Data Mining Text Classification Naive Bayes

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- Text Mining is data mining applied to text data.
- Often uses well-known data mining algorithms.
- Text data requires substantial pre-processing.
- This typically results in a large number of attributes (for example, the size of the vocabulary/dictionary).

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- Predict the class(es) of text documents.
- Can be single-label or multi-label.
- Multi-label classification is often performed by building multiple binary classifiers (one for each possible class).
- Examples of text classification:
 - topics of news articles,
 - spam/no spam for e-mail messages,
 - sentiment analysis (e.g. positive/negative review),
 - opinion spam (e.g. fake reviews),
 - music genre from song lyrics

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Is this Rap, Blues, Metal, or Country?

Blasting our way through the boundaries of Hell No one can stop us tonight We take on the world with hatred inside Mayhem the reason we fight Surviving the slaughters and killing we've lost Then we return from the dead Attacking once more now with twice as much strength We conquer then move on ahead

[Chorus:] Evil My words defy Evil Has no disguise Evil Will take your soul Evil My wrath unfolds

Satan our master in evil mayhem Guides us with every first step Our axes are growing with power and fury Soon there'll be nothingness left Midnight has come and the leathers strapped on Evil is at our command We clash with God's angel and conquer new souls Consuming all that we can

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A probabilistic classifier assigns a probability to each class. In case a class prediction is required we typically predict the class with highest probability:

$$\hat{c} = \arg \max_{c \in C} P(c \mid d) = \arg \max_{c \in C} \frac{P(d \mid c)P(c)}{P(d)}$$

where d is a document, and C is the set of all possible class labels.

Since $P(d) = \sum_{c \in C} P(c, d)$ is the same for all classes, we can ignore the denominator if we only need to know the most likely class:

$$\hat{c} = \arg \max_{c \in C} P(c \mid d) = \arg \max_{c \in C} P(d \mid c)P(c)$$

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Represent document as set of features:

$$\hat{c} = \arg \max_{c \in C} P(d \mid c) P(c) = \arg \max_{c \in C} P(x_1, \dots, x_m \mid c) P(c)$$

Naive Bayes assumption:

$$P(x_1,\ldots,x_m \mid c) = P(x_1 \mid c)P(x_2 \mid c) \cdot \ldots \cdot P(x_m \mid c)$$

The features are assumed to be independent within each class (avoiding the curse of dimensionality).

$$c_{\scriptscriptstyle \mathrm{NB}} = rg\max_{c \in \mathcal{C}} P(c) \prod_{i=1}^m P(x_i \mid c)$$

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Independence Graph of Naive Bayes



The features are independent given the class label.

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Bag Of Words Representation of a Document



Figure 6.1 Intuition of the multinomial naive Bayes classifier applied to a movie review. The position of the words is ignored (the *bag of words* assumption) and we make use of the frequency of each word.

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Bag Of Words Representation of a Document



Not matter, the order and position do.

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Represent document *d* as a sequence of words: $d = \langle w_1, w_2, \dots, w_n \rangle$.

$$c_{\scriptscriptstyle \mathrm{NB}} = rg\max_{c \in \mathcal{C}} P(c) \prod_{k=1}^n P(w_k \mid c)$$

Notice that $P(w \mid c)$ is independent of word position or word order, so d is truly represented as a bag-of-words. Taking the log we obtain:

$$c_{\scriptscriptstyle \mathrm{NB}} = rg\max_{c\in C}\log P(c) + \sum_{k=1}^n\log P(w_k\mid c)$$

By the way, why is it allowed to take the logarithm?

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Consider the text (perhaps after some pre-processing)

catch as catch can

We have $d = \langle \text{catch}, \text{as}, \text{catch}, \text{can} \rangle$, with $w_1 = \text{catch}, w_2 = \text{as}, w_3 = \text{catch}, \text{ and } w_4 = \text{can}$. Suppose we have two classes, say $C = \{+, -\}$, then for this document:

$$\begin{split} c_{\text{\tiny NB}} &= \arg \max_{c \in \{+,-\}} \log P(c) + \log P(\texttt{catch} \mid c) + \log P(\texttt{as} \mid c) \\ &+ \log P(\texttt{catch} \mid c) + \log P(\texttt{can} \mid c) \\ &= \arg \max_{c \in \{+,-\}} \log P(c) + 2 \log P(\texttt{catch} \mid c) + \log P(\texttt{as} \mid c) \\ &+ \log P(\texttt{can} \mid c) \end{split}$$

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Training Multinomial Naive Bayes

Class priors:

$$\hat{P}(c) = \frac{N_c}{N_{doc}}$$

Word probabilities within each class:

$$\hat{P}(w_i \mid c) = rac{ ext{count}(w_i, c)}{\sum_{w_j \in V} ext{count}(w_j, c)} \quad ext{ for all } w_i \in V,$$

where V (for Vocabulary) denotes the collection of all words that occur in the training corpus (after possibly extensive pre-processing).

count(w, c) is the number of times word $w \in V$ occurs in a document of class $c \in C$.

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Interpretation of word probabilities

Word probabilities within each class:

$$\hat{P}(w_i \mid c) = rac{ ext{count}(w_i, c)}{\sum_{w_j \in V} ext{count}(w_j, c)} \quad ext{ for all } w_i \in V$$

Interpretation: if we draw a word at random from a document of class c, the probability that we draw w_i is $\hat{P}(w_i | c)$.

Verify that

$$\sum_{\mathsf{w}_i\in V}\hat{P}(\mathsf{w}_i\mid c)=1,$$

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as required.

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Training Multinomial Naive Bayes: Smoothing

Perform *smoothing* to avoid zero probability estimates.

Word probabilities within each class with Laplace smoothing are:

$$\hat{P}(w_i \mid c) = \frac{\mathsf{count}(w_i, c) + 1}{\sum_{w_j \in V} (\mathsf{count}(w_j, c) + 1)} = \frac{\mathsf{count}(w_i, c) + 1}{\sum_{w_j \in V} \mathsf{count}(w_j, c) + |V|}$$

Verify that again

$$\sum_{w_i \in V} \hat{P}(w_i \mid c) = 1,$$

as required.

The +1 is also called a pseudo-count: pretend you already observed one occurrence of each word in each class.

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Worked Example: Sentiment of Movie Reviews

	Cat	Documents
Training	-	just plain boring
	-	entirely predictable and lacks energy
	-	no surprises and very few laughs
	+	very powerful
	+	the most fun film of the summer
Test	?	predictable with no fun

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The Vocabulary

The vocabulary consists of all words that occur in the training documents:

1.	just	11.	very
2.	plain	12.	few
З.	boring	13.	laughs
4.	entirely	14.	powerful
5.	predictable	15.	the
6.	and	16.	most
7.	lacks	17.	fun
8.	energy	18.	film
9.	no	19.	of
10.	surprises	20.	summer

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Class Prior Probabilities

Recall that:

$$\hat{P}(c) = rac{N_c}{N_{doc}}$$

So we get:

$$\hat{P}(+) = \frac{2}{5}$$
 $\hat{P}(-) = \frac{3}{5}$

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Word Conditional Probabilities

To classify the test example, we need the following probability estimates:

$$\hat{P}(\text{predictable} \mid -) = \frac{1+1}{14+20} = \frac{1}{17} \qquad \hat{P}(\text{predictable} \mid +) = \frac{0+1}{9+20} = \frac{1}{29}$$
$$\hat{P}(\text{no} \mid -) = \frac{1+1}{14+20} = \frac{1}{17} \qquad \hat{P}(\text{no} \mid +) = \frac{0+1}{9+20} = \frac{1}{29}$$
$$\hat{P}(\text{fun} \mid -) = \frac{0+1}{14+20} = \frac{1}{34} \qquad \hat{P}(\text{fun} \mid +) = \frac{1+1}{9+20} = \frac{2}{29}$$

Note that the word "with" in the test document is ignored because it doesn't occur in the training corpus.

Classification:

$$\hat{P}(-)\hat{P}(\text{predictable no fun} \mid -) = \frac{3}{5} \times \frac{1}{17} \times \frac{1}{17} \times \frac{1}{34} = \frac{3}{49,130}$$
$$\hat{P}(+)\hat{P}(\text{predictable no fun} \mid +) = \frac{2}{5} \times \frac{1}{29} \times \frac{1}{29} \times \frac{2}{29} = \frac{4}{121,945}$$

 The model predicts class negative for the test review.
 Image: Comparison of the test review.
 Image: Comparison of the test review.

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Why smoothing?

If we don't use smoothing, the estimates are:

$$\hat{P}(predictable \mid -) = \frac{1}{14} \qquad \hat{P}(predictable \mid +) = \frac{0}{9} = 0$$
$$\hat{P}(no \mid -) = \frac{1}{14} \qquad \hat{P}(no \mid +) = \frac{0}{9} = 0$$
$$\hat{P}(fun \mid -) = \frac{0}{14} = 0 \qquad \hat{P}(fun \mid +) = \frac{1}{9}$$

Classification:

$$\hat{P}(-)\hat{P}(\text{predictable no fun} \mid -) = \frac{3}{5} \times \frac{1}{14} \times \frac{1}{14} \times 0 = 0$$
$$\hat{P}(+)\hat{P}(\text{predictable no fun} \mid +) = \frac{2}{5} \times 0 \times 0 \times \frac{1}{9} = 0$$
have estimated probability undefined. (division by zero)

Both classes have estimated probability undefined! (division by zero)

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Multinomial Naive Bayes: Training

TRAINMULTINOMIALNB(C, D)

- 1 $V \leftarrow \text{ExtractVocabulary}(D)$
- 2 $N_{doc} \leftarrow \text{CountDocs}(D)$
- 3 for each $c \in C$
- 4 do $N_c \leftarrow \text{COUNTDOCSInCLASS}(D, c)$
- 5 $prior[c] \leftarrow N_c/N_{doc}$
- 6 $text_c \leftarrow CONCATENATETEXTOFALLDOCSINCLASS(D, c)$
- 7 for each $w \in V$
- 8 **do** count_{*cw*} \leftarrow COUNTWORDOCCURRENCE(*text_c*, *w*)
- 9 for each $w \in V$
- 10 **do** $condprob[w][c] \leftarrow \frac{count_{cw}+1}{\sum_{w'}(count_{cw'}+1)}$
- 11 return V, prior, condprob

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Predict the class of a document d.

APPLYMULTINOMIALNB(C, V, prior, condprob, d)

- 1 $W \leftarrow \text{ExtractWordOccurrencesFromDoc}(V, d)$
- 2 for each $c \in C$
- 3 **do** $score[c] \leftarrow \log prior[c]$
- 4 for each $w \in W$
- 5 **do** $score[c] + = \log condprob[w][c]$
- 6 **return** $\arg \max_{c \in C} score[c]$

|白子 |田子 |田子

The multinomial naive Bayes model makes two kinds of independence assumptions:

1 Word occurrences are independent within each class:

$$P(\langle w_1,\ldots,w_n\rangle|c) = \prod_{k=1}^n P(W_k = w_k|c)$$

• Positional independence: $P(W_{k_1} = w|c) = P(W_{k_2} = w|c)$

These independence assumptions do not really hold for documents written in natural language.

How can naive Bayes get away with such heroic assumptions?

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Why does Naive Bayes work?

- Naive Bayes can work well even though independence assumptions are *badly* violated.
- Example:

	<i>c</i> ₁	<i>c</i> ₂	predicted
true probability $P(c d)$	0.6	0.4	<i>C</i> 1
$\hat{P}(c)\prod\hat{P}(w_k c)$	0.00099	0.00001	
NB estimate $\hat{P}(c d)$	0.99	0.01	<i>c</i> ₁

- Double counting of evidence causes underestimation (0.01) and overestimation (0.99).
- Classification is about predicting the correct class, *not* about accurate estimation.

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Double counting of evidence

- Suppose the words *special* and *effects* always occur together in a movie review: either both occur in the review, or neither occurs.
- The independence assumption is badly violated!
- Let $P(special \ effects \mid pos) = 0.01$ and $P(special \ effects \mid neg) = 0.001$.
- Evidence in favor of the positive class when *special effects* occurs in a review, because probability for the positive class is 10 times as big as for the negative class.
- But naive Bayes will count this evidence twice, namely when is sees *special* and when it sees *effects*.

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- Probability estimates may be way off, but that doesn't have to hurt classification performance (much).
- Requires the estimation of relatively few parameters, which may be beneficial if you have a small training set.
- Fast, low storage requirements

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The vocabulary of a training corpus may be huge, but not all words will be good class predictors.

How can we reduce the number of features?

- Feature utility measures:
 - Frequency exclude sparse terms.
 - Mutual information select the terms that have the highest mutual information with the class label.
 - Chi-square test of independence between term and class label.
- Sort features by utility and select top k.
- Can we miss good sets of features this way?

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Entropy

Entropy is the average amount of information generated by observing the value of a random variable:

$$H(X) = \sum_{x} P(x) \log_2 \frac{1}{P(x)} = -\sum_{x} P(x) \log_2 P(x)$$

We can also interpret it as a measure of the uncertainty about the value of X prior to observation.

Compare the weather forecast in the Netherlands (P(sunny) = 0.5, P(rain) = 0.5):

$$P(sunny) \log_2 \frac{1}{P(sunny)} + P(rain) \log_2 \frac{1}{P(rain)} = 0.5 \log_2 2 + 0.5 \log_2 2 = 1$$
 bit.

On the Canary islands (P(sunny) = 0.9, P(rain) = 0.1):

$$0.9 \log_2 1.11 + 0.1 \log_2 10 = 0.47$$
 bits.

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Conditional entropy:

$$H(X \mid Y) = \sum_{x,y} P(x,y) \log_2 \frac{1}{P(x \mid y)} = -\sum_{x,y} P(x,y) \log_2 P(x \mid y)$$

- Measure of the uncertainty about the value of X after observing the value of Y.
- If X and Y are independent, then H(X) = H(X | Y).
- Example: gender and eye color.

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For random variables X and Y, their mutual information is given by

$$I(X;Y) = H(X) - H(X | Y) = \sum_{x} \sum_{y} P(x,y) \log_2 \frac{P(x,y)}{P(x)P(y)}$$

- Mutual information measures the reduction in uncertainty about X achieved by observing the value of Y (and vice versa).
- If X and Y are independent, then for all x, y we have P(x, y) = P(x)P(y), so I(X; Y) = 0.
- Otherwise I(X; Y) is a positive quantity, and the larger its value the stronger the association.

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Estimated Mutual Information

To estimate I(X; Y) from data we compute

$$I(X;Y) = \sum_{x} \sum_{y} \hat{P}(x,y) \log_2 \frac{\hat{P}(x,y)}{\hat{P}(x)\hat{P}(y)},$$

where

$$\hat{P}(x,y) = rac{n(x,y)}{N}$$
 $\hat{P}(x) = rac{n(x)}{N}$

and n(x, y) denotes the number of records with X = x and Y = y. Plugging-in these estimates we get:

$$I(X;Y) = \sum_{x} \sum_{y} \frac{n(x,y)}{N} \log_2 \frac{n(x,y)/N}{(n(x)/N)(n(y)/N)}$$
$$= \sum_{x} \sum_{y} \frac{n(x,y)}{N} \log_2 \frac{N \times n(x,y)}{n(x) \times n(y)}$$

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Estimated Mutual Information

Mutual information between occurrence of the word "bad" in a movie review and class (negative/positive review):

bad/class	0	1	Total
0	5243	7080	12323
1	2757	920	3677
Total	8000	8000	16000

$$I(\text{bad}; \text{class}) = \frac{5243}{16000} \log_2 \frac{16000 \times 5243}{12323 \times 8000} + \frac{7080}{16000} \log_2 \frac{16000 \times 7080}{12323 \times 8000} \\ + \frac{2757}{16000} \log_2 \frac{16000 \times 2757}{3677 \times 8000} + \frac{920}{16000} \log_2 \frac{16000 \times 920}{3677 \times 8000} \\ \approx 0.056$$

Fun fact: the estimated mutual information is equal to the deviance of the independence model divided by 2N (if we take the log with base 2 in computing the deviance).

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Movie Reviews: IMDB Review Dataset

- Collection of 50,000 reviews from IMDB, allowing no more than 30 reviews per movie.
- Contains an even number of positive and negative reviews, so random guessing yields 50% accuracy.
- Considers only highly polarized reviews. A negative review has a score \leq 4 out of 10, and a positive review has a score \geq 7 out of 10.
- Neutral reviews are not included in the dataset.

Andrew L. Maas et al., *Learning Word Vectors for Sentiment Analysis*, Proceedings of the 49th Annual Meeting of the Association for Computational Linguistics: Human Language Technologies, pages 142–150,2011.

Data available at:

http://ai.stanford.edu/~amaas/data/sentiment/

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```
# load the tm package
> library(tm)
# Read in the data using UTF-8 encoding
> reviews.neg <- VCorpus(DirSource("D:/MovieReviews/train/neg",</pre>
                         encoding="UTF-8"))
> reviews.pos <- VCorpus(DirSource("D:/MovieReviews/train/pos",</pre>
                         encoding="UTF-8"))
# Join negative and positive reviews into a single Corpus
> reviews.all <- c(reviews.neg,reviews.pos)</pre>
# create label vector (0=negative, 1=positive)
> labels <- c(rep(0,12500),rep(1,12500))</pre>
> reviews.all
<<VCorpus>>
Metadata: corpus specific: 0, document level (indexed): 0
Content: documents: 25000
```

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The first review before pre-processing:

> as.character(reviews.all[[1]])

[1] "Story of a man who has unnatural feelings for a pig. Starts out with a opening scene that is a terrific example of absurd comedy. A formal orchestra audience is turned into an insane, violent mob by the crazy chantings of it's singers. Unfortunately it stays absurd the WHOLE time with no general narrative eventually making it just too off putting. Even those from the era should be turned off. The cryptic dialogue would make Shakespeare seem easy to a third grader. On a technical level it's better than you might think with some good cinematography by future great Vilmos Zsigmond. Future stars Sally Kirkland and Frederic Forrest can be seen briefly."

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Analysis of Movie Reviews: Pre-Processing

- # Remove punctuation marks (comma's, etc.)
- > reviews.all <- tm_map(reviews.all,removePunctuation)</pre>
- # Make all letters lower case
- > reviews.all <- tm_map(reviews.all,content_transformer(tolower))</pre>
- # Remove stopwords
- # Remove numbers
- > reviews.all <- tm_map(reviews.all,removeNumbers)</pre>
- # Remove excess whitespace
- > reviews.all <- tm_map(reviews.all,stripWhitespace)</pre>

Not done: stemming, part-of-speech tagging, ...

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The first review after pre-processing:

> as.character(reviews.all[[1]])

[1] "story man unnatural feelings pig starts opening scene terrific example absurd comedy formal orchestra audience turned insane violent mob crazy chantings singers unfortunately stays absurd whole time general narrative eventually making just putting even era turned cryptic dialogue make shakespeare seem easy third grader technical level better might think good cinematography future great vilmos zsigmond future stars sally kirkland frederic forrest can seen briefly"

```
# draw training sample (stratified)
# draw 8000 negative reviews at random
> index.neg <- sample(12500,8000)
# draw 8000 positive reviews at random
> index.pos <- 12500+sample(12500,8000)
> index.train <- c(index.neg,index.pos)
# create document-term matrix from training corpus
> train.dtm <- DocumentTermMatrix(reviews.all[index.train])</pre>
```

```
> dim(train.dtm)
```

```
[1] 16000 92819
```

We've got 92,819 features. Perhaps this is a bit too much.

```
# remove terms that occur in less than 5% of the documents
# (so-called sparse terms)
```

```
> train.dtm <- removeSparseTerms(train.dtm,0.95)
> dim(train.dtm)
[1] 16000 306
```

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```
# view a small part of the document-term matrix
> in grant (turning dtm [100:110.00.05])
```

```
> inspect(train.dtm[100:110,80:85])
```

```
<<DocumentTermMatrix (documents: 11. terms: 6)>>
Non-/sparse entries: 7/59
Sparsity
              : 89%
Maximal term length: 6
Weighting
             : term frequency (tf)
Sample
         Terms
        family fan far father feel felt
Docs
 10099_1.txt
             0 0
                   1
                         0
                            0
                                0
 1033_4.txt
             0 0 0
                         0
                            1
                                0
 10718_4.txt 0 0 0
                         0 0 0
 11182_3.txt 0 0 0
                         0
                            0
                                0
 11861_4.txt 1 0 0
                            0 0
                         0
 3014_4.txt
             0 1
                   0
                         0
                            1 2
             0 0 0
 315 1.txt
                         0
                            0
                                0
             0 0 0
 6482 2.txt
                         0
                            1
                                0
 9577 1.txt
              0 0 0
                         0
                            0
                                0
 9674_3.txt
              0
                 0
                   0
                         0
                            0
                                0
```

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Multinomial naive Bayes in R: Training

```
> train.mnb
function (dtm.labels)
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call <- match.call()
V <- ncol(dtm)
N \leq -nrow(dtm)
prior <- table(labels)/N
labelnames <- names(prior)
nclass <- length(prior)</pre>
cond.probs <- matrix(nrow=V,ncol=nclass)</pre>
dimnames(cond.probs)[[1]] <- dimnames(dtm)[[2]]
dimnames(cond.probs)[[2]] <- labelnames
index <- list(length=nclass)</pre>
for(i in 1:nclass){
 index[[j]] <- c(1:N)[labels == labelnames[j]]</pre>
}
for(i in 1:V){
 for(j in 1:nclass){
    cond.probs[i,j] <- (sum(dtm[index[[j]],i])+1)/(sum(dtm[index[[j]],])+V)</pre>
 }
}
list(call=call,prior=prior,cond.probs=cond.probs)}
```

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Multinomial naive Bayes in R: Prediction

```
> predict.mnb
function (model,dtm)
{
    classlabels <- dimnames(model$cond.probs)[[2]]
    logprobs <- dtm %*% log(model$cond.probs)
    N <- nrow(dtm)
    nclass <- ncol(model$cond.probs)
    logprobs <- logprobs+matrix(nrow=N,ncol=nclass,log(model$prior),byrow=T)
    classlabels[max.col(logprobs)]
}</pre>
```

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Application of Multinomial naive Bayes to Movie Reviews

Train multinomial naive Bayes model

```
> reviews.mnb <- train.mnb(as.matrix(train.dtm),labels[index.train])</pre>
```

```
# create document term matrix for test set
# we only extract words from the training vocabulary!
> test.dtm <- DocumentTermMatrix(reviews.all[-index.train],</pre>
               list(dictionary=dimnames(train.dtm)[[2]]))
> dim(test.dtm)
[1] 9000 306
> reviews.mnb.pred <- predict.mnb(reviews.mnb,as.matrix(test.dtm))</pre>
> table(reviews.mnb.pred.labels[-index.train])
reviews.mnb.pred 0 1
               0 3473 849
               1 1027 3651
# compute accuracy on test set: about 79% correct
> (3473+3651)/9000
[1] 0.7915556
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```

Feature Selection with Mutual Information

The top-10 features (terms) according to mutual information are:

term	MI(term, class)
bad	0.056
worst	0.052
waste	0.035
awful	0.032
great	0.028
terrible	0.020
excellent	0.020
wonderful	0.018
boring	0.018
stupid	0.018

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Computing Mutual Information

- # load library "entropy"
- > library(entropy)
- # convert document term matrix to binary (term present/absent)
- > train.dtm.bin <- as.matrix(train.dtm)>0

compute mutual information of each term with class label
> train.mi <- apply(as.matrix(train.dtm.bin),2,
 function(x,y){mi.plugin(table(x,y)/length(y),unit="log2")},
 labels[index.train])</pre>

sort the indices from high to low mutual information
> train.mi.order <- order(train.mi,decreasing=T)</pre>

show the five terms with highest mutual information
> train.mi[train.mi.order[1:5]]

bad worst waste awful great 0.05568853 0.05161474 0.03456289 0.03168221 0.02807607

Using the top-50 features

```
# train on the 50 best features
> revs.mnb.top50 <- train.mnb(as.matrix(train.dtm)[,train.mi.order[1:50]],</pre>
                          labels[index.train])
# predict on the test set
> revs.mnb.top50.pred <- predict.mnb(revs.mnb.top50,</pre>
    as.matrix(test.dtm)[,train.mi.order[1:50]])
# show the confusion matrix
> table(revs.mnb.top50.pred,labels[-index.train])
revs.mnb.top50.pred 0
                            1
                  0 3429 996
                  1 1071 3504
# accuracy is a bit worse compared to using all features
> (3429+3504)/9000
```

```
[1] 0.7703333
```

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Feature score in model

$$\operatorname{score}(c,d) = \log \hat{P}(c) + \sum_{k=1}^{n} \log \hat{P}(w_k \mid c)$$

Score difference of word w_k in favour of positive class is

$$\log \hat{P}(w_k \mid \mathsf{pos}) - \log \hat{P}(w_k \mid \mathsf{neg})$$

The top-20 feature score differences sorted by absolute value are:

word	score diff.	word	score diff.
waste	-2.71	poor	-1.24
awful	-2.28	loved	1.18
worst	-2.17	beautiful	0.93
terrible	-1.72	minutes	-0.91
wonderful	1.61	great	0.90
stupid	-1.55	money	-0.83
boring	-1.49	nothing	-0.81
excellent	1.41	best	0.76
bad	-1.33	performances	0.76
perfect	1.33	script	-0.74

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- # load the required packages
- > library(rpart)
- > library(rpart.plot)
- # grow the tree
- > reviews.rpart <- rpart(label~.,</pre>

data=data.frame(as.matrix(train.dtm),

label=labels[index.train]),cp=0,method="class")

- # plot cv-error of pruning sequence
- > plotcp(reviews.rpart)

Cross-Validation Error of Pruning Sequence



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Classification Trees

- # simple tree for plotting > reviews.rpart.pruned <- prune(reviews.rpart,cp=5.0000e-03)</pre> > rpart.plot(reviews.rpart.pruned) # tree with lowest cv error > reviews.rpart.pruned <- prune(reviews.rpart,cp=5.833333e-04)</pre> # make predictions on the test set > reviews.rpart.pred <- predict(reviews.rpart.pruned,</pre> newdata=data.frame(as.matrix(test.dtm)),type="class") # show confusion matrix > table(reviews.rpart.pred,labels[-index.train]) reviews.rpart.pred 0 1 0 3150 1021 1 1350 3479 # accuracy is worse than naive Bayes!
- > (3150+3479)/9000
- [1] 0.7365556

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The Simple Tree



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49 / 50

2

Random Forests

- # load the required packages
- > library(randomForest)
- # train random forest with default settings: 500 trees and mtry = 17
- # make predictions
 > reviews.rf.pred <- predict(reviews.rf,newdata=data.frame(as.matrix(test.dtm)))</pre>

```
# show confusion matrix
> table(reviews.rf.pred,labels[-index.train])
```

```
reviews.rf.pred 0 1
0 3483 824
1 1017 3676
```

compute accuracy: only slightly better than naive Bayes!
> (3483+3676)/9000
[1] 0.7954444