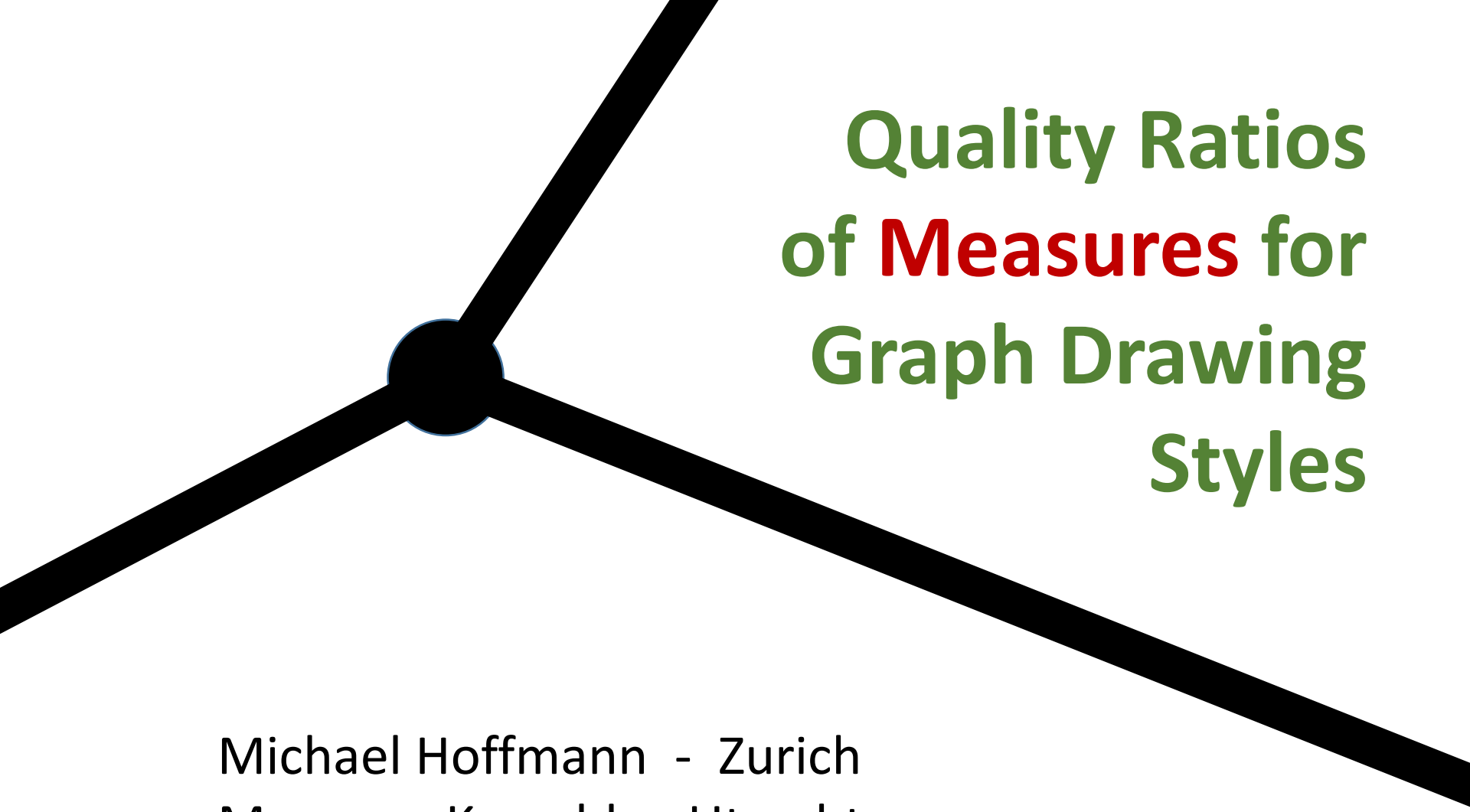


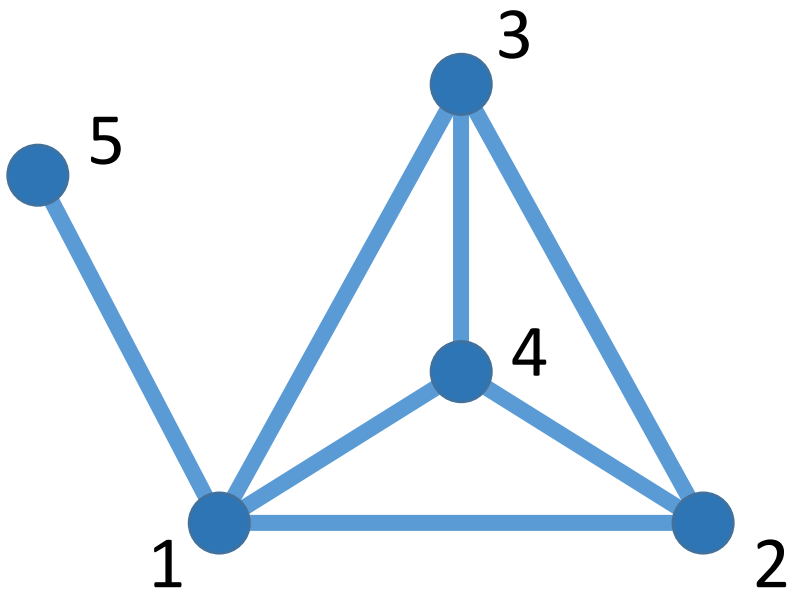
Quality Ratios of Measures for Graph Drawing Styles

Michael Hoffmann - Zurich
Marc van Kreveld - Utrecht
Vincent Kusters - Zurich
Günter Rote - Berlin

V = {1,2,3,4,5}

E = { (1,2), (1,3),
(1,4), (1,5),
(2,3), (2,4),
(3,4) }

Graph Drawing



V = {1,2,3,4,5}

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(1,4), (1,5),
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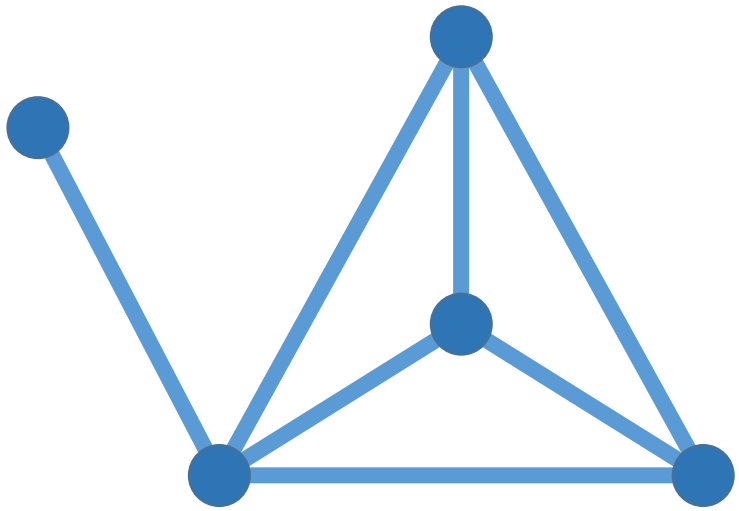
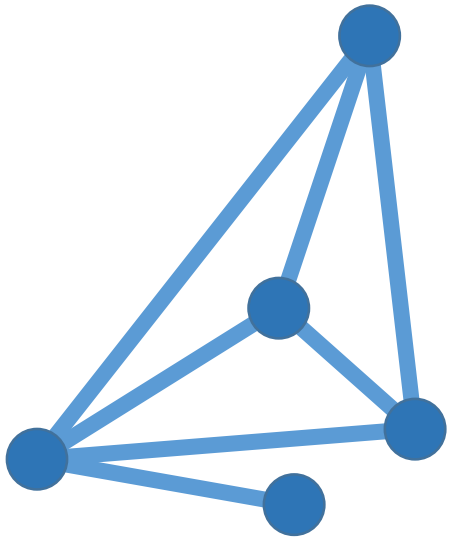
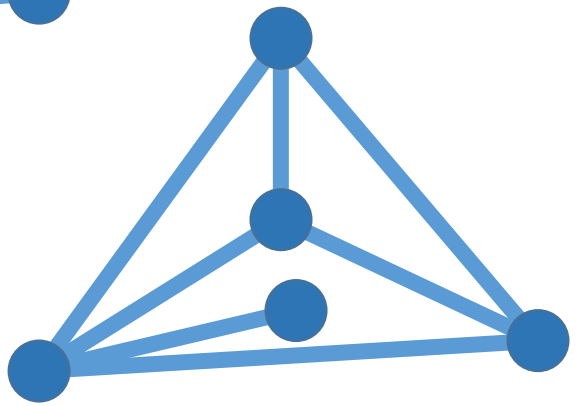
Graph Drawing







V = {1,2,3,4,5}

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(1,4), (1,5),
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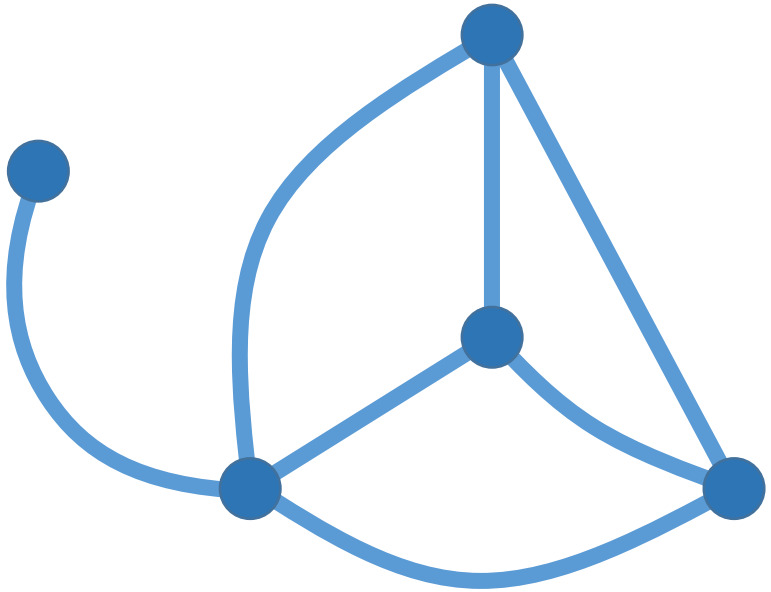
(3,4) }

Graph Drawing



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(1,4), (1,5),
(2,3), (2,4),
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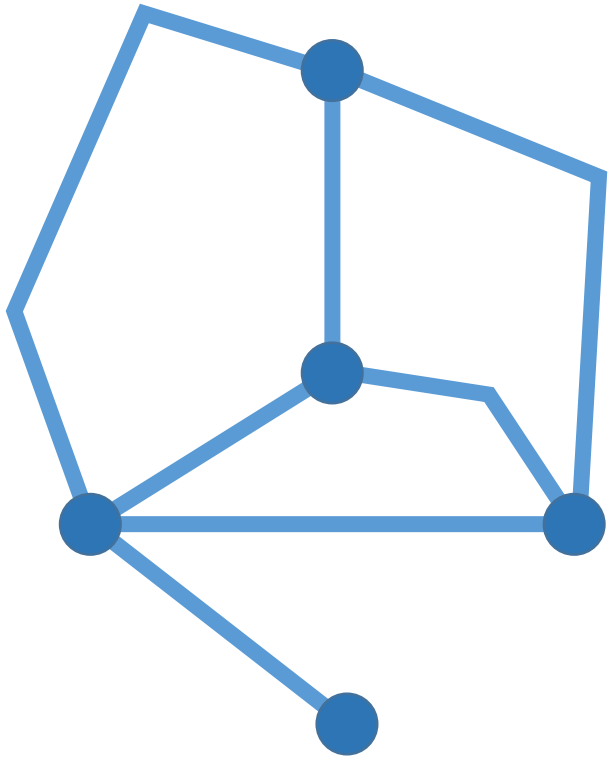
Graph Drawing



V = {1,2,3,4,5}

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(1,4), (1,5),
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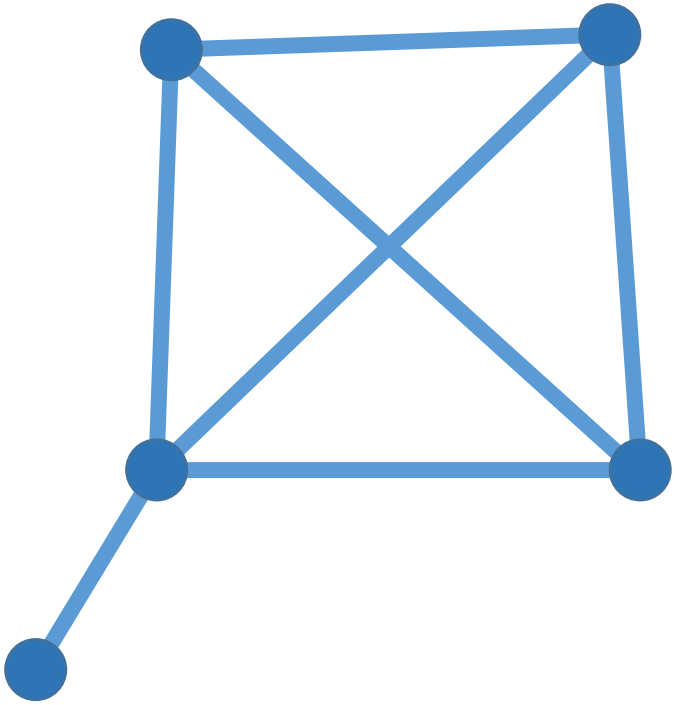
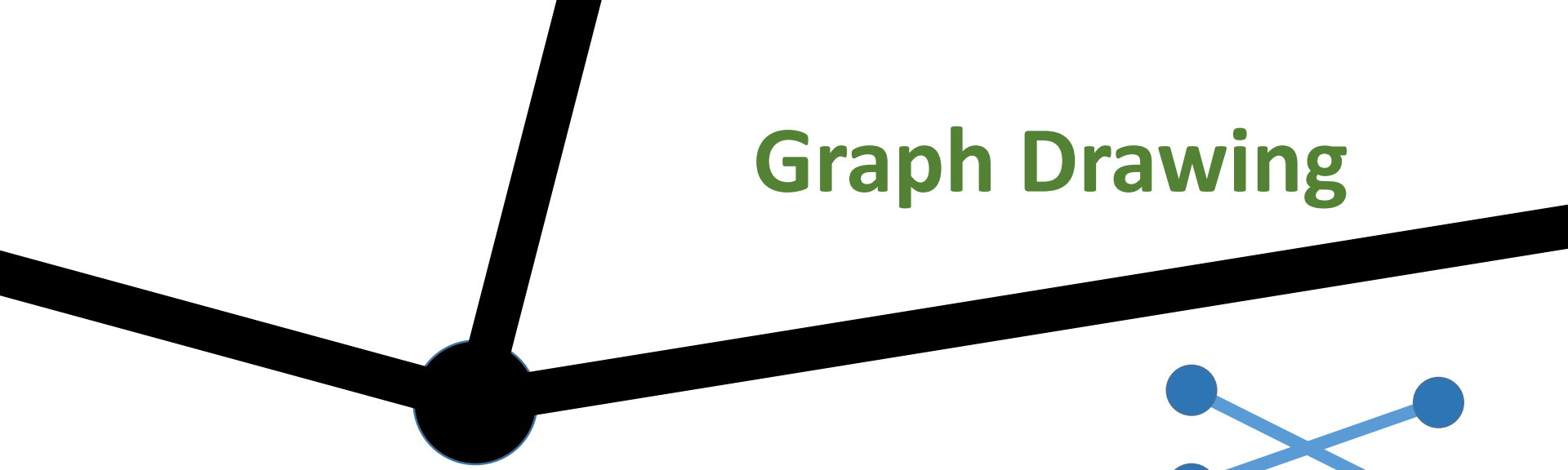
Graph Drawing



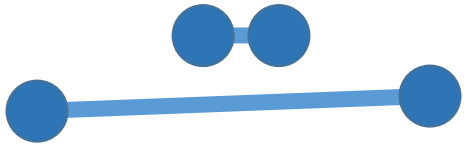
Graph Drawing



Don’t like:

- edge-edge crossings

- small angles between incident edges

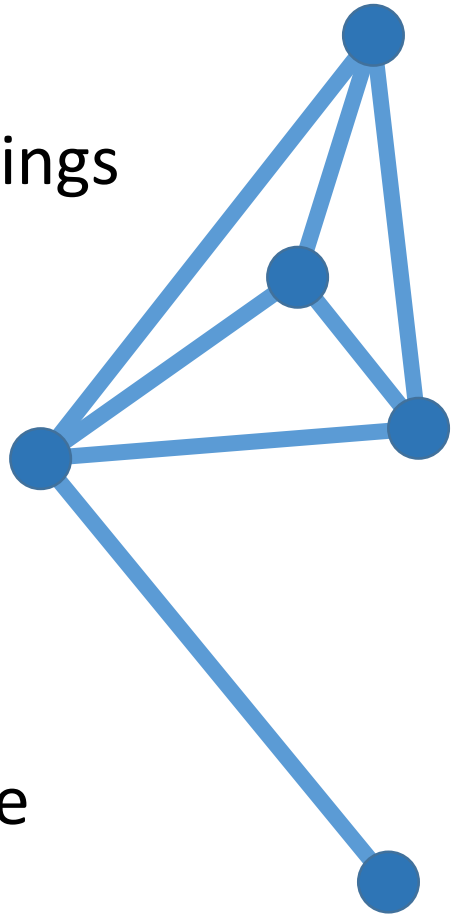


- both long and short edges

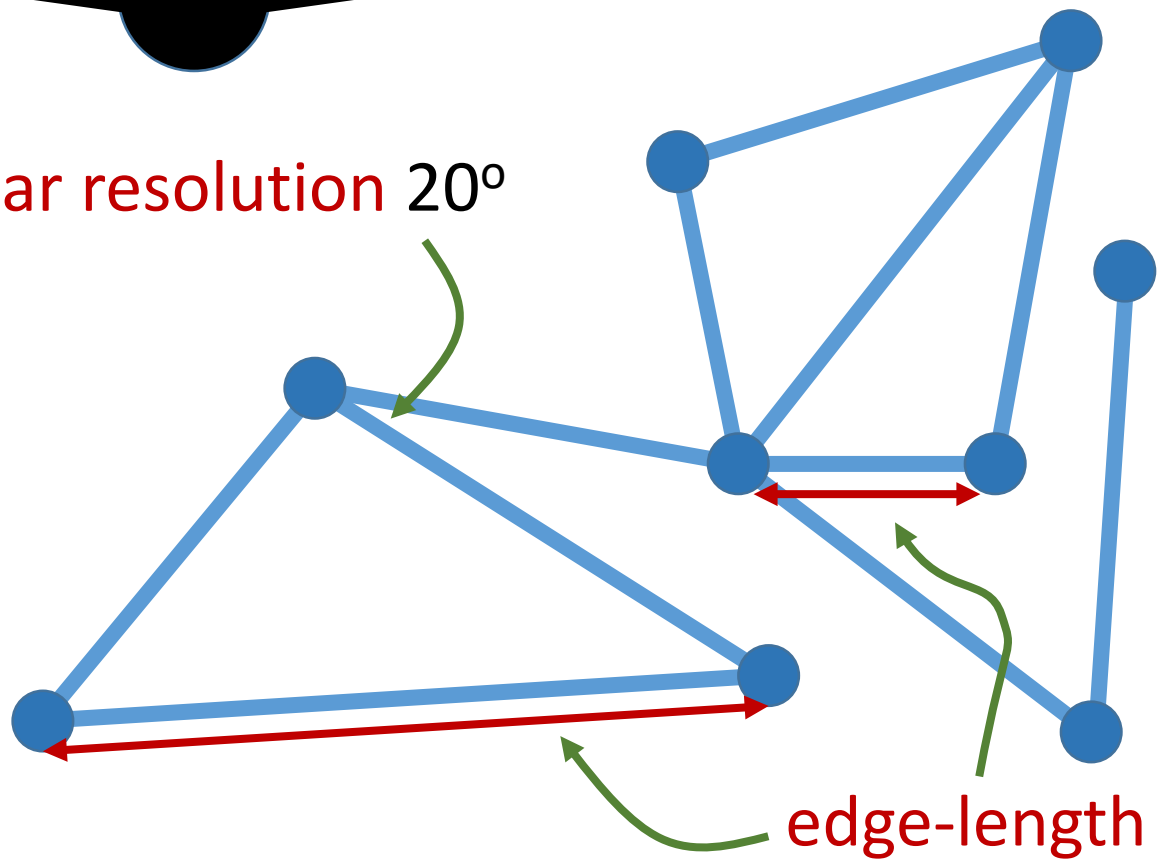
- edges close to non-incident vertices

Quality Measures

- **crossing number**: number of edge crossings
- **angular resolution**: smallest angle between incident edges
- **edge-length ratio**: ratio of longest to shortest edge length
- **area requirement**: grid size needed
- **feature resolution** : ratio of longest edge to shortest vertex-edge distance



Quality Measures

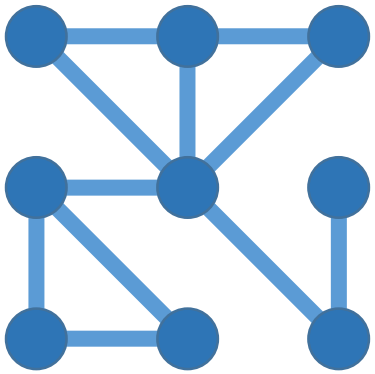


Quality Measures

angular resolution

edge-length ratio

area requirement

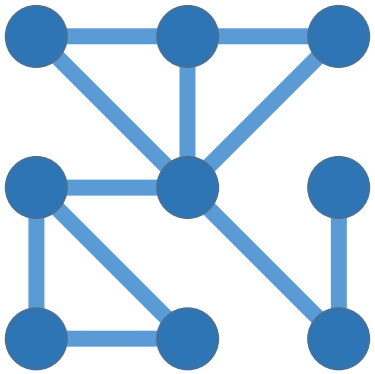


Quality Measures

angular resolution 45°

edge-length ratio $\sqrt{2}$

area requirement 9

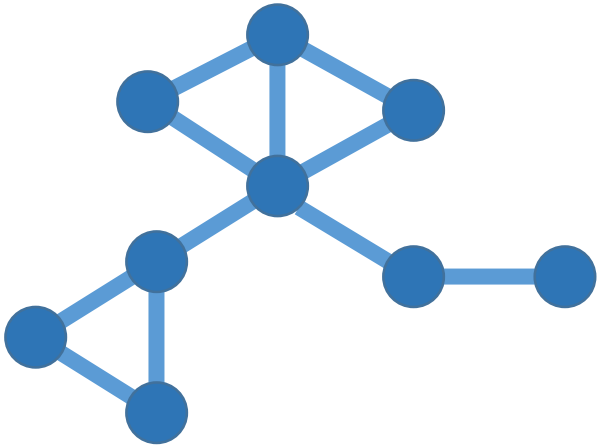


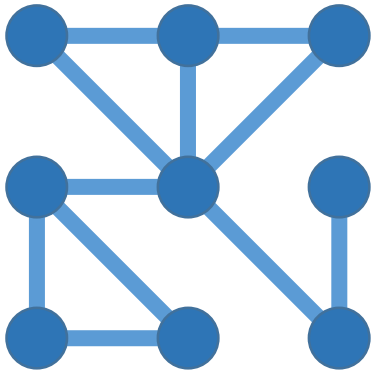
Quality Measures

angular resolution 60^o

edge-length ratio 1

area requirement 9





Quality Measures

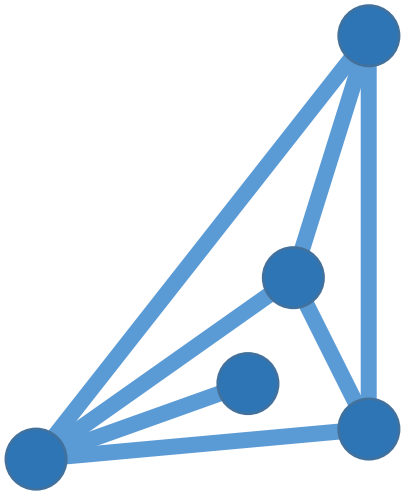
quality measure of a drawing of a graph

not the same thing as

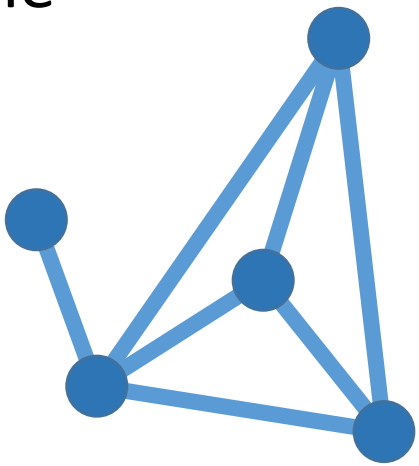
quality measure of a graph

Drawing Styles

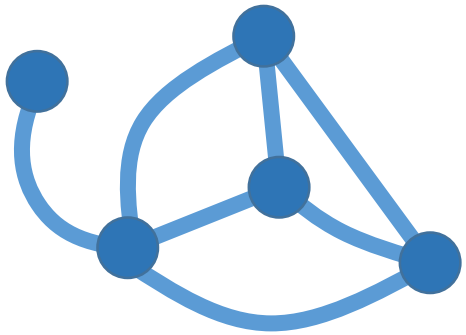
fixed embedding
planar straight line



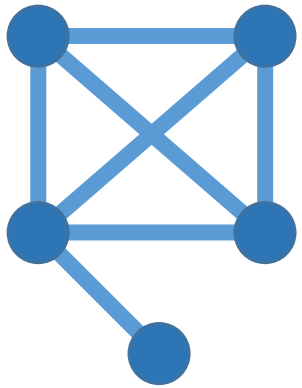
free embedding
planar straight line



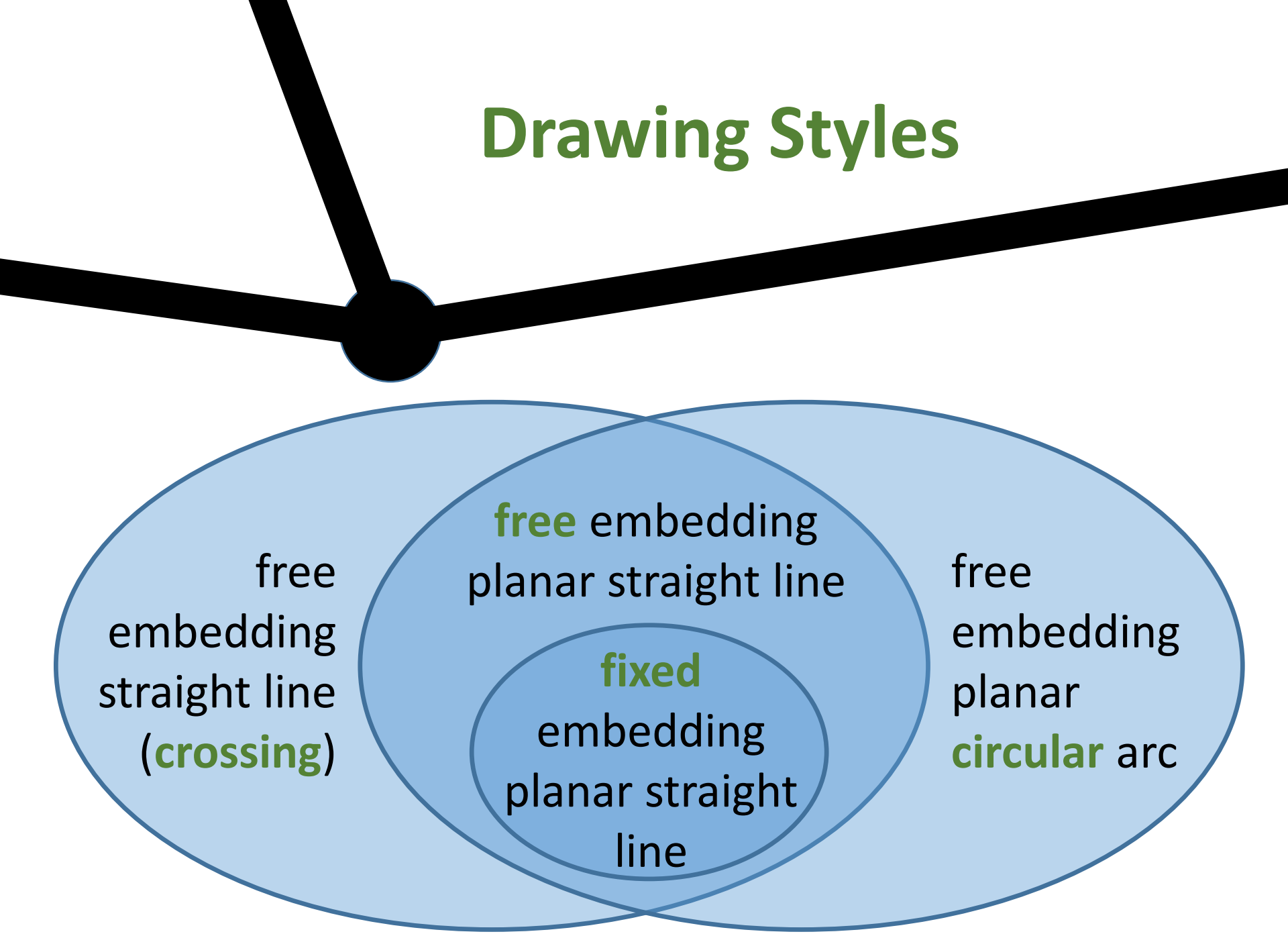
free embedding
planar **circular** arc



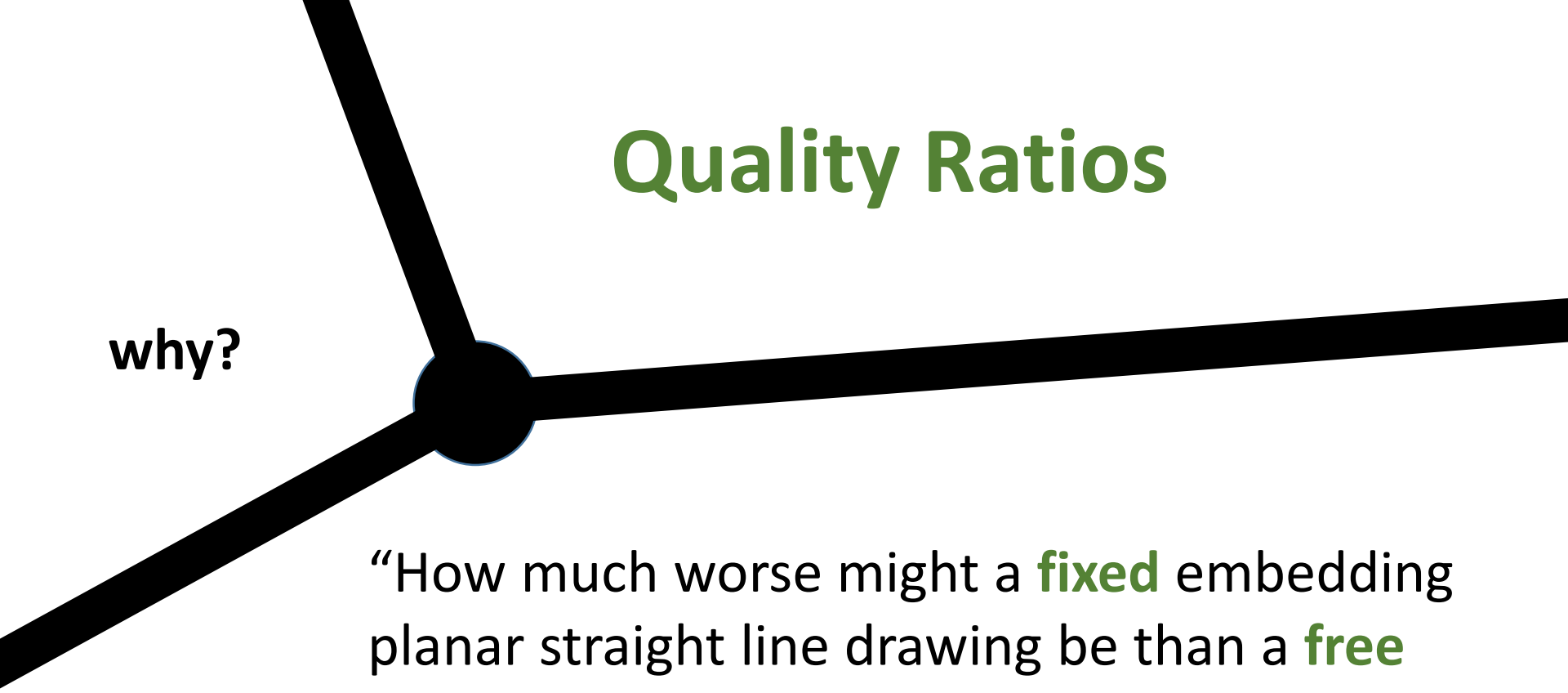
free embedding
straight line (**crossing**)



Drawing Styles



Quality Ratios



"How much worse might a **fixed** embedding planar straight line drawing be than a **free** embedding planar straight line drawing in terms of angular resolution?"

"How much better can a **circular** arc drawing of a graph be than a **free** planar straight line drawing in terms of area requirement?"

Quality Ratios

definition

for angular resolution; circular versus free plane drawings

$$\mathrm{QR}(\text{circular}, \text{free}) = \sup_{\text{planar graph } G} \frac{\text{angular resolution of circular plane drawing of } G}{\text{angular resolution of free straight plane drawing of } G}$$

Quality Ratios

definition

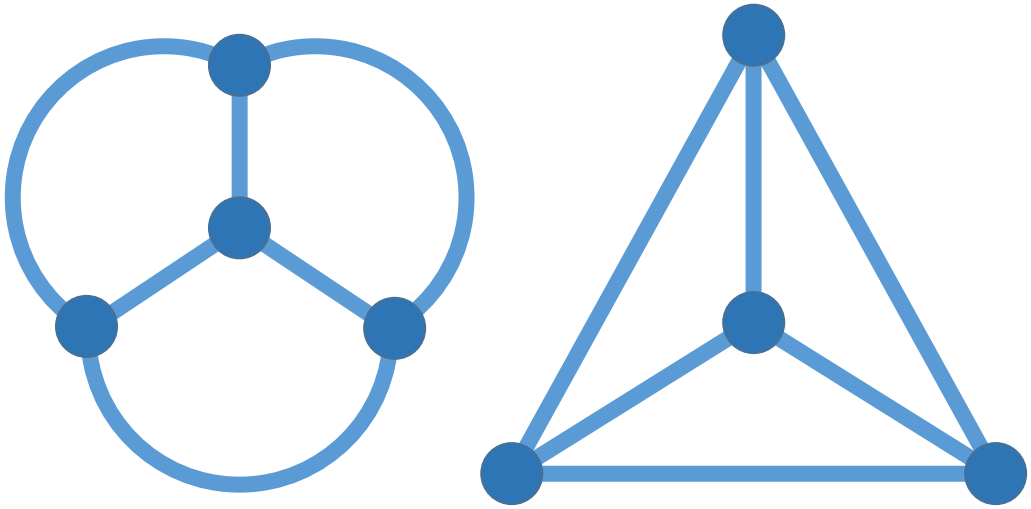
for area requirement; free versus fixed embedding straight plane drawings

$$QR(\text{free}, \text{fixed}) = \sup_{\text{planar graph } G} \frac{\text{area requirement of free plane drawing of } G}{\text{area requirement of fixed plane drawing of } G}$$

Quality Ratios

example

Angular resolution; circular versus free plane drawings



QR $\geq$ 120/30 = 4

Results

planar graphs

	free : fixed	circ. : free	crossing : free
angular resolution		≥ 4.8	
area requirement			
edge-length ratio			
feature resolution			

Results

planar graphs

	free : fixed	circ. : free	crossing : free
angular resolution	≥ 12	≥ 4.8	∞
area requirement	∞	∞	∞
edge-length ratio	∞	∞	∞
feature resolution	∞	$\geq 3\sqrt{3} / \pi$	≥ 2.509

free : fixed circ. : free crossing : free

angular resolution

area requirement

edge-length ratio

feature resolution

Results

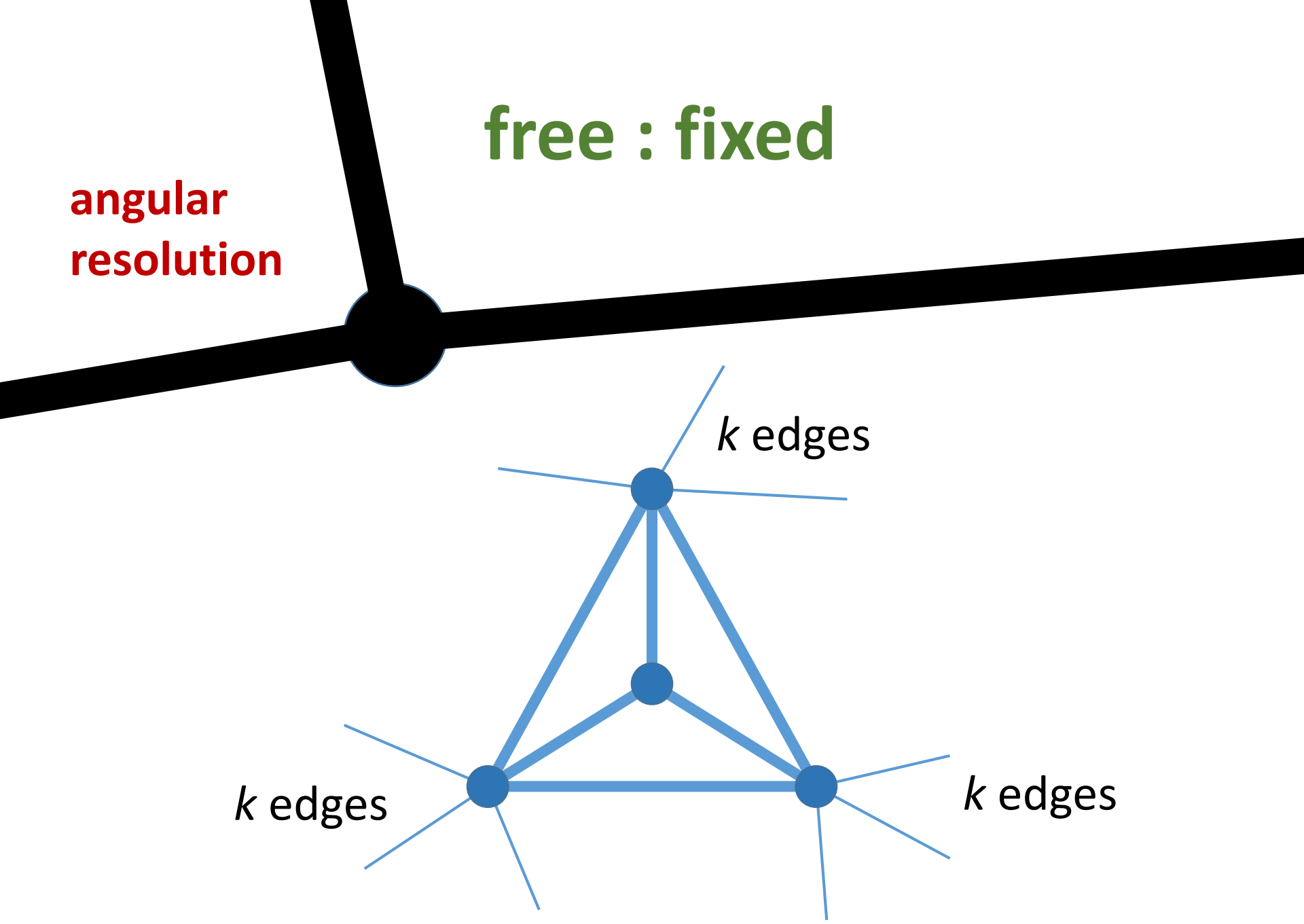
	free : fixed	circ. : free	crossing : free
angular resolution	1	1	1
area requirement	$\geq 16/15$	≥ 1.5	$\geq 22/21$
edge-length ratio	1	1	1
feature resolution	$\geq 1+\varepsilon$	?	?

Results

	free : fixed	circ. : free	crossing : free
angular resolution	≥ 12	≥ 4.8	∞
area requirement	∞	∞	∞
edge-length ratio	∞	∞	∞
feature resolution	∞	$\geq 3\sqrt{3} / \pi$	≥ 2.509

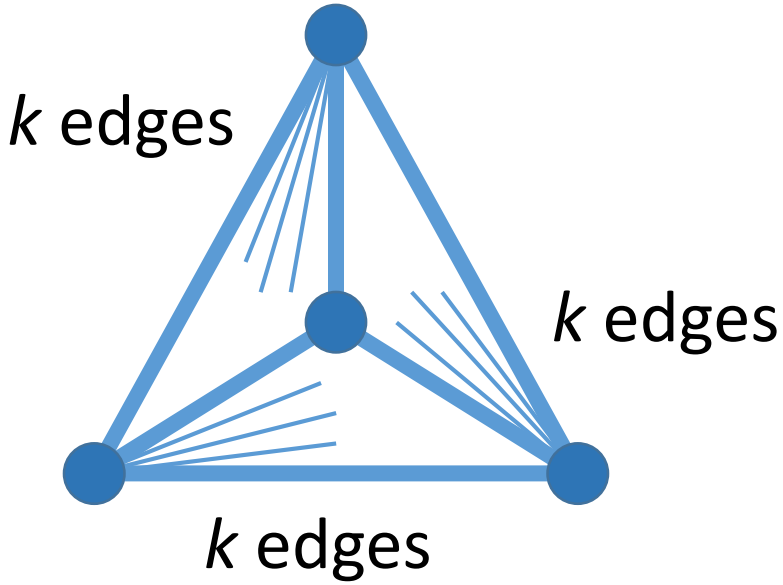
free : fixed

angular resolution



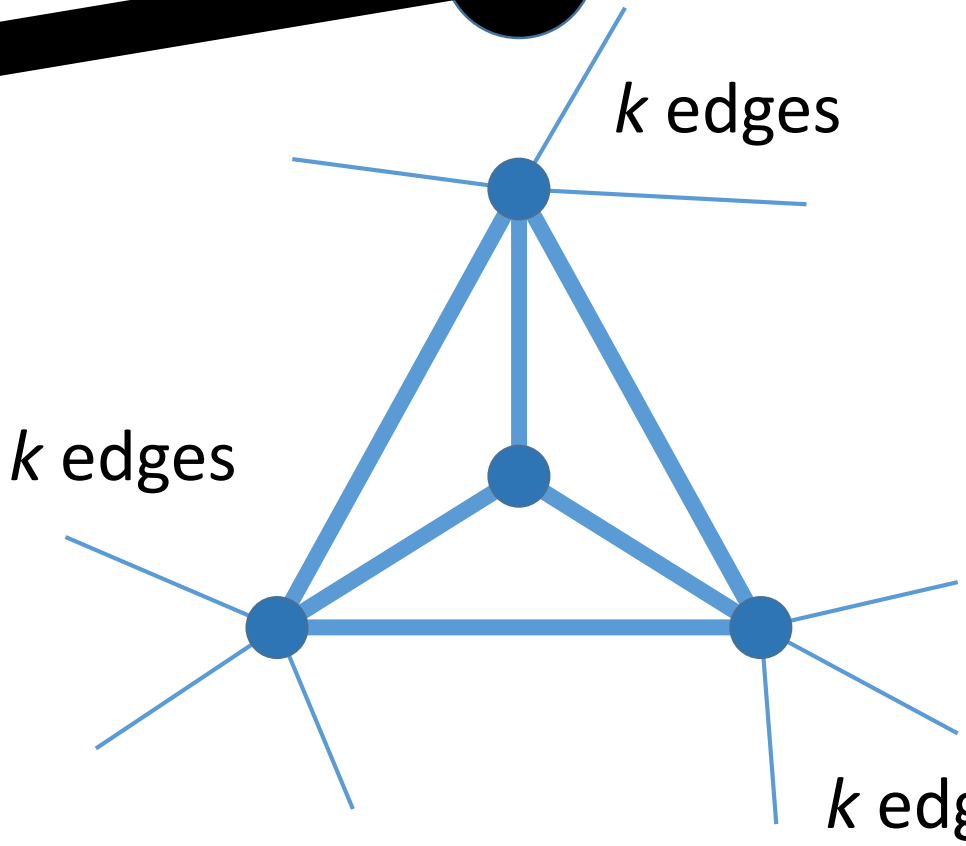
free : fixed

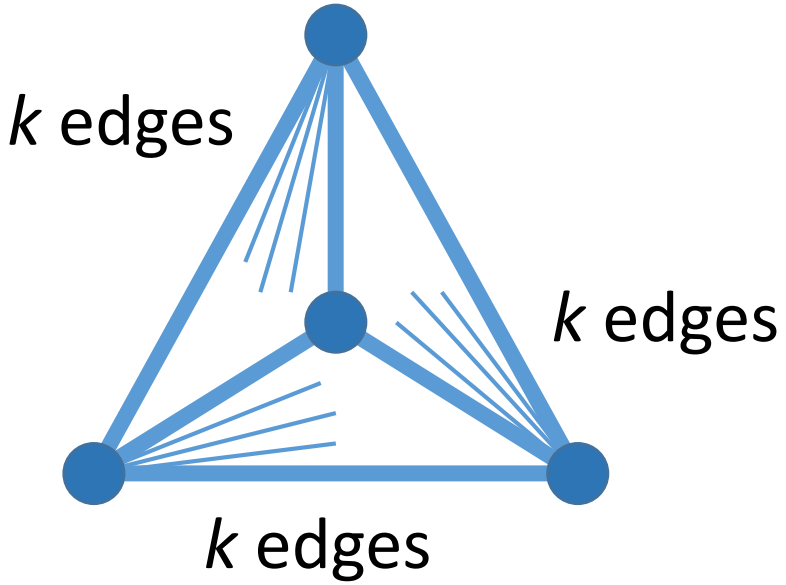
angular resolution



free : fixed

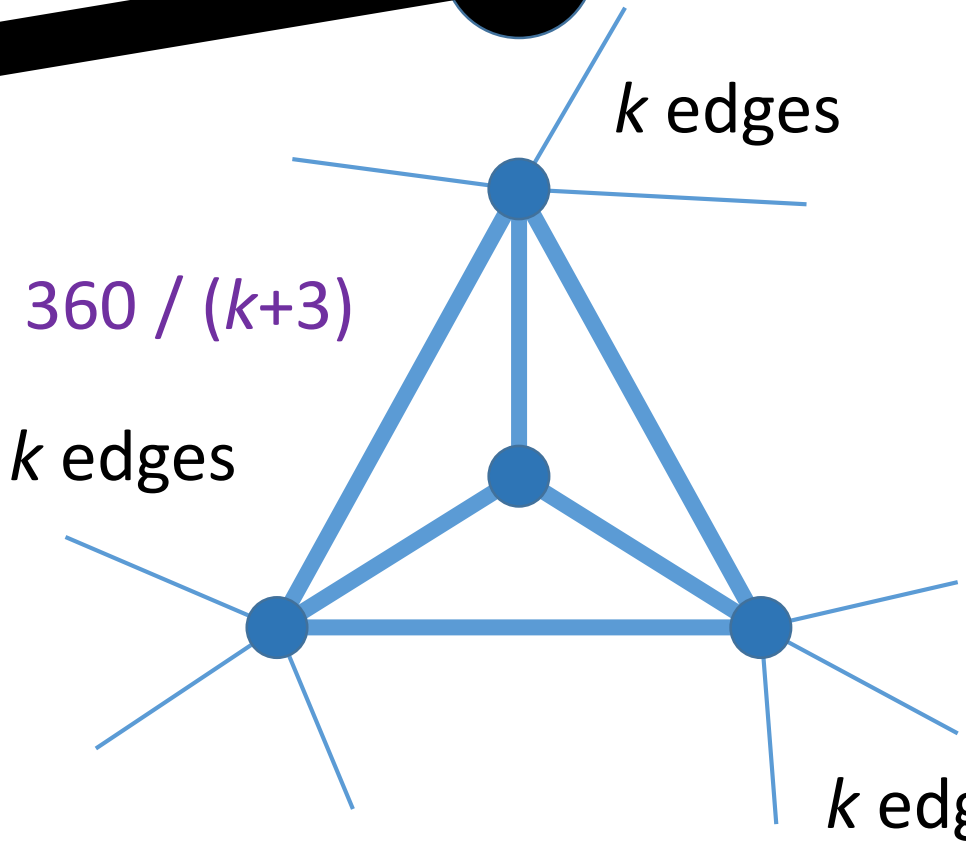
angular resolution

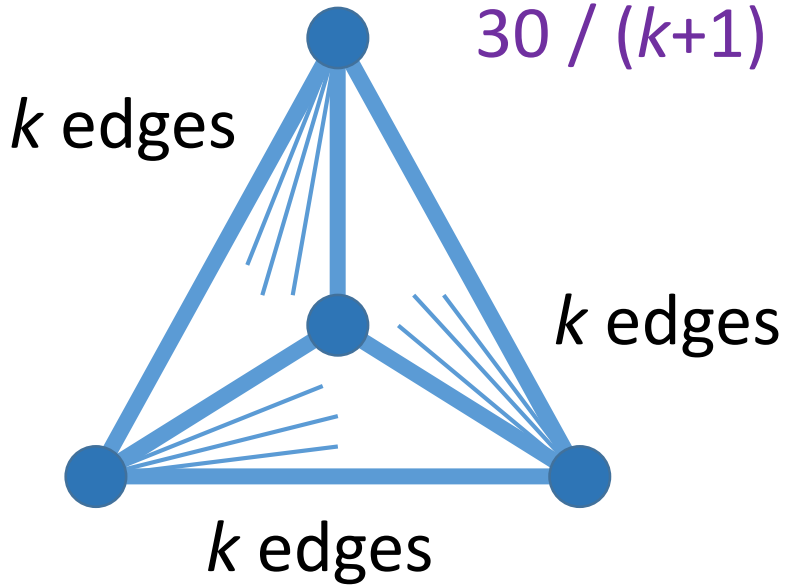




free : fixed

angular resolution





free : fixed

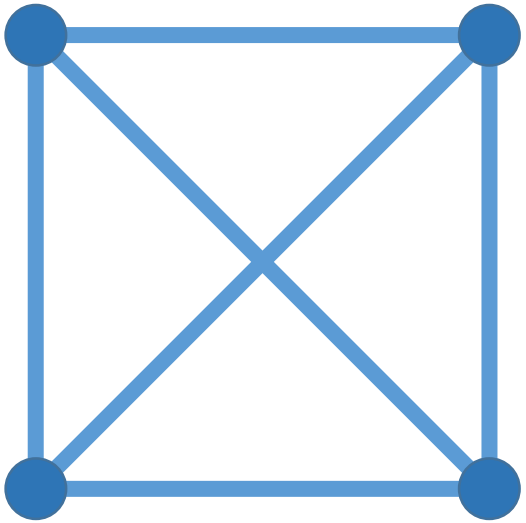
angular resolution

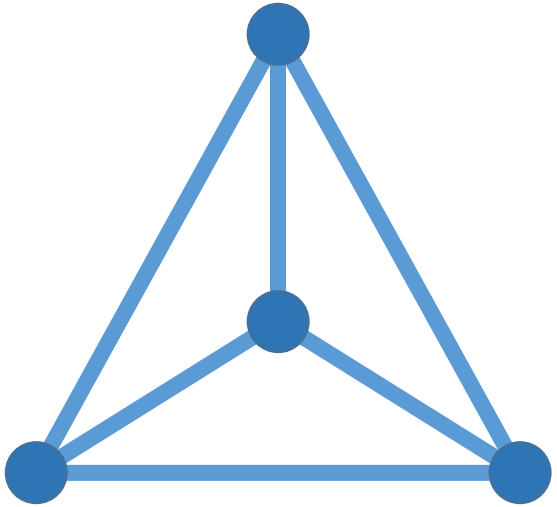
$$\mathrm{QR}(\text{free}, \text{fixed}) = \sup_{\substack{\text{planar} \\ \text{graph } G}} \frac{\text{angular resolution of free plane drawing of } G}{\text{angular resolution of fixed plane drawing of } G}$$

$$\geq \sup_{k} \frac{360 / (k+3)}{30 / (k+1)} = 12$$

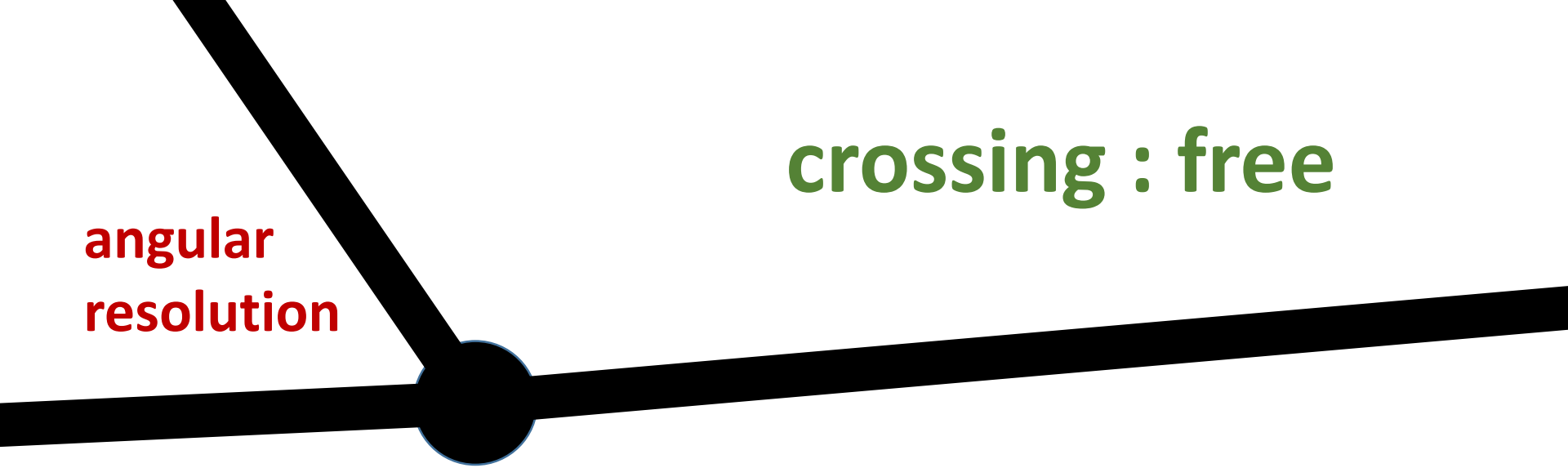
angular resolution

crossing : free





QR $\geq$ 1.5

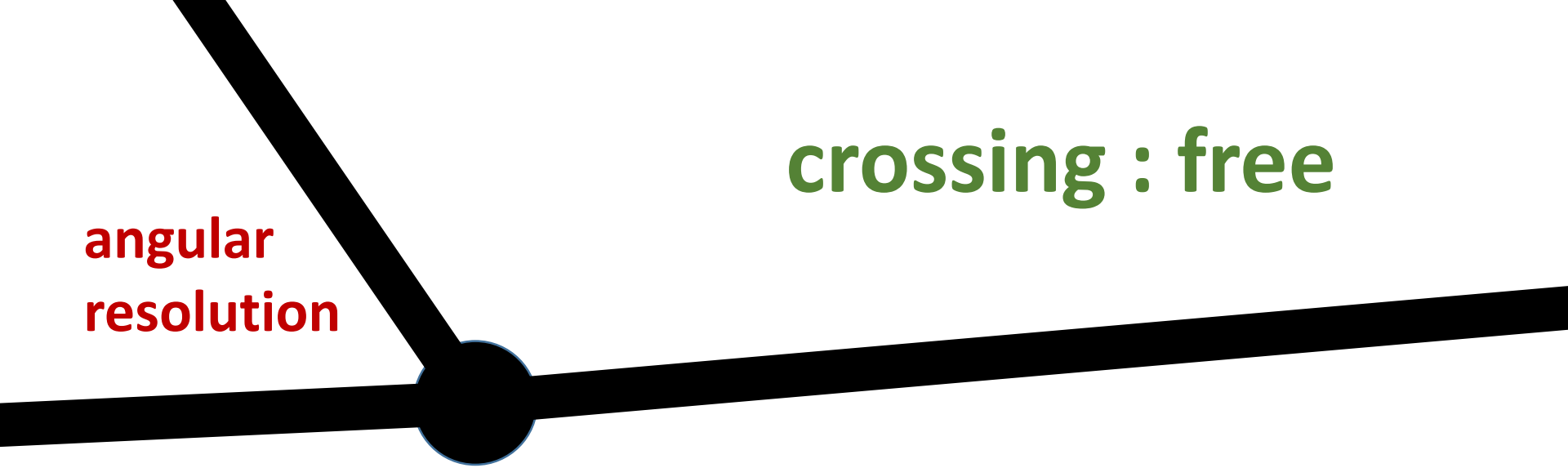


angular resolution

crossing : free

Formann et al. ‘93: Every planar graph can be drawn with angular resolution $\Omega(1/d)$; the drawing may be non-planar

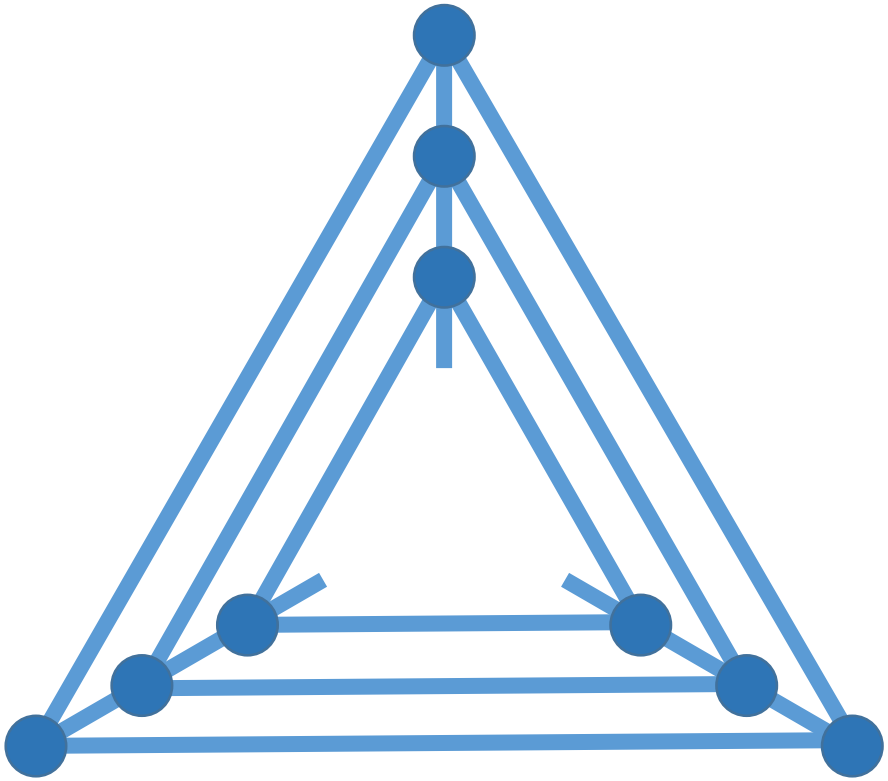
Garg & Tamassia ‘94: There exists a family of planar graphs with max degree d for which any plane straight-line drawing has angular resolution $O(\sqrt{((\log d) / d^3)})$



→ The quality ratio of crossing versus free drawings for angular resolution grows with at least $\sim (1/d) / \sqrt{((\log d) / d^3)}$, so it goes to ∞ as d goes to ∞

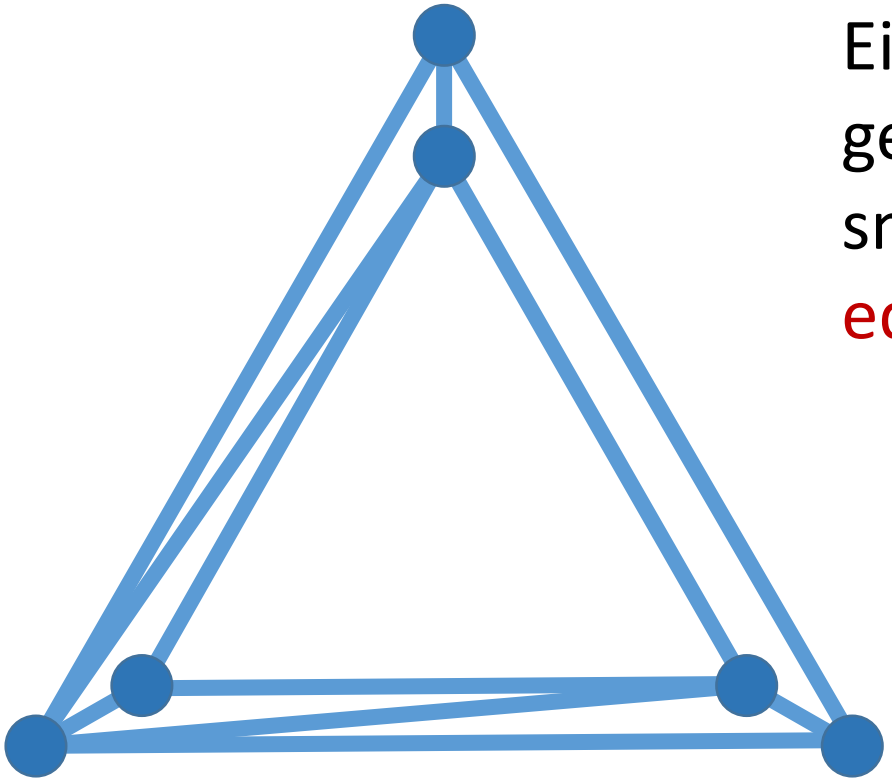
edge-length ratio

circular : free



edge-length ratio

circular : free

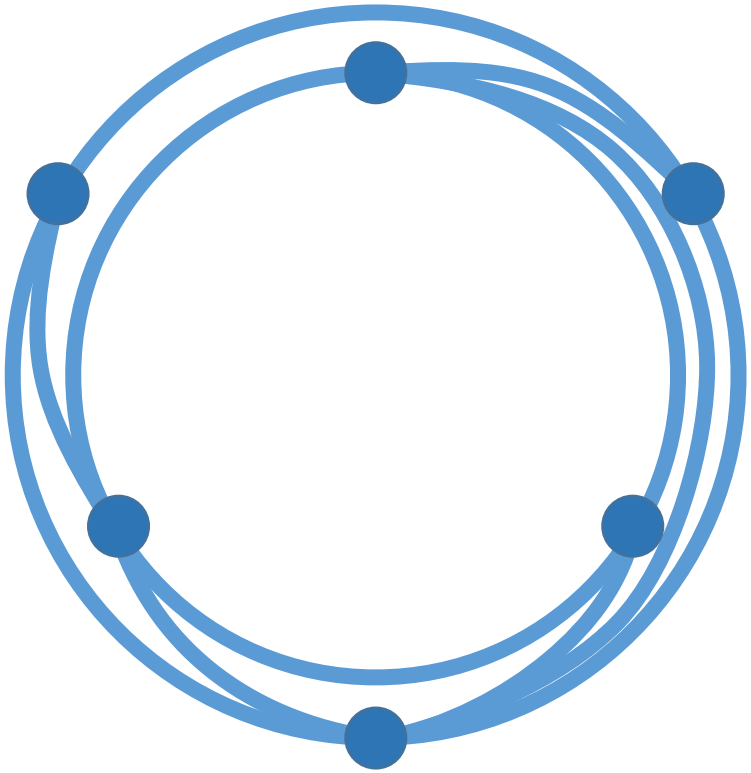


Either the nested triangles get significantly smaller and smaller, giving unbounded edge-length ratio, …

or the edges between the triangles must be short, also giving an unbounded edge-length ratio

edge-length ratio

circular : free

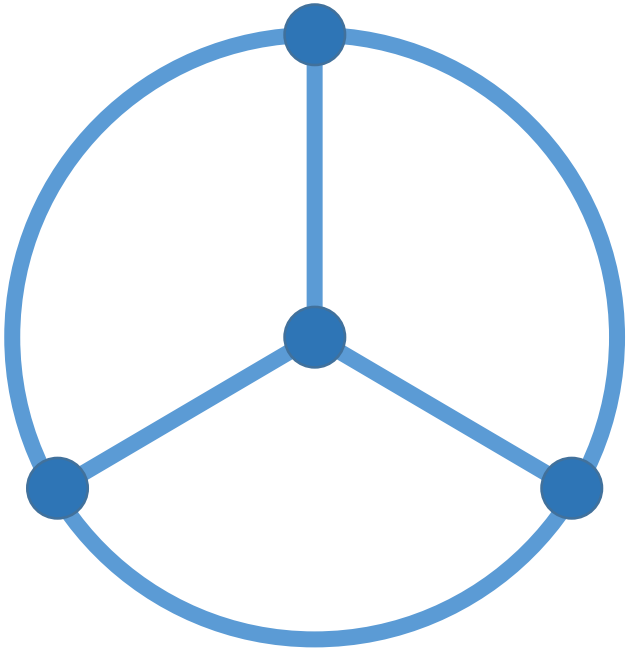


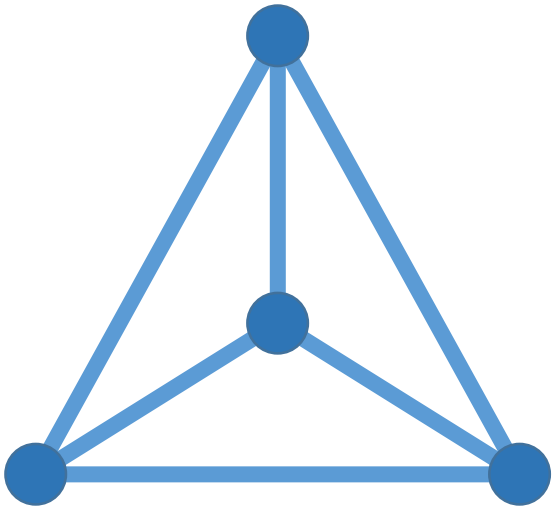
Nested circles can have radii that are arbitrarily close, and the edge-length ratio remains ~3

→ The quality ratio is unbounded

feature
resolution

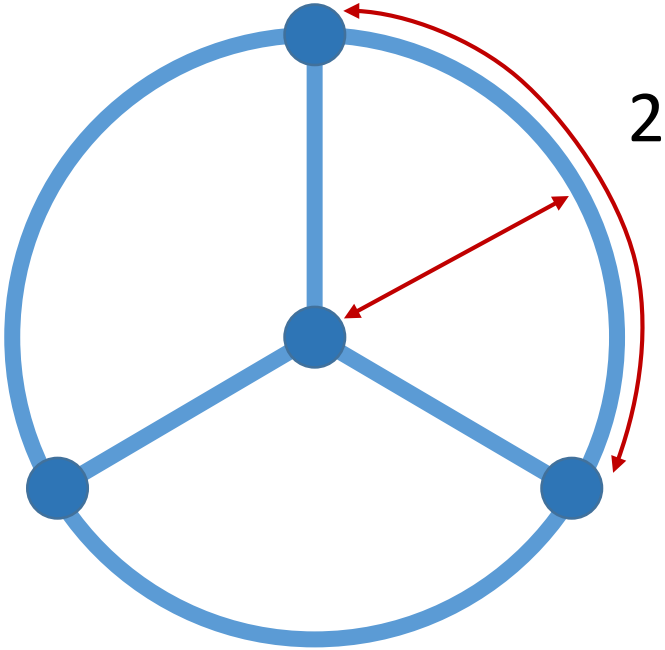
circular : free



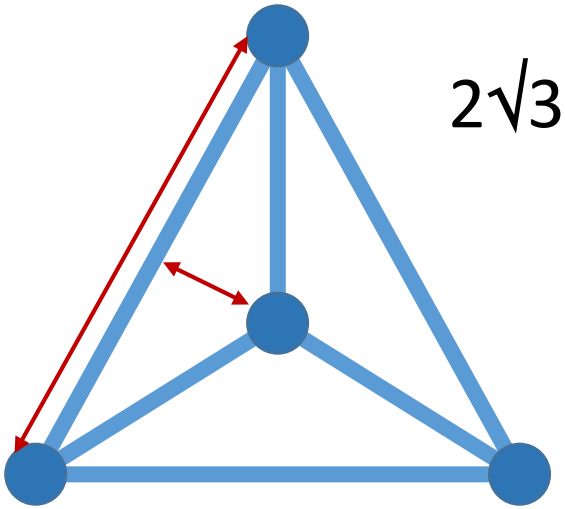


circular : free

feature resolution

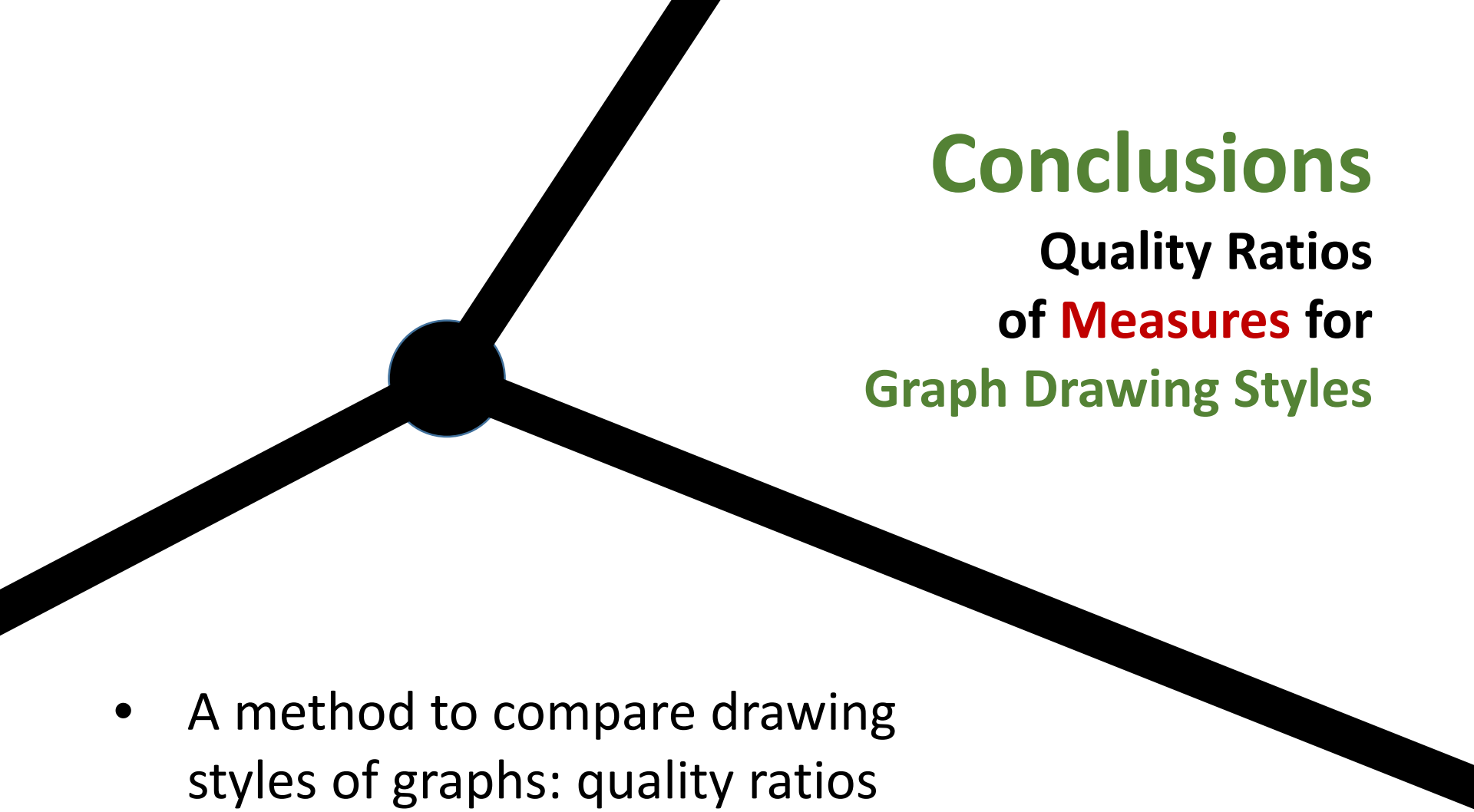


$2\pi / 3$



$2\sqrt{3}$

$QR \geq 3\sqrt{3} / \pi$



Conclusions

Quality Ratios of Measures for Graph Drawing Styles

- A method to compare drawing styles of graphs: quality ratios
- Various results for four quality measures and four drawing styles
- 10 of the 24 table entries are open problems