	1st-Price	2nd-Price
Duration	#bidders, increment	starting price, clock speed	#bidders, increment	fixed	fixed
Info Revealed	2 nd -highest val; bounds on others	winner's bid	all val's but winner's	none	none
Jump bids	yes	n/a	no	n/a	n/a
Price Discovery	yes	no	yes	no	no
Regret	no	yes	no	yes	no

Auctions as games

Let X be a set of allocations of goods. An auction can be viewed as a game $\langle N, A, O, \chi, \rho \rangle$

- ▶ N is a set of agents;
- ▶ $A = A_1 \times \dots \times A_n$ is the strategy space (each player's possible moves);
- ▶ $O = X \times \mathcal{R}^n$ is a set of outcomes (allocation of goods with payments);
- ▶ $\chi : A \rightarrow O$ is the choice function, which associates an outcome to action profile;
- ▶ $\rho : A \rightarrow \mathcal{R}^n$ is the payment function, which associates a payment for each agent to an action profile;

Second-price, sealed bid auction

Proposition

*In a second-price auction where bidders have independent private values, **truth telling is a dominant strategy**.*

Proof.

Assume that the other bidders bid in some arbitrary way. We must show that i 's best response is always to bid truthfully. We'll break the proof into two cases:

1. Bidding honestly, i would win the auction
2. Bidding honestly, i would lose the auction



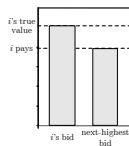
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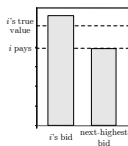
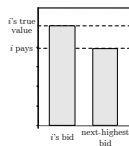
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- ▶ If i bids higher, he will still win and still pay the same amount

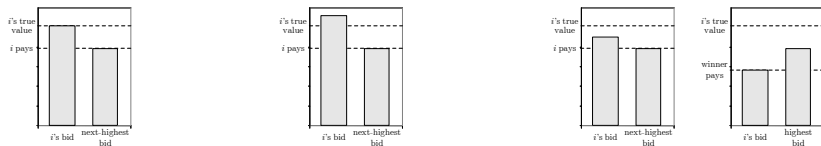
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- ▶ If i bids higher, he will still win and still pay the same amount
- ▶ If i bids lower, he will either still win and still pay the same amount or lose and get utility of zero.

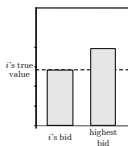
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Bidding honestly, i is not the winner



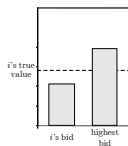
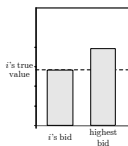
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In a second-price auction where bidders have independent private values, truth telling is a dominant strategy.

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Bidding honestly, i is not the winner



- ▶ If i bids lower, he will still lose and still pay nothing

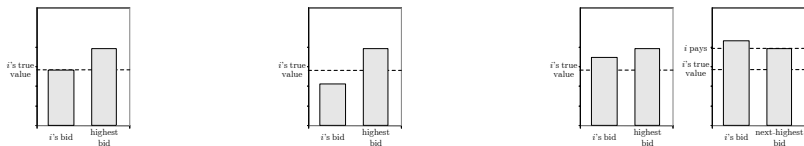
Second-price, sealed bid auction

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In a second-price auction where bidders have independent private values, truth telling is a dominant strategy.

Proof.

Bidding honestly, i is not the winner



- ▶ If i bids lower, he will still lose and still pay nothing
- ▶ If i bids higher, he will either still lose and pay nothing or win and pay more than his valuation.

Second-price, Japanese and English auctions

Assuming Independent Private Value (IPV)

- ▶ Second-price and Japanese auctions are closely related. Each bidder selects a number and the bidder with the highest bid wins and pays (something near) the second-highest bid.
- ▶ Second-price and English auctions are closely related as well. Use Proxy bidding.
 - ▶ A much **more complicated** strategy space for ascending-bid auctions
 - ▶ extensive form game
 - ▶ bidders are able to condition their bids on information revealed by others
 - ▶ in the case of English auctions, the ability to place jump bids
 - ▶ intuitively, though, the revealed information does not make any difference in the IPV setting.

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Theorem

*Under the independent private values model (IPV), it is a **dominant strategy** for bidders to bid up to (and not beyond) their valuations in both Japanese and English auctions.*

First-Price and Dutch auctions

Theorem

*First-Price and Dutch auctions are **strategically equivalent**, i.e., they are the same auction, regardless of valuation model or risk attitudes.*

- ▶ In both first-price and Dutch auctions, a bidder must decide on the amount he's willing to pay without knowing the other agents' selections. The highest bidder wins and pay its announced bid.
- ▶ Both auction types are used in practice for different reasons.
 - ▶ First-price auctions can be held **asynchronously**
 - ▶ Dutch auctions are fast, and require **minimal communication**: only one bit needs to be transmitted from the bidders to the auctioneer.
- ▶ How should bidders bid in these auctions?
 - ▶ Bidders do not have a dominant strategy any more.
 - ▶ They should clearly bid **less than their valuations**.
 - ▶ There's a tradeoff between:
 - ▶ probability of winning
 - ▶ amount paid upon winning

Truth revelation and Revenue equivalence

Proposition

In a first-price auction participants are better off by not telling the truth, i.e., truth telling is not rewarding.

Proposition

In a first-price sealed-bid auction with n agents the unique equilibrium is given by the strategy profile $(\frac{n-1}{n}v_1, \dots, \frac{n-1}{n}v_n)$.

Proposition

*Under certain assumptions (risk neutral agents with independent private valuation), English, Japanese, Dutch and all sealed-bid auctions are **revenue equivalent**, i.e., the seller receives the same revenue from all auctions.*

Risk-neutral, IPV	Jap	=	Eng	=	2nd	=	1st	=	Dutch
Risk-averse, IPV	Jap	=	Eng	=	2nd	<	1st	=	Dutch
Risk-seeking, IPV	Jap	=	Eng	=	2nd	>	1st	=	Dutch

Multiunit Auctions

- ▶ We have so far considered the problem of selling a single good to one winning bidder;
- ▶ In practice there will often be **more than one good to allocate**, and different goods may end up going to different bidders;
- ▶ Multiunit auctions consider now **multiple copies of a good**;
- ▶ The type of the good remains however the same.

Sealed-bid auctions

- ▶ If there are three items for sale, and each of the top three bids requests a single unit, then each bid will win one good. However the price paid may vary:
 - ▶ **Discriminatory pricing rule:** everyone pays his own bid;
 - ▶ **Uniform pricing rule:** all winners pay the same amount (a function of the highest bids);
- ▶ Bidders may place bids on multiple units. But the bid types can vary:
 - ▶ **all or nothing bid:** bidders will buy no less than the number of units they bid for;
 - ▶ **divisible bid:** bidders are willing to purchase a fewer number of units each for individual bidding price;
- ▶ Further tie-breaking rules could be enacted to weight different types of bids (e.g., larger, earlier bids win).

Single-unit demand on Multiunit auctions

- ▶ Consider a setting in which k identical goods are for sale;
- ▶ Consider n bidders with independent private value, each willing one unit of good;

We have seen that in single good second price auctions truth telling was a **dominant strategy**: is there an equivalent result for Multiunit auctions?

- ▶ The auction mechanism is to sell the units to the k highest bidders for the same price, and to set this price at the amount offered by the highest losing bid. Thus, instead of a second-price auction we have a $k + 1$ st-price auction;
- ▶ The proof can be generalized.

English auctions

- ▶ It faces the same problems discussed for sealed-bid auctions;
- ▶ Bidders can revise their bids from one round to another. This is often not allowed, i.e., bidders can specify only one number of units considered as a divisible bid.
- ▶ As it works with minimum increments, it may become problematic to define this notion for multiple units.

Japanese auctions

- ▶ After each price increase each agent calls out a number rather than the simple in/out declaration, signifying the number of units he is willing to buy at the current price;
- ▶ The number must decrease over time;
- ▶ The auction is over when the supply equals or exceeds the demand. In the latter case, goods can go unsold.

Dutch auctions

- ▶ The seller calls out descending per unit prices;
- ▶ Agents must declare a quantity they want to buy;
- ▶ If that is not the entire available quantity, the auction continues.

Combinatorial Auctions

- ▶ We allow for a **variety of goods** to be available in the market;
- ▶ Goods may no longer be **interchangeable**.
- ▶ Consider a set of bidders $N = \{1, \dots, n\}$ and a set of goods $G = \{1, \dots, m\}$;
- ▶ Let (v_1, \dots, v_n) denote the true valuation functions of the different bidders, where $v_i : 2^G \rightarrow \mathcal{R}$.
- ▶ Remember that each agent's valuation depends only on the goods he wins.

Auctioning related goods

- ▶ Auctioning related bundles of goods may be problematic for the **exposure problem**: a bidder might bid aggressively for a set of goods in the hopes of winning a bundle, but succeed in winning only a subset of the goods and therefore pay too much.
- ▶ Combinatorial auctions solve the problem: they allow bidders to **bid directly on combinatorial bundles of goods**.
- ▶ A simple combinatorial auction is to compute an allocation that maximizes the social welfare of the declared valuations and charge the winners with their bids. **Truth telling is not dominant**.

Bidder1

$$v_1(x, y) = 100$$

$$v_1(x) = v_1(y) = 0$$

Bidder2

$$v_2(x) = 75$$

$$v_2(x, y) = v_2(y) = 0$$

Bidder3

$$v_3(y) = 40$$

$$v_3(x, y) = v_3(x) = 0$$

Expressing succinct bids

- ▶ We have so far assumed that bidders will specify a valuation for every subset of the goods at auction. But this number grows rapidly.
- ▶ We need to express bids (valuation functions) in a succinct manner. The language should be:
 - ▶ **expressive**, i.e. powerful enough to talk about the actual bids;
 - ▶ **natural**, i.e. understandable and easy to use;
 - ▶ **tractable**, i.e. questions asked in that language can be answered positively or negatively in a polynomial amount of time.
- ▶ Valuation functions v_i are assumed to have the following properties:
 - ▶ **Free-disposal**: for $S, T \subseteq G$, we have that $S \subseteq T$ implies $v_i(S) \leq v_i(T)$, i.e. goods have non-negative value;
 - ▶ **Nothing for nothing**: $v_i(\emptyset) = \emptyset$, i.e. getting no goods is getting no utility.

Substitutability

- ▶ However goods can fully or partially substitute each other (for instance CD player and MP3 player);
- ▶ The value of winning two goods which substitute each other may be less than the sum of the value of winning them separately.

Definition

Bidder i 's valuation exhibits **substitutability** if there exist two sets of goods $G_1, G_2 \subseteq G$ such that $G_1 \cap G_2 = \emptyset$ and $v_i(G_1 \cup G_2) < v_i(G_1) + v_i(G_2)$. When this condition holds, we say that the valuation function v_i is **subadditive**.

Complementarity

- ▶ Opposite to substitutable goods we can have goods which fully or partially complete each other (for instance left shoe and right shoe);
- ▶ The value of winning two goods which complement each other may be at times bigger than the sum of the value of winning them separately.

Definition

Bidder i 's valuation exhibits **complementarity** if there exist two sets of goods $G_1, G_2 \subseteq G$ such that $G_1 \cap G_2 = \emptyset$ and $v_i(G_1 \cup G_2) > v_i(G_1) + v_i(G_2)$. When this condition holds, we say that the valuation function v_i is **superadditive**.

Atomic bids

- ▶ The most basic bid language we consider associates an offer to a set of goods. We call this an **atomic** bid. It is a pair (S, p) where $S \subseteq G$ and p is the price agent i is willing to pay for S . We write then $v_i(S) = p$.
- ▶ Notice that atomic bids are implicitly AND bids.

OR bids

- ▶ Atomic bids cannot express disjunctive bids (i.e. 10 euros for a CD player or 20 euros for a MP3 player).
- ▶ An **OR** bid is a disjunction of atomic bids, i.e., $(S_1, p_1) \vee \dots \vee (S_k, p_k)$.
- ▶ Intuition: A bidder is willing to obtain any number of disjoint atomic bids for the sum of their respective prices, i.e.,
$$v((S_1, p_1) \vee \dots \vee (S_k, p_k)) = p_1 + \dots + p_k.$$

Proposition

OR bids can express all valuation functions that exhibit no substitutability, i.e., for valuation functions v where for all $S \cap T = \emptyset$, $v(S \cup T) \geq v(S) + v(T)$, and only these.

XOR bids

XOR bids do not have this limitation. They are an exclusive OR of atomic bids $(S_1, p_1) \parallel \dots \parallel (S_k, p_k)$, to mean that the agent is willing to **accept exactly one of the atomic bids**.

Proposition

XOR bids can represent all possible valuation functions.

Proposition

*Any **additive valuations** on m items can be represented by OR bids of size m , but can be represented by XOR bids of size 2^m .*

Combining OR and XOR together does not add **expressivity**, but may add **compactness**.