Assignment 3: Fundamental Research on Quality Ratios

You get a lot of time for each assignment. Therefore, we ask you to take:

- Proper time for thinking: do not get stuck on the first reasonable idea, but think further if there are other options. Weigh the advantages and disadvantages.
- Proper time for careful execution: make sure that you write up your results correctly. Having the correct idea but writing it up wrong is very common; avoid this.
- Proper time for reporting: avoid typos, substandard layout, inconsistent terminology, etc.

Research is all about being careful and taking time to do it right. All assignments can be solved in half the time, but this is not what we expect you to do.

Handing in assignments: Use e-mail and send to Marc as a pdf. CC all team members. Use a specific format for the file name: SPGMT-3-Y.pdf where Y is the number of your team. The pdf itself should start with the names and student numbers of everyone in the team. **Deadline: Thursday, October 3 at 11am.**

This assignment is on quality ratios for graph drawing. This project is rather open: I will not specify what exactly you should do. You determine this yourself. Therefore, a time investment cannot be given, but I expect 10-15 hours per person, including a hand-in of 2-5 pages with the findings for each task and with proper illustrations. This assignment is more like a collaborative take-home test. Do not write a paper or report. There are many tasks and questions. Do not do them all.

Assignment background

Besides the four drawing styles in the paper, there is another interesting drawing style: the planar 1-bend drawings. Here every edge is drawn by two straight-line segments that join in a bend; the two other ends of these line segments lie at the vertices, of course. We are interested in planar 1-bend drawings. We will focus on the measure angular resolution. Since 1-bend drawings have an extra angle on each edge, we will count the angle of these bends too when assessing the angular resolution of a 1-bend drawing.

Since 1-bend drawings are a style, we can compare it to other styles. It is natural to compare 1-bend planar drawings to free straight-line planar drawings: How much better can 1-bend planar drawings be for a quality measure than straight-edge planar drawings for that measure? Since an edge need not have a bend in a 1-bend drawing, planar 1-bend drawings are more general than free straight-line planar drawings.

From now on we assume that we are considering planar graphs only, and only planar drawings of such graphs.

Here is an easy example showing why bends can help to improve angular resolution. Imagine a graph that has three vertices and three edges. In a straight-line drawing, it will be a triangle. No matter how we draw it, it will have an angle of 60 degrees or less. This is easy to prove: the sum of the three angles of a triangle is always exactly 180 degrees (known fact), so it cannot be that all three angles are larger than 60 degrees. If we allow a bend in every edge, then we can draw this graph as a regular hexagon where the corners alternate between graph vertices and bends. For this graph, we can guarantee that all of the angles contributing to the angular resolution are 120 degrees. So allowing bends improves the best possible angular resolution from 60 to 120 degrees, and hence the quality ratio of angular resolution for the two drawing styles is at least 2. Can you find a graph that gives a bigger ratio? You need to prove 2 things for this graph G: (i) Without bends it is not possible to find a drawing where all angles are better than some value s. (ii) With bends there is a drawing whose smallest angle is at least b. Then the quality ratio is at least b/s. The idea is to find the graph for which we can find the largest ratio.

Exploration: draw some small planar graphs and try to obtain the best possible straight-line drawing (for angular resolution), and also try to get the best 1-bend drawing. This helps to acquire intuition. The research activity itself is sitting together and posing ideas or observations to each other, which others can comment on or add to.

Below a number of tasks are given. You need not “solve” each task, but I expect some form of answer to at least three of the tasks.

Task 1: Give the planar graph you found that has the largest quality ratio. Give the planar straight-line drawing with the best angular resolution, and argue that this is indeed the best you can do, over all possible straight-line planar drawings. Then give the best 1-bend drawing that you found. Give the angular resolution of both drawings of that graph, and argue that your research shows that the quality ratio of angular resolution for 1-bend drawings with respect to straight-line drawings is at least “something”.

We can allow more bends in edges: two bends, three bends, k bends, arbitrarily many bends, …

Suppose we allow 2 bends per edge. We can now compare straight-line planar drawings of planar graphs with 2-bend planar drawings of planar graphs, and determine the quality ratio of angular resolution.

Task 2: Do the same as in task 1, but now to compare 2-bend drawings with free straight-line drawings. Also, compare 2-bend drawings with 1-bend drawings and try to obtain a bound on the quality ratio for angular resolution.

Task 3: What angular resolution can be realized when you are allowed arbitrarily many bends? The graphs under consideration here are all planar graphs. Write the result in a well-formulated theorem.

Task 4: Can you bound the number of bends that is needed to realize your result in the theorem? For example, for a graph with n vertices, can you already realize your result with at most n bends in every edge?

Task 5: All of the above tasks were in the free embedding version of the problem. Do you get the same or alternative results in the fixed embedding setting for any of the tasks 1-4?

Task 6: Suppose we do not count the angle at the bend (because we can just round it and then there is no corner anymore). Can you get new results for tasks 1-5 in this model?

Bonus task: We can consider the other quality measures edge length ratio and graph resolution as well for 1-bend drawings. The question is how to interpret the length of an edge when it has a bend. There are a few possible options: (i) the sum of the lengths of the two segments in a bend is the length of the edge; (ii) the individual lengths of the segments in a 1-bend edge are used for the length of an edge (as if an edge with a bend were two edges); and (iii) both sum and individual lengths are used when considering the edge length (for the edge-length ratio and for the feature resolution).

Make a document where you report on any result you have obtained, inclusive of good illustrations. State your results as theorems and provide a proof, if you can. For this purpose, proofs are careful and complete argumentations that avoid intuition and examples. If you cannot give a proof for a result, state that result as an observation or conjecture and explain why you think it is correct, even though you do not have a proof. Make sure that it is always clear which quality measure you are giving a result on. I am not expecting the best possible results for all tasks, but I am expecting some interesting examples and results. Since this is fundamental research, it is even acceptable to state that you did not discover any relevant results for several of the tasks. This assignment is not about producing a paper or full report, so do not write an introduction or motivation, nor conclusions. Just give the results for several tasks in a properly worked out manner, like in a take-home exam. I expect at least three theorems, and results for at least three tasks. Results can be simple, but they must be presented formally and correctly, and be well illustrated.

Evaluation criteria: The quantity of the results obtained (20%), the quality of the results obtained (20%), the quality of the write-up including completeness of arguments (40%), quality and suitability of the illustrations (20%). Note that theorems with proofs can lead to higher grades than observations and conjectures, but if a theorem or proof is not correct, that does not help. There are of course degrees in which a theorem statement or proof is incorrect; a forgotten proof step or missed extra restriction on the validity of the result in the theorem statement is less critical than a completely wrong theorem or proof that is fundamentally flawed.

State how much time each of the team members spent on this assignment individually and how much time you spent together, with a very short description of the work.

For illustrations I recommend ipe combined with Latex. For ipe, see https://ipe.otfried.org/