

# Communication in Games

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# Cheap Talk

In cheap talk, players can communicate before taking actions.

- ▶ Communication is **costless**
- ▶ Communication does **not need to be truthful**
- ▶ Communication does **not imply commitment**

Cheap talk can be seen as a two-stage game.

- ▶ **First stage**: Players communicate
- ▶ **Second stage**: Players decide actions

	<i>NotConfess</i>	<i>Confess</i>
<i>NotConfess</i>	2, 2	0, 3
<i>Confess</i>	3, 0	1, 1

Let players be able to communicate before the game, e.g., row player says he will not confess. Does this change the game's outcome?

# Cheap Talk

How about this game? Does the game's outcome change if the Row player declares to play U?

	<i>L</i>	<i>R</i>
<i>U</i>	1,1	0,0
<i>D</i>	0,0	1,1

# Cheap Talk

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Cheap talk is **not Worthless**. It can affect the outcome of a game by changing one player's beliefs about another player's actions, and so selecting one equilibrium out of multiple equilibria.

# Cheap Talk

*Signaling game* is a type of game with communication where one player (the sender) has private information that is not directly observable by another player (the receiver). The sender can communicate its private information and the receiver makes choices based on the received message.

A signaling game is an extensive game in which the receiver of the message has incomplete information.

A practical example:

- ▶ Anna has a company with a Demanding and an Undemanding position
- ▶ Bob is seeking for a job and his ability may be High or Low
- ▶ Anna and Bob benefit if Bob gets the appropriate job

<i>Bob</i> \ <i>Anna</i>	Demanding	Undemanding
High	2, 1	0, 0
Low	0, 0	1, 3

Note: If Bob tells Anna that he has low ability, Anna can believe him since Bob has no incentive to lie.

# Signaling Games with Imperfect Information

Row player knows which game is selected by the nature (equal probability). Row player chooses his message (U or D) and Column player (who does not know which game is being chosen by the nature) will choose his action (L or R).

	<i>L</i>	<i>R</i>
<i>U</i>	4, -4	1, -1
<i>D</i>	3, -3	0, 0

	<i>L</i>	<i>R</i>
<i>U</i>	1, -1	3, -3
<i>D</i>	2, -2	5, -5

Which signal should row player sends to column player?

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Which signal should row player sends to column player?

Equilibria are in dominant strategies. So, column player will play R if row player selects U, and L if row player selects D.

$$EU(\text{Row}) = (0.5)1 + (0.5)2 = 1.5$$

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Which signal should row player send to column player?

A better strategy for row player is to always select D.

$$EU(\text{Row}) = (0.5)(3p + 0(1 - p)) + (0.5)(2p + 5(1 - p)) = 2.5$$

given column player plays L with probability  $p$  and R with probability  $1 - p$ .

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Which signal should row player sends to column player?

It is not always an advantage to exploit the privileged information.

# Cheap Talk: Credibility

**Key Question:** Can cheap talk influence the outcomes of a game, i.e., for every Nash equilibrium of the **game without communication**, is there a (subgame) perfect Nash equilibrium in the **game with communication** that differs (and perhaps Pareto superior)?

Under which condition does a cheap talk influence the outcome of a game? I.e., when does a cheap talk **conveys information** and is **credible**?

Note that:

- ▶ Players may not tell the truth.
- ▶ Players' messages may not be believed.

# Cheap Talk: Credibility

Under which condition does a cheap talk influence the outcome of a game? I.e., when does a cheap talk **conveys information** and is **credible**?

A declared type (action) is

- ▶ **Self-Committing**: if Sender believes that Receiver believes that Sender has (is going to do) the declared type (action), then the declared type (action) is optimal.

Or alternatively, once uttered, and assuming it is believed, the declared action is the optimal one.

- ▶ **Self-Revealing**: Sender wants Receiver to believe that he has (is going to do) the declared type (action) if and only if the Sender has (is going to do) the declared type (action).

Or alternatively, assuming it is uttered with the expectation that it will be believed, it is uttered only when it was the intention to act that way.

# Cheap Talk in Signaling Games

Under which condition does a cheap talk influence the outcome of a game? I.e., when does a cheap talk is **credible** and **conveys information**?

	Demanding	Undemanding
High	2, 1	0, 0
Low	0, 0	1, 3

Is the utterance "I have high ability" by Bob self-committing and self-revealing?

# Cheap Talk in Signaling Games

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Is the utterance "I have high ability" by Bob self-committing and self-revealing?  
Yes, it is self-committing.

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High	2, 1	0, 0
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Is the utterance "I have high ability" by Bob self-committing and self-revealing?

Yes, it is self-committing.

Yes, it is self-revealing.

# Cheap Talk in Signaling Games

Suppose now that Bob wants to have the Demanding position regardless of its ability.

	Demanding	Undemanding
High	2, 1	0, 0
Low	2, 0	1, 3

Exercise 1: Is the utterance "I have high ability" by Bob self-committing and self-revealing? why?

Exercise 2: Is the utterance "I have low ability" by Bob self-committing and self-revealing? why?

# Cheap Talk in Games

What if Sender has no fixed type?

	<i>L</i>	<i>R</i>
<i>U</i>	1,1	0,0
<i>D</i>	0,0	1,1

Exercise 3: Is the utterance "I play U" by the row player self-committing and self-revealing? why?

# Self-Commitment and Self-revelation

- ▶ **Self-Committing Utterance:** if Sender believes that Receiver believes that Sender is going to do the declared action, then the declared action is optimal.
- ▶ **Self-Revealing Utterance:** Sender wants Receiver to believe that he is going to do the declared action if and only if the Sender is going to do the declared action.

	<i>Stag</i>	<i>Hare</i>
<i>Stag</i>	9, 9	0, 8
<i>Hare</i>	8, 0	7, 7

The row player declares to play Stag.

- ▶ Is this self-revealing utterance?
- ▶ Is it a self-committing utterance?

# Self-Commitment and Self-revelation

	<i>Stag</i>	<i>Hare</i>
<i>Stag</i>	9, 9	0, 8
<i>Hare</i>	8, 0	7, 7

The row player's utterance to play Stag is **not self-revealing** because row player would like column player to believe the utterance no matter what row player is going to play. This is because if the column player plays Stag, then the row player receives more payoff no matter what he plays. Note that in addition to the two pure Nash equilibria, there exists a mixed strategy equilibrium (playing Stag with probability  $7/8$ ) with expected utility 7.875. If column player believes the utterance, the expected utility increases to 8.875. Thus, row player has an incentive to make the column player to believe the utterance even if he has no plan to play Stag before announcing it.

# Self-Commitment and Self-revelation

	<i>Stag</i>	<i>Hare</i>
<i>Stag</i>	9,9	0,8
<i>Hare</i>	8,0	7,7

The utterance is **self-committing** because if row player thinks the column player believes it will play Stag, then row player prefers to play Stag.

# Self-Commitment and Self-revelation

	<i>Stag</i>	<i>Hare</i>
<i>Stag</i>	9,9	0,8
<i>Hare</i>	8,0	7,7

Self-commitment alone, without self-revelation, is not credible.

# Payoff & Risk Dominance

(9,9) payoff dominates (7,7)

	Stag	Hare
Stag	9,9	0,8
Hare	8,0	7,7

(Stag , Stag) payoff dominates (Hare , Hare) if  $A \geq D$ ,  $a \geq d$ , and at least one of the two is a strict inequality:  $A > D$  or  $a > d$

	Stag	Hare
Stag	A, a	C, b
Hare	B, c	D, d

# Payoff & Risk Dominance

(9,9) payoff dominates (7,7)

(7,7) risk dominates (9,9)

	Stag	Hare
Stag	9, 9	0, 8
Hare	8, 0	7, 7

(Hare , Hare) risk dominates (Stag , Stag) if  $(C - D)(c - d) \geq (B - A)(b - a)$

	Stag	Hare
Stag	A, a	C, b
Hare	B, c	D, d

# Payoff & Risk Dominance

(9,9) payoff dominates (7,7)

(7,7) risk dominates (9,9)

	<i>Stag</i>	<i>Hare</i>
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**An agreement is effective only if it changes the payoffs or the players' information (Aumann, 1990).**

# Self-Commitment and Self-revelation

	<i>Stag</i>	<i>Hare</i>
<i>Stag</i>	100, 100	0, 8
<i>Hare</i>	8, 0	7, 7

The row player declares to play Stag. Is this self-revealing utterance? Is it a self-committing utterance?

# Self-Commitment and Self-revelation

	<i>Stag</i>	<i>Hare</i>
<i>Stag</i>	100, 100	0, 8
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The row player declares to play Stag. Is this self-revealing utterance? Is it a self-committing utterance?

The strategy profile (**Stag, Stag**) is both payoff and risk dominant strategy profile. Players will therefore chose this outcome regardless of agreement.

# Self-Commitment and Self-revelation

	<i>Stag</i>	<i>Hare</i>
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The row player declares to play Stag. Is this self-revealing utterance? Is it a self-committing utterance?

The strategy profile (**Stag, Stag**) is both payoff and risk dominant strategy profile. Players will therefore choose this outcome regardless of agreement.

**Therefore, a signal to play Stag does not change the players' information.**

# Self-Commitment and Self-revelation

	<i>Stag</i>	<i>Hare</i>
<i>Stag</i>	9, 9	0, 8
<i>Hare</i>	8, 0	7, 7

Self-commitment with self-revelation is certainly credible. However, self-commitment alone is not credible.

# Revealing and Babbling Equilibria with Risk Analysis

- ▶ **Babbling Equilibrium:** An equilibrium that is not affected by a cheap talk, i.e., Row player sends a meaningless message and column player ignores it.
- ▶ **Revealing Equilibrium:** An equilibrium which is not a babbling equilibrium.

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## Revealing Equilibria:

- ▶ **Column player says (honestly) 'Stag' and both Stag.** Most likely if column player has a reputation for honest (risk analysis is required).
- ▶ **Column player says (dishonestly) 'Hare' and both Stag.** Most likely if column player has a reputation for dishonesty (risk analysis is required).

Perfectly dishonest people are worth listening to for the same reasons as listening to perfectly honest people.

# Revealing and Babbling Equilibria with Risk Analysis

- ▶ **Babbling Equilibrium:** An equilibrium that is not affected by a cheap talk, i.e., Row player sends a meaningless message and column player ignores it.
- ▶ **Revealing Equilibrium:** An equilibrium which is not a babbling equilibrium.

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## Babbling Equilibria:

- ▶ **Column player says (dishonestly) 'Stag' and both Stag.** Most likely if column player is often ignored (risk analysis is required).
- ▶ **Column player says (dishonestly) 'Stag' and both Hare.** Most likely if column player is often ignored (risk analysis is required).

Every cheap talk game has a babbling equilibrium.